NOTES ON HEAT CALCULATIONS

HEAT TRANSFER TO (OR FROM) AN OBJECT

$$Q = mc\Delta T \qquad (1)$$

Note that this is not a "**change** in heat" and *there is no such thing as ΔQ*. Heat is energy in transit, or energy being transferred, and it makes no sense to speak of a *change* in energy being transferred. It's either being transferred or it isn't. Just like there is no such thing as a "change in work."

The factors in Eq(1) are the mass *m*, the specific heat capacity *c*, and the temperature difference *ΔT*. In SI units the specific heat is in joules per kilogram per kelvin, or J / (kg K). Since a temperature *difference* is the same in Kelvins as in degrees Celsius, we can use either measure; degrees C are commonly used.

If the heat *Q* is positive we say that heat flows into the object, and its temperature is increased. If *Q* is negative, heat has flowed out of the object and its temperature has decreased. Clearly, since the mass and specific heat both have positive values, the sign of *Q* depends only on the sign of *ΔT*.

Observe that if we solve Eq(1) for the temperature change for some given heat input, we see that the temperature change will be *inversely* proportional to the mass and to the specific heat. So, a larger specific heat means that more of the input energy is going into the internal energy of the object without increasing the kinetic energy of the molecules. The latter is, of course, directly related to the temperature.

The term "specific heat capacity" is unfortunate, since *we cannot store heat*. Some other term that implies resistance to a change in temperature, like "thermal inertia," might be better. (The term "specific" just means that a quantity is normalized to be on a per-unit-mass basis, that is, per kg.)

The "calorie" is another, widely used, unit for heat. This is the amount of heat needed to change the temperature of 1 gram of water by one degree Celsius. *The calorie is not an SI unit!* In SI, energy and work are always measured in joules. (Also, the old CGS metric system of units is not part of SI.)

For water, the specific heat is 1 cal / g or 1000 cal / kg, which is convenient when solving problems involving water (or liquids that can be considered water). It can be shown that the conversion from calories to joules leads to **4.186 J / cal or 4186 J / kcal**; the latter is used when the mass is given in kg.

HEAT OF FUSION OR VAPORIZATION (HEAT OF TRANSFORMATION)

$$Q = \pm\, m h_{fusion} \qquad Q = \pm\, m h_{vaporization} \qquad (2)$$

This is the energy input needed to change a substance from solid to liquid, or from liquid to gas. *This energy input does not result in an increase in temperature*. This heat is considered positive when it is input to the substance (melts) and negative when it is given up by the substance when it solidifies (freezes). That is, we must *add* energy to melt ice, and we must *remove* energy to freeze water; similarly for boiling (positive) and condensing (negative).

This heat of transformation is also referred to as "latent" (or hidden) heat; "hidden" because it is not manifested in an increased temperature. The units of latent heat are joules per kilogram (J / kg).

SOME NUMERICAL VALUES

specific heat of liquid water	4186	joules per kilogram per degree Celsius (or K)
specific heat of ice	2090	joules per kilogram per degree Celsius (or K)
heat of fusion of water	3.35E05	joules per kilogram
heat of vaporization of water	2.26E06	joules per kilogram

Note that the specific heat of ice is needed if we had some ice whose temperature was less than 0˚C. As it warms up toward 0, this is the specific heat we would use, not the value for liquid water!

CALORIMETRY

This is the study of heat flow in a thermally-isolated system. That is, there is no energy input to or lost from the objects under consideration other than that contained by the objects themselves. For example, suppose we have a well-insulated container, with some water in it, and the container and water are in thermal equilibrium; so, they have the same temperature. Then we place a heated object, like a chunk of metal taken from a hot oven, into the water. The heat that flows from the metal into the water/container system is the source of thermal energy, and we assume that none is lost to the environment. What is the final, equilibrium, temperature of this system (the water, the container, and the metal)?

The best way to analyze problems like this is to express the conservation of energy as

$$Q_{net} = Q_1 + Q_2 + Q_3 + \cdots = 0 \quad (3)$$

This says that the net *algebraic* sum of the heat flows between the objects must be zero; it is a "zero-sum" game! Thermal energy will be lost by some objects, *and that same energy will be gained by other objects*, such that the overall change in this isolated system's energy content is zero.

