EXERCISES [MAA 2.2]

QUADRATICS

Compiled by Christos Nikolaidis

O. Practice questions

1.   [Maximum mark: 10]   ***[without GDC]***
Solve the equations

(a) $x^2 - 3x - 4 = 0$.   [4]

(b) $x^2 - 3x = 0$.   [3]

(c) $x^2 - 4 = 0$.   [3]

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2. [Maximum mark: 15]   *[without GDC]*

Let $f(x) = x^2 - 2x - 3$ and $g(x) = 2x^2 - 4x - 6$ [that is $g(x) = 2f(x)$]

(a) Find the discriminant of f and **hence** solve the equation $f(x) = 0$.   [4]

(b) Find the discriminant of g and **hence** solve the equation $g(x) = 0$.   [4]

(c) Find the vertex of each function.   [4]

(d) Sketch the graphs of the two functions on the same diagram.   [3]

3. [Maximum mark: 33]   *[with GDC]*

Complete the following table for the three quadratic functions below

	$f(x) = 2x^2 - 12x + 10$	$f(x) = 2x^2 - 12x + 18$	$f(x) = 2x^2 - 12x + 23$
Discriminant			
y-intercept			
Roots			
Factorisation (if possible)			
axis of symmetry			
Vertex			
Vertex form $f(x) = a(x-h)^2 + k$			
Solve $f(x) \geq 0$			
Solve $f(x) > 0$			
Solve $f(x) \leq 0$			
Solve $f(x) < 0$			

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4. [Maximum mark: 12] *[with GDC]*

Consider the quadratic $y = 4x^2 - 120x + 800$

(a) (i) Find the roots.

    (ii) **Hence** express the quadratic in the form $y = a(x - x_1)(x - x_2)$ [3]

(b) (i) Find the coordinates of the vertex.

    (ii) **Hence** express the quadratic in the form $y = a(x - h)^2 + k$

    (iii) Write down the equation of the axis of symmetry

    (iv) Write down the minimum value of y [5]

(c) Write down the y - intercept of the quadratic [1]

(d) Draw the graph of the quadratic on the diagram below. [3]

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5. [Maximum mark: 14]   *[with GDC]*

Consider the quadratic $y = -4x^2 + 120x - 800$

(a) (i) Find the roots.

   (ii) **Hence** express the quadratic in the form $y = a(x - x_1)(x - x_2)$ [3]

(b) (i) Find the coordinates of the vertex.

   (ii) **Hence** express the quadratic in the form $y = a(x - h)^2 + k$

   (iii) Write down the equation of the axis of symmetry

   (iv) Write down the maximum value of y [5]

(c) Write down the y - intercept of the quadratic [1]

(d) Draw the graph of the quadratic on each of the diagrams below. [5]

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6. [Maximum mark: 8]   *[without GDC]*

The parabola of a quadratic $y = f(x)$ is shown below.

The x-intercepts are $x = 2$ and $x = 6$.

The y-intercept is $y = 12$. The curve passes through the point A(7,5).

(a) By using the symmetry of the graph

(i) Write down the equation of the axis of symmetry.

(ii) Find the value of $f(8)$. Justify your answer.

(iii) Find the value of $f(1)$. Justify your answer. [5]

(b) Find the equation of the curve in the form $f(x) = ax^2 + bx + c$. [3]

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7*. [Maximum mark: 12]   *[with / without GDC]*

Consider the quadratic $y = x^2 - 3x + 4$.

(a)   Find the points of intersection of its curve with each of the following lines.

