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More on rules of derivatives
By: Designing team

Tecnológico de Monterrey Preparatoria

Name: Laura M. Londoño Hidalgo Id: A01570224 Date: 26/09/17

Product (derivative): $(UV)' = UV' + VU'$

Quotient (derivative): $\left(\frac{U}{V}\right)' = \frac{VU' - UV'}{V^2}$

1. If $f(5)=1$, $f'(5)=6$, $g(5)=-3$, $g'(5)=2$. Find the values of

a) $(f\cdot g)'(5) = (1)(2)+(-3)(6) = 2-18 = (f\cdot g)'(5) = -16$

b) $(f/g)'(5) = \frac{(-3)(6)-(1)(2)}{(-3)^2} = \frac{-18-2}{9} = \frac{-20}{9} = (f/g)'(5) = \frac{-20}{9}$

c) $(g/f)'(5) = \frac{(1)(2)-(-3)(6)}{(1)} = \frac{2+18}{1} = (g\cdot f)'(5) = 20$

2. If $f(3)=4$, $g(3)=2$, $f'(3)=-6$ and $g'(3)=5$, find the following values

a) $(f+g)'(3) = (-6)+(5) = (f+g)'(3) = -1$

b) $(f\cdot g)'(3) = (4)(5)+(2)(-6) = 20-12 = (f\cdot g)'(3) = 8$

c) $(f/g)'(3) = \frac{(2)(-6)-(4)(5)}{(2)^2} = \frac{-12-20}{4} = \frac{-32}{4} = (f/g)'(3) = -8$

3. If $h(x) = f(x)g(x)$, use the table to find $h'(-1)$, $h'(0)$ and $h'(1)$

original derivative original derivative

x	$f(x)$ U	$f'(x)$ U'	$g(x)$ V	$g'(x)$ V'
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

① $h'(-1) = (2)(2)+(1)(1) = 4+1 = 5$ → $h'(-1) = 5$

② $h'(0) = (-1)(3)+(-1)(0) = -3+0$ → $h'(0) = -3$

③ $h'(1) = (2)(5)+(0)(-1) = 10+0$ → $h'(1) = 10$

4. If $h(x) = f(x)/g(x)$, use the table to find $h'(-1)$, $h'(0)$ and $h'(1)$

original derivative original derivative

x	$f(x)$ U	$f'(x)$ U'	$g(x)$ V	$g'(x)$ V'
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	2	5

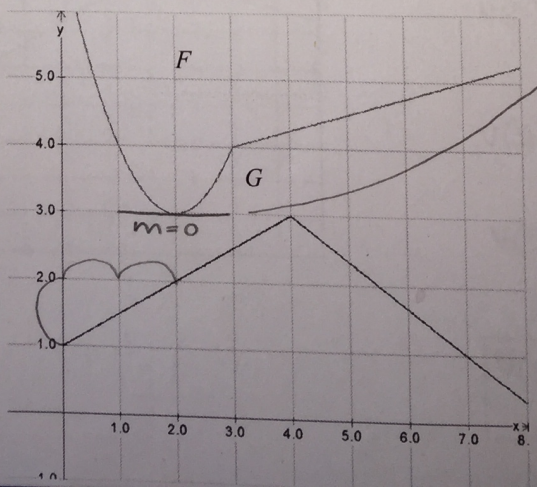
① $h'(-1) = \frac{(1)(1)-(2)(2)}{(1)^2} = \frac{1-4}{1}$ → $h'(-1) = -3$

② $h'(0) = \frac{(-1)(0)-(-1)(3)}{(1)^2} = \frac{-(-3)}{1}$ → $h'(0) = 3$

③ $h'(1) = \frac{(2)(-1)-(2)(5)}{(2)^2} = \frac{(-2)-10}{4} = \frac{-12}{4}$ → $h'(1) = -3$

