Domaći rad Matija Koprivica 3-3

Zadatak 1080. a)

1+2+3+...+n = $\frac{n(n+1)}{2}$

Za n=1

1 = $\frac{1(1+1)}{2}$ = $\frac{2}{2}$ T

Tvrdnja je tačna za barem jedan pridodan broj.

Ako je tvrdnja tačna za neki prirodan broj, onda treba pokazati da je tačna i za sledeći.

n=n+1

$\underbrace{1+2+3+\cdots+n}$+n+1=$\frac{(n+1)(n+1+1)}{2}$ T

$\frac{n(n+1)}{2}$ + n+1= $\frac{n(n+1)+2(n+1)}{2}$=$\frac{(n+1)(n+2)}{2}$

Zadatak 1081. b)

$$\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+\cdots+\frac{n^2+n+1}{n(n+1)}=\frac{n(n+2)}{n+1}$$

I. n = 1 $\frac{1^2+1+1}{1(1+1)}=\frac{1(1+2)}{1+1}$

$\frac{3}{1\cdot 2}=\frac{3}{2}$ T

II. $\boldsymbol{S_n=F(n)\rightarrow S_{n+1}=F(n+1)}$

$$\underbrace{\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+\cdots+\frac{n^2+n+1}{n(n+1)}}_{S_n = F(n)} + \underbrace{\frac{(n+1)^2+(n+1)+1}{(n+1)(n+2)}}_{a_{n+1}} = \underbrace{\frac{(n+1)(n+3)}{n+2}}_{F(n+1)} \quad \mathbf{T}$$

$$\frac{n(n+2)}{n+1}+\frac{n^2+3n+3}{(n+1)(n+2)}=\frac{n(n+2)^2+n^2+3n+3}{(n+1)(n+2)}=\frac{n^3+5n^2+7n+3}{(n+1)(n+2)}=$$

$$\frac{n^2(n+1)+4n(n+1)+3(n+1)}{(n+1)(n+2)}=\frac{(n+1)(n^2+4n+3)}{(n+1)(n+2)}=\frac{n^2+4n+3}{n+2}=\frac{(n+1)(n+3)}{n+2}$$

T

Zadatak 1084. b)

$$133|11^{n+2}+12^{2n+1} \quad ,n\geq 0$$

(1)n=0

$$a_0=11^{0+2}+12^{2\cdot 0+1} = 11^2+12^1=121+12=133\rightarrow \quad 133|a_0=133$$

(2)

$$133\,\Big|a_n=\underbrace{11^{n+2}+12^{2n+1}}_{a_n} \;\rightarrow\; 19\Big|\,a_{n+1}= 11^{n+1+2}+12^{2(n+1)+1}$$

$$a_{n+1}= 11^{n+1+2}+12^{2(n+1)+1}= 11^{n+2}\cdot 11+ 12^2\cdot 12^{2n+1}=11^{n+2}\cdot 11 + 144\cdot 12^{2n+1}= 11^{n+2}\cdot 11 +(11+133)\cdot 12^{2n+1}= 11^{n+2}\cdot 11 + 11\cdot 12^{2n+1}+133\cdot 12^{2n+1} = 11\cdot(\underbrace{11^{n+2}+12^{2n+1}}_{a_n})+\underbrace{133\cdot 12^{2n+1}}_{\text{deljivo sa 133}} \rightarrow 133|a_{n+1}$$

1084. g)

$17|6^{2n} + 19^n - 2^{n+1}, n \geq 0$

(1) n=0

$a_0 = 6^{2\cdot 0} + 19^0 - 2^{0+1} = 1 + 1 - 2 = 0$

$a_0 = 0 \rightarrow T$

(2)

$17\left|a_n = 6^{2n} + 19^n - 2^{n+1} \rightarrow 17\right|a_{n+1} = 6^{2(n+1)} + 19^{n+1} - 2^{n+1+1}$

$a_{n+1} = 6^{2(n+1)} + 19^{n+1} - 2^{n+1+1} = 6^2 \cdot 6^{2n} + 19 \cdot 19^n - 2 \cdot 2^{n+1} = 36 \cdot 6^{2n} + 19 \cdot 19^n - 2 \cdot 2^{n+1} = (19 + 17) \cdot 6^{2n} + 19 \cdot 19^n - (19 - 17) \cdot 2^{n+1} = 19(\underbrace{6^{2n} + 19^n - 2^{n+1}}_{a_n}) + 17(\underbrace{6^{2n} + 2^{n+1}}_{\text{deljivo sa 17}}) \rightarrow 17|a_{n+1}$

Zadatak 1084.đ)

$11|\underbrace{30^n + 4^n(3^n - 2^n)}_{a_n} - 1, n \geq 0$

$a_n = 30^n + 12^n - 8^n - 1$

(1) n = 0

$a_0 = 30^0 + 12^0 - 8^0 - 1 = 0$

$a_0 = 0 \rightarrow T$

(2)

$$11|a_n = 30^n + 12^n - 8^n - 1 \;\rightarrow\; 11|a_{n+1} = 30^{n+1} + 12^{n+1} - 8^{n+1} - 1$$

$$a_{n+1} = 30^{n+1} + 12^{n+1} - 8^{n+1} - 1 = 30 \cdot 30^n + 12 \cdot 12^n - 8 \cdot 8^n - 1 =$$

$$1 \cdot 30^n + 29 \cdot 30^n + 1 \cdot 12^n + 11 \cdot 12^n - 1 \cdot 8^n - 7 \cdot 8^n - 1$$

$$= \underbrace{(30^n + 12^n - 8^n - 1)}_{a_n} + \underline{29 \cdot 30^n} + \underbrace{11 \cdot 12^n}_{\text{deljivo sa 11}} - \underline{7 \cdot 8^n}$$

Dalje je potrebno dokazati da je $\mathbf{11}|29 \cdot 30^n - 7 \cdot 8^n$

(1) n = 0

$$a'_0 = 29 \cdot 30^0 - 7 \cdot 8^0 = 29 - 7 = 22 \rightarrow 11|a_0 = 22$$

(2)

$$11|a'_n = 29 \cdot 30^n - 7 \cdot 8^n \rightarrow 11|a'_{n+1} = 29 \cdot 30^{n+1} - 7 \cdot 8^{n+1}$$

$$a'_{n+1} = 29 \cdot 30^{n+1} - 7 \cdot 8^{n+1} = 29 \cdot 30 \cdot 30^n - 7 \cdot 8 \cdot 8^n = 29 \cdot 8 \cdot 30^n +$$

$$29 \cdot 22 \cdot 30^n - 7 \cdot 8 \cdot 8^n = 8\Big(\underbrace{29 \cdot 30^n - 7 \cdot 8^n}_{a'_n}\Big) + \underbrace{29 \cdot 22 \cdot 30^n}_{\text{deljivo}}$$

$$\rightarrow 11|a_{n+1}$$