(i) $y = x + 1$ (ii) $y = x$ (iii) $y = x - 1$ [9]

(b)   The graph of the quadratic is shown on the diagram below. On the same diagram represent the results of question (a).   [3]

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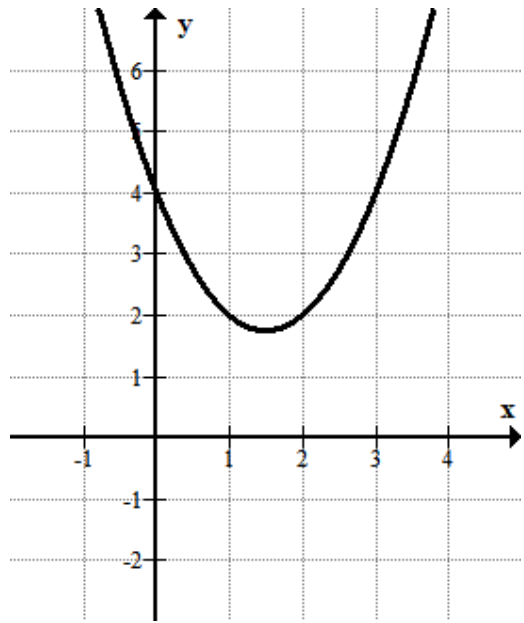
A. Exam style questions (SHORT)

8. [Maximum mark: 4]   *[with / without GDC]*

(b)   Solve the equation $x^2 - 3x - 10 = 0$.   [2]

(a)   Factorize $x^2 - 3x - 10$.   [2]

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9. [Maximum mark: 4]   *[without GDC]*

The diagram represents the graph of the function $f : x \mapsto (x - p)(x - q)$.
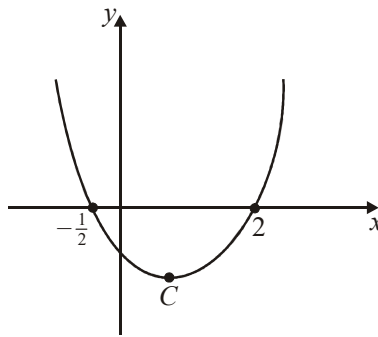
(a)   Write down the values of p and q .   [2]

(b)   The function has a minimum value at the point C. Find the x -coordinate of C .   [2]

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10. [Maximum mark: 6]   *[without GDC]*

The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.
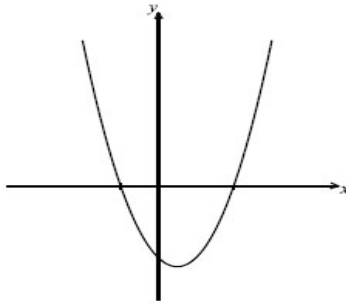
(a) Find both x-intercepts.   [4]

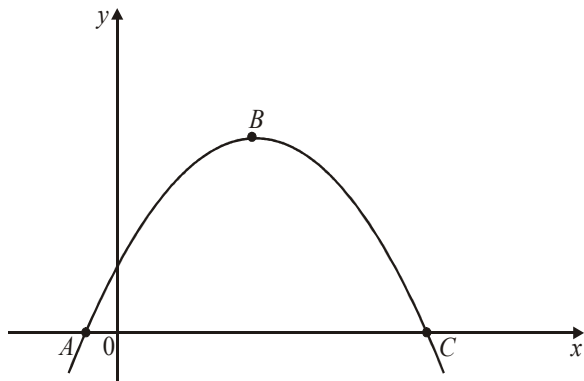(b) Find the x-coordinate of the vertex.   [2]

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11. [Maximum mark: 4]   *[without GDC]*

The diagram shows the parabola $y = (7-x)(1+x)$. The point B is the maximum point

and the points A, C are the x-intercepts. Find the coordinates of A, B and C.

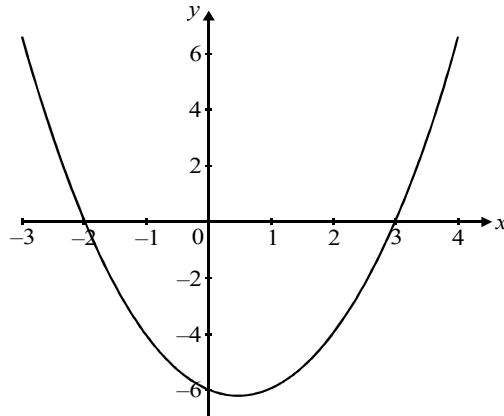..

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12. [Maximum mark: 4] **[without GDC]**

The diagram shows part of the graph with equation $y = x^2 + px + q$. The graph cuts the x - axis at -2 and 3.