5. Considering that $P(x) = F(x)G(x)$ y $Q(x) = F(x)/G(x)$, where F and G are functions whose graphs are shown below.

a) Find $P'(2)$ b) Find $Q'(7)$



$F(2) = 3$ $G(2) = 2$
$F'(2) = 0$ $G'(2) = \frac{1}{2}$

$F(7) = 5$ $G(7) = 1$
$F'(7) = \frac{1}{4}$ $G'(7) = \frac{-2}{3}$

$P'(2) = F(2)G'(2) + G(2)F'(2)$

$P'(2) = (3)\left(\frac{1}{2}\right)+(2)(0)$

$= \frac{3}{2}+0$

$P'(2) = \frac{3}{2}$

$Q'(7) = \frac{(1)\left(\frac{1}{4}\right)-(5)\left(\frac{-2}{3}\right)}{1^2}$

$= \frac{1}{4}+\frac{10}{3} =$

$Q'(7) = \frac{13}{12}$

$h'(x) = f'(g(x))\, g'(x)$

6. Consider that $h(x) = f(g(x))$, find $h'(-1)$, $h'(0)$, and $h'(1)$.

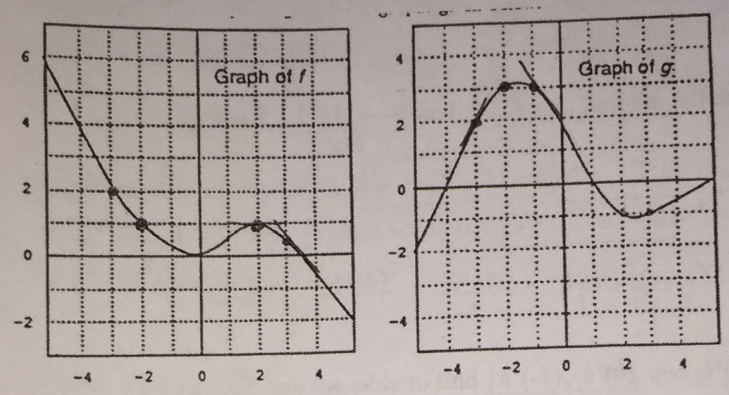
x	$f(x)$ ↓ original	$f'(x)$ ↓ derivative	$g(x)$ ↓ original	$g'(x)$ ↓ derivative
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

① $h'(-1) = f'(1)(2)$
$h'(-1) = (-1)(2)$
$h'(-1) = -2$

② $h'(0) = f'(-1)(3)$
$h'(0) = (1)(3)$
$h'(0) = 3$

③ $h'(1) = f'(0)(5)$
$h'(1) = (0)(5)$
$h'(1) = 0$

7. Consider that $h(x) = f(g(x))$, where f and g are functions whose graphs are shown below.



a) Evaluate $h(-2)$ and $h(3)$
b) Is $h'(-3)$ positive, negative or zero? Explain your answer.
c) Is $h'(-1)$ positive, negative or zero? Explain your answer.

a) $h(-2) = f(g(x))$
$h(-2) = f(3)$
$h(-2) = 0.5$

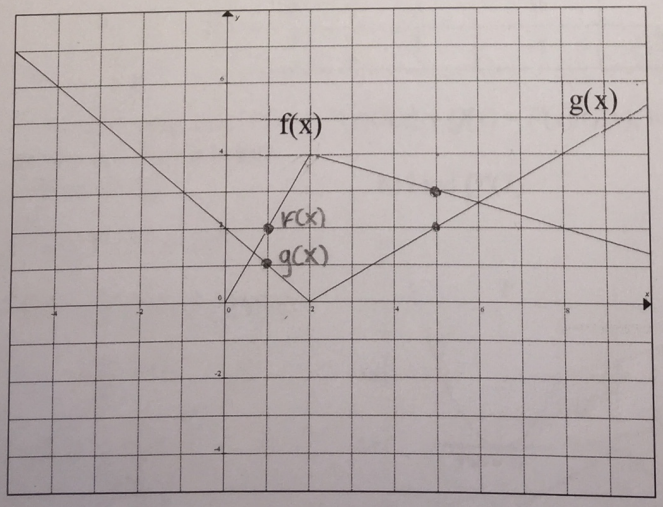
$h(3) = f(g(x))$
$h(3) = f(-1)$
$h(3) = 0.25$

b) $h'(-3) = f'(g(x))\, g'(-3)$
$h'(-3) = f'(2)\, g'(-3)$
$h'(-3) = (0)(+) = 0$

c) $h'(-1) = f'(g(x))\, g'(-1)$
$h'(-1) = f'(3)\, g'(-1)$
$h'(-1) = (-)(-) = +$

8. If $f(x)$ and $g(x)$ are the functions whose graphs are shown, let $u(x) = f(x) \cdot g(x)$ and $v(t) = f(x)/g(x)$

a) Find $u'(1)$ b) Find $v'(5)$



$f(1) = 2$
$g(1) = 1$
$f'(1) = 2$
$g'(1) = -1$

$f(5) = 3$
$f'(5) = -\frac{1}{3}$
$g(5) = 2$
$g'(5) = \frac{2}{3}$

a) $u'(1) = f(1)\, g'(1) + g(1)\, f'(1)$
$u'(1) = (2)(-1) + (1)(2)$
$u'(1) = 0$

b) $v(t) = \frac{f(5)}{g(5)}$

$v(5) = \frac{(2)(-\frac{1}{3}) - (3)(\frac{2}{3})}{(2)^2}$

$v(5) = \frac{(-\frac{2}{3}) - (2)}{4}$

$v(5) = \frac{-\frac{8}{3}}{4}$

$v(5) = -\frac{2}{3}$