We can express Eq(3) using Eq(1), to have

$$m_1 c_1 \left[T_f - T_1 \right] + m_2 c_2 \left[T_f - T_2 \right] + m_3 c_3 \left[T_f - T_3 \right] + \cdots = 0$$

where T_f is the final equilibrium temperature of all the objects in the system, and T_1 etc. are the initial temperatures of the respective objects. Note that *all these temperature differences are written as final minus initial*. The signs of these terms will take care of themselves. Expanding, collecting terms, and factoring leads to the solution for the final temperature, for n objects:

$$T_f = \frac{\sum_{i=1}^{n} m_i c_i T_i}{\sum_{i=1}^{n} m_i c_i} \quad (4)$$

This is just a weighted average of the initial temperatures. To be a little more general we can observe that the mass can be written as the product of the density ρ and the volume V of each object, so that Eq(4) expressed a different way is

$$T_f = \frac{\sum_{i=1}^{n} \rho_i V_i c_i T_i}{\sum_{i=1}^{n} \rho_i V_i c_i} \quad (5)$$

This form can be useful for problems involving liquids, where a volume might be given, not a mass. *These solutions only apply if there is no phase change* (melting, boiling, etc.).

SPECIAL CASES

Here are some re-expressions of Eq(4) and (5) for certain commonly-occurring problem types. What happens in these is that some parameters are constant and can thus be factored out of the numerator and denominator summations in Eq(4) or (5), and then they cancel out.

Two volumes of the same material (e.g., water)

$$T_f \;=\; \frac{V_1 T_1 \;+\; V_2 T_2}{V_1 + V_2} \tag{6}$$

Two masses of the same material

$$T_f \;=\; \frac{m_1 T_1 \;+\; m_2 T_2}{m_1 + m_2} \tag{7}$$

Two masses of different materials

$$T_f \;=\; \frac{m_1 c_1 T_1 \;+\; m_2 c_2 T_2}{m_1 c_1 \;+\; m_2 c_2} \tag{8}$$

Many other variations are possible; use Eq(4) or (5) as needed for a specific problem.

THE "KEEP ΔT POSITIVE" APPROACH

Some physics texts use what appears to be a different method for these calculations. In this, the fundamental statement is that

$$Q_{gained} \;=\; Q_{lost} \qquad \text{so that} \qquad Q_{gained} - Q_{lost} \;=\; 0 \tag{9}$$

Notice that the only way Eq(9) can be true is if both quantities are positive. In fact this is no different than Eq(3), the conservation of energy, but it is constructed (and applied) in a way that is potentially confusing. We are to ensure that the temperature differences used for Q are always positive. However, consider this statement from a college-level physics text:

> You may have learned to solve calorimetry problems ... by balancing heat gained with heat lost. That approach works in simple problems, but it has two drawbacks. First, you often have to "fudge" the signs to make them work. Second, and more serious, you can't extend this approach to a problem with three or more interacting systems. Using $Q_{net} = 0$ is much preferred.
>
> R. D. Knight, *Physics for Scientists and Engineers*, Pearson (2004), p532.

We will now show that this method leads to exactly the same solution for the final temperature that we derived above, for two objects. Let us assume that object 1 is "hotter." Then we are supposed to write

$$m_1 c_1 \left[T_1 - T_f \right] \;=\; m_2 c_2 \left[T_f - T_2 \right]$$

and both temperature differences are positive. The fact is, if we solve this for T_f by expanding, collecting terms, and factoring, just as we did in deriving Eq(4), we will get Eq(8)! And if we choose object 2 as the "hotter" one, we get exactly the same result. In the texts that use this method, they do not do the algebra, they put in the numbers and create a confusing mess that needs to be solved for T_f. Using Eq(8) is a faster, more reliable way to get this job done. And you can see from Eq(4) or (5) that we can find the equilibrium temperature T_f for *any* number of objects, not just two.

It would be interesting to see how the temperature of two objects varies with time. To do this we use Newton's Law of Cooling, which says that the rate of change of the temperature difference between an object and its surroundings is proportional to the current temperature difference. This is a calculus problem, and when we solve it, we can find the time-dependent temperatures. (The solutions for the two temperatures involve a function that many of you have not studied yet, so they will not be shown here.)

The graph below shows this time variation for the coffee and milk in a worksheet problem with these parameters (the milk is added to the coffee; assume no external gains or losses):

Coffee: $m_1 = 0.15$ kg $c_1 = 4187$ $T_1 = 70$ C Milk: $m_2 = 0.01$ kg $c_2 = 3800$ $T_2 = 5$ C

Using this information in Eq(8), we find the final temperature

$$T_f = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2} = \frac{0.15(4187)(70) + 0.01(3800)(5)}{0.15(4187) + 0.01(3800)} = 66.3$$

This temperature is the horizontal dotted line in the figure below. We see, as we would expect, that the milk temperature rises dramatically and the coffee temperature drops only slightly. The time scale is based on an unknown parameter; this was just picked arbitrarily. We could do an experiment to gather some data that could be used to estimate a more physically-realistic value for that parameter.

Notice in the graph that the slope of the curves is not constant-- it is steepest at the start, when the temperature difference is largest. As the temperature difference becomes smaller, the slope approaches zero (a horizontal line). This slope is the rate of change of temperature, sort of like the slope of a position vs. time graph is the velocity. Same idea.