Find the value of   (i) p ;   (ii) q .

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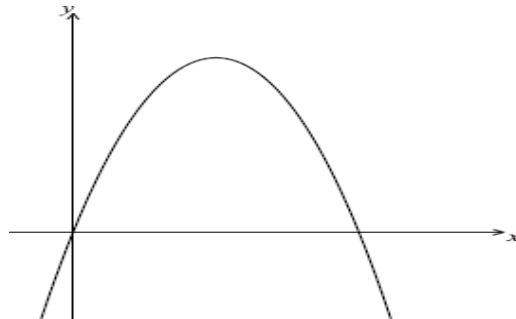
13. [Maximum mark: 6] **[without GDC]**

Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.

(a)   Find the x -intercepts of the graph.   [3]

(b)   (i) Write down the equation of the axis of symmetry.

(ii) Find the y -coordinate of the vertex.   [3]

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14. [Maximum mark: 5]   *[without GDC]*

The following diagram shows the graph of function $y = ax^2 + bx + c$.

Complete the table next to the graph to show whether each expression is positive (+), negative (–) or zero (0).

Expression	+	–	0
a			
c			
$b^2 - 4ac$			
$-\dfrac{b}{2a}$			
b			

15. [Maximum mark: 5]   *[without GDC]*

The following diagram shows the graph of function $y = ax^2 + bx + c$.

Complete the table next to the graph to show whether each expression is positive (+), negative (–) or zero (0).

Expression	+	–	0
a			
c			
$b^2 - 4ac$			
$-\dfrac{b}{2a}$			
b			

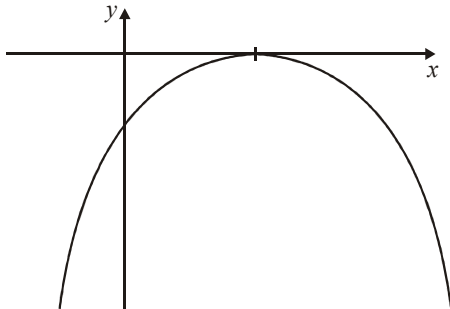**16.** [Maximum mark: 5]   *[without GDC]*

The following diagram shows the graph of function $y = ax^2 + bx + c$.

Complete the table next to the graph to show whether each expression is positive (+), negative (–) or zero (0).
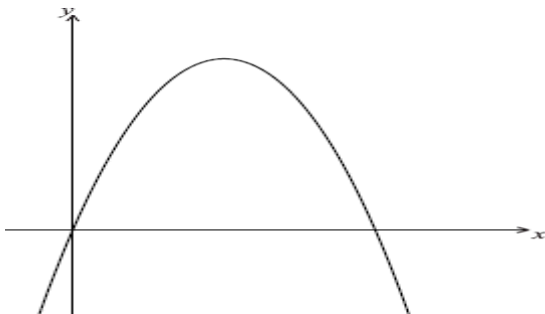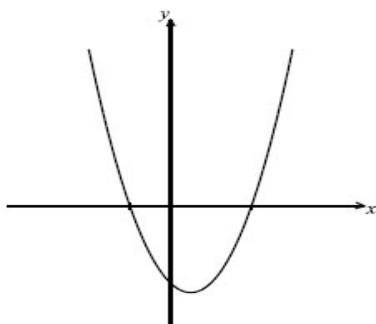

Expression	+	–	0
a			
c			
$b^2 - 4ac$			
$-\dfrac{b}{2a}$			
b			

17. [Maximum mark: 4]   *[with / without GDC]*

(a) Find the vertex of $f(x) = x^2 - 6x + 14$.   [2]

(b) Express the function in the form $f(x) = (x-h)^2 + k$.   [2]

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18. [Maximum mark: 4]   *[with / without GDC]*

Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x-p)^2 + q$, where $a, p, q \in \mathbb{Z}$.   [3]

(b) Find the minimum value of $f(x)$.   [1]

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19. [Maximum mark: 4]   *[with / without GDC]*

(a) Find the vertex of $f(x) = 2x^2 + 2x + 2$.   [2]

