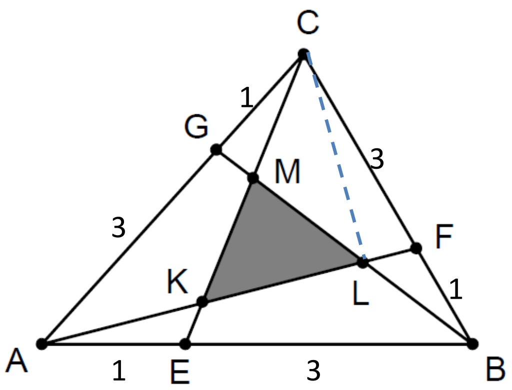
(IMO 1998 Hong Kong Preliminary Selection Contest)

In ΔABC, E, F, G are points on AB, BC, CA respectively such that $AE : EB = BF : FC = CG : GA = 1 : 3$. K, L, M are the intersection points of the lines AF and CE, BG and AF, CE and BG, respectively. Suppose the area of ΔABC is 1; find the area of ΔKLM.



Solution

Let the area of ΔABL = s

Then area of ΔCAL = 3s [Using $\frac{\text{Area } BAL}{\text{Area } CAL} = \frac{1}{3}$]

and area of ΔBCL = $\frac{1}{3}s$ [Using $\frac{\text{Area } BCL}{\text{Area } ABL} = \frac{1}{3}$]

Area of ΔABC = Area ΔABL + ΔCAL + ΔBCL = s + 3s + $\frac{1}{3}s = \frac{13}{3}s = 1$

$\Rightarrow$ Area of ΔABL = $\frac{3}{13}$

Similarly, using the same argument Area of ΔBCM = $\frac{3}{13}$ and Area of ΔCAK = $\frac{3}{13}$

Thus Area of ΔKLM = $1 - \frac{9}{13} = \frac{4}{13}$

$$\frac{\text{Area } ABC}{\text{Area } KLM} = \frac{13}{4}$$