(b) Express the function in the form $f(x) = a(x-h)^2 + k$.   [2]

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20. [Maximum mark: 4]   *[with / without GDC]*

(a)   Find the vertex of $f(x) = -x^2 - x - 1$.   [2]

(b)   Express the function in the form $f(x) = a(x - h)^2 + k$.   [2]

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21. [Maximum mark: 6]   *[without GDC]*

Let $f(x) = p(x - q)(x - r)$. Part of the graph of f is shown below.
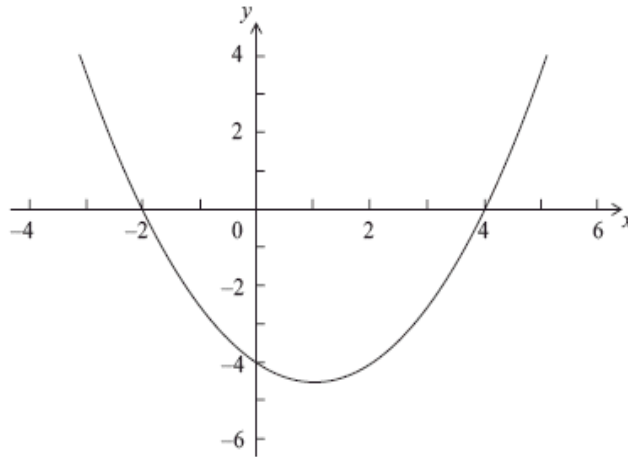
The graph passes through the points (–2, 0), (0, –4) and (4, 0).

(a)   Write down the value of q and of r .   [2]

(b)   Write down the equation of the axis of symmetry.   [1]

(c)   Find the value of p .   [3]

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22. [Maximum mark: 6] **[without GDC]**

The diagram shows part of the graph of the curve $y = a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$.

(a) The vertex is at the point $(3, 1)$. Write down the value of h and of k. [2]

(b) The point P $(5, 9)$ is on the graph. Show that $a = 2$. [3]

(c) Hence show that the equation of the curve can be written as $y = 2x^2 - 12x + 19$. [1]

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23. [Maximum mark: 5] *[without GDC]*

The function $f(x)$ is defined as $f(x) = -(x-h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is P (3, 2).
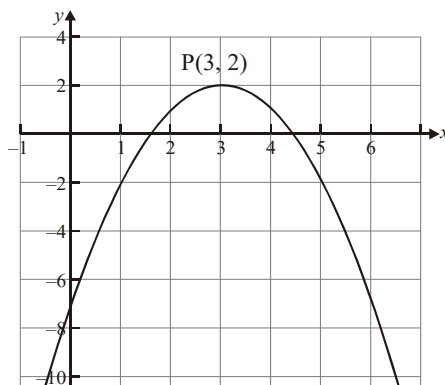
(a) Write down the value of (i) h (ii) k.   [2]

(b) Write down the possible values of $f(x)$.   [2]

(c) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.   [1]

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24. [Maximum mark: 6] *[without GDC]*

The diagram shows part of the graph of $y = a(x-h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates (1, 0).
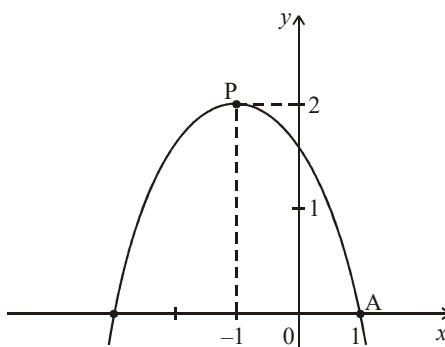
(a) Write down the value of (i) h (ii) k [2]

(b) Calculate the value of a.   [3]

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25. [Maximum mark: 6]   *[without GDC]*

The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.
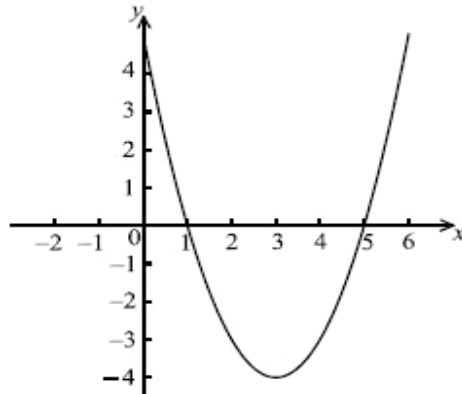
(a) (i) Write down the value of p and of q

(ii) Write down the equation of the axis of symmetry of the curve   [3]

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$.   [3]

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26. [Maximum mark: 5]   *[without GDC]*

Part of the graph of the function $y = d(x - m)^2 + p$ is given in the diagram below.

The x-intercepts are $(1, 0)$ and $(5, 0)$. The vertex is $V(m, 2)$.
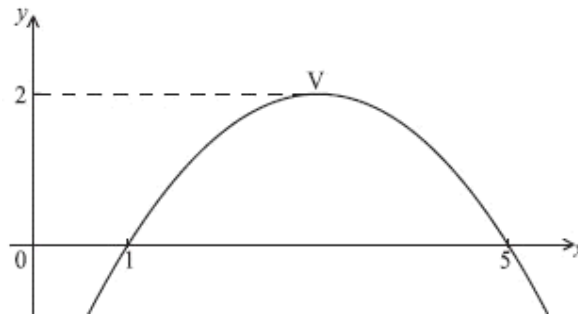
(a) Write down the value of (i) m (ii) p [3]

(b) Find d.   [2]

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27. [Maximum mark: 8]   *[with / without GDC]*

The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve

intersects the x-axis at A$(-2, 0)$ and B$(4, 0)$. The curve of $y = f(x)$ is shown in the

diagram below.

(a)   Write down the value of p and of q.   [2]

(b)   Given that the point $(6, 8)$ is on the curve, find the value of a.   [2]

(c)   Write the equation of the curve in the form $y = ax^2 + bx + c$.   [1]

(d)   Write the equation of the curve in the form $y = a(x - h)^2 + k$, where $h, k \in \mathbb{Z}$..   [3]

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28. [Maximum mark: 8]   *[without GDC]*

The following diagram shows part of the graph of a quadratic function f.

The x-intercepts are at $(-4, 0)$ and $(6, 0)$ and the y-intercept is at $(0, 240)$.

(a) Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$. [2]

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$. [4]

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2]

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29. [Maximum mark: 6]   *[with / without GDC]*

Find the points of intersection between $y = x^2 - 5x + 3$ and $y = 3x - 9$

and sketch a graph to demonstrate the result.

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30. [Maximum mark: 6]   *[with / without GDC]*

Find the point of intersection between $y = x^2 - 5x + 3$ and $y = 3x - 13$

and sketch a graph to demonstrate the result.

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31. [Maximum mark: 6]   *[with / without GDC]*

Find the points of intersection (if any) between $y = x^2 - 5x + 3$ and $y = 3x - 15$

and sketch a graph to demonstrate the result.

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32. [Maximum mark: 6]   *[with / without GDC]*

Find the points of intersection (if any) between $y = x^2 - 3$ and $y = 5 - x^2$

and sketch a graph to demonstrate the result.

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33*. [Maximum mark: 4]   *[with / without GDC]*

The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x.

Find the value of k.

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34*. [Maximum mark: 6]   *[with / without GDC]*

The equation $kx^2 + 3x + 1 = 0$ has exactly one solution. Find the value of k.

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35*. [Maximum mark: 7]   *[without GDC]*

The quadratic equation $kx^2 + (k-3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k . [5]

(b) Write down the values of k for which $x^2 + (k-3)x + k = 0$ has two equal real roots. [2]

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36*. [Maximum mark: 7]   *[with GDC]*

Consider $f(x) = 2kx^2 - 4kx + 1 = 0$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k . [5]

(b) The line $y = p$ intersects the graph of f . Find all possible values of p . [2]

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37*. [Maximum mark: 6]   **[with / without GDC]**

The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

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38*. [Maximum mark: 6]   **[with / without GDC]**

A family of functions is given by $f(x) = x^2 + 3x + k$, where $k \in \{1, 2, 3, 4, 5, 6, 7\}$.

Find the possible values of k if the curve of this function crosses the x-axis.

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39*. [Maximum mark: 6]   *[with GDC]*

The equation $kx^2 - 3x + (k+2) = 0$ has two distinct real roots. Find the set of possible values of k.

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40*. [Maximum mark: 6]   *[with GDC]*

The function f is defined by $f(x) = x^2 - 2x + k(3k+2)$ where $k \in \mathbb{R}$. Find the set of values of k for which $f(x) = 0$ has two distinct real roots.

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41*. [Maximum mark: 6]   *[with GDC]*

Consider the equation $(1+2k)x^2 - 10x + k - 2 = 0$, $k \in \mathbb{R}$. Find the set of values of k for which the equation has real roots.

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42*. [Maximum mark: 6]   *[with GDC]*

The quadratic function Q is defined by $Q(x) = kx^2 - (k-3)x + (k-8)$, $k \in \mathbb{R}$.
Determine the values of k for which $Q(x) = 0$ has no real roots.

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43*. [Maximum mark: 6]   *[with / without GDC]*

Given $f(x) = x^2 + x(2-k) + k^2$, find the range of values of k for which $f(x) > 0$ for all real values of x.

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44**. [Maximum mark: 6]   *[without GDC]*

Find the range of values of m such that for all x, $m(x+1) \leq x^2$.

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45*. [Maximum mark: 5]   *[without GDC]*

For what values of m is the line $y = mx + 5$ a tangent to the parabola $y = 4 - x^2$?

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46**. [Maximum mark: 5]   *[without GDC]*

The parabolas $y = 2x^2 + 2x - 1$ and $y = x^2 - m$ have only one common point (they are tangent to each other). Find the value of m.

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B. Exam style questions (LONG)

47. [Maximum mark: 12]   ***[without GDC]***

Let $f(x) = a(x-4)^2 + 8$.

(a) Write down the coordinates of the vertex of the curve of f.   [2]

(b) Given that $f(7) = -10$, find the value of a.   [2]

(c) Hence find the y-intercept of the curve of f.   [2]

(d) Find the x-intercepts of the curve of f.   [4]

(e) Sketch the graph of the curve of f.   [2]

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48*. [Maximum mark: 11]   *[with GDC]*

Let $f(x) = ax^2 + bx + c$ where a, b and c are rational numbers.

(a)   The point P(–4, 3) lies on the curve of f. Show that $16a - 4b + c = 3$.   [2]

(b)   The points Q(6, 3) and R(–2, –1) also lie on the curve of f. Write down two other

linear equations in a, b and c.   [4]

(c)   **Hence** find $f(x)$.   [2]

(d)   Write $f(x)$ in the form $f(x) = a(x - h)^2 + k$, where a, h and k are rational numbers.   [3]

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49*. [Maximum mark: 10]   *[with / without GDC]*

Consider two different quadratic functions of the form $f(x) = 4x^2 - qx + 25$. The graph of each function has its vertex on the x-axis.

(a) Find both values of q.   [3]

(b) For the greater value of q, solve $f(x) = 0$.   [2]

(c) Find the coordinates of the point of intersection of the two graphs.   [2]

(d) Sketch the graphs of the two functions on the same diagram.   [3]

50*. [Maximum mark: 12]   *[without GDC]*

Let $f(x) = 3x^2 - x + 4$.

(a)   Find the values of m for which the line $y = mx + 1$

   (i)   is a tangent to the graph of f.

   (ii)   meets the graph of f.

   (iii) does not meet the graph of f.   [6]

(b)   For each value of m found in question (a)(i), find the common point of the two

   graphs.   [6]

51**. [Maximum mark: 10]   *[with GDC]*

Consider the graph *G* of the function $f(x) = x^2$ and the point M(5,2) on the Cartesian plane.

(a) Find the coordinates of a point A on *G* and a point B on *x*-axis, given that M is the midpoint of the line segment [AB]. [5]

(b) Find the coordinates of a point C on *G*, given that the distance between M and C is 23. [5]

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