Subject: Mathematics **F.M. : 100**
Class: XI **Time: 3:00 Hours.** **P.M. : 40**

SET-1

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) Write truth table for $p \wedge q \Leftrightarrow q \wedge p$. Hence draw a conclusion from the truth table.
   b) If A = {1, 2, 3 } find the cartesian product $A \times A$.
   c) Test periodicity and symmetricity of the function $y = \text{Cos } x$.
2. a) Find the value of x for which $2\text{Sin } 2x - \text{Sin } x = 0$.
   b) Using mathematical induction, prove that
   $1 + 3 + 5 + \ldots\ldots + (2n - 1) = n^2$.
   c) Find the inverse of the matix $A = \begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$.
3. a) Solve the following system of linear equations by inverse matrix method, if possible: $5x - 3y = 6$
   $10x - 6y = 16$
   b) Find the value of the real numbers x and y if
   $(x + iy) = (3 + i)(1 + 2i)$
   c) Determine the nature of the roots of the equation $2x^2 + 3x - 2 = 0$.
4. a) Show that the points (1, 2) and (2, 3) lie on the opposite side of the line $5x - 2y - 3 = 0$.
   b) Find the equation of the circle concentric with $x^2 + y^2 - 8x - 12y + 14 = 0$ and passing through (5, 4).
   c) Find the limit of $f(x) = \frac{x^2-4}{x-2}$ as $x \to 2$. Is $f(x)$ continuous? If not, find the point of discontinuity.
5. a) Find the derivative of $\sec^2(\tan\sqrt{x})$.
   b) Determine whether the curve $y = 3x^2 - 2x$ is increasing or decreasing at $x = 1$.
   c) Evaluate: $\int \frac{x}{(1-x^2)^{3/2}}\, dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) If A, B and C are any three non-empty sets, prove that
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   OR

   Define the absolute value of a real number. Prove that
   $|x + y| \le |x| + |y|$.
   b) Sketch the graph of $y = x^2 - 6x + 9$ indicating its different characteristics.
7. a) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

   OR

   State and prove sine law.
   b) Prove that
   $$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$$
8. a) Applying Cramer's rule or Row equivalent method, solve the system of linear equations :
   $3x + y + 2z = -1$
   $2x + 3y + z = 5$
   $x + 2y - z = 8$.
   b) If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p - q = 0$ or $p + q + 1 = 0$.
9. a) Prove that the line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1+m^2}$. Also show that $3x + 4y = 20$ is tangent to the circle $x^2 + y^2 = 16$.
   b) Evaluate: $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$.

   OR

   A function $f(x)$ is defined as follows:
   $$f(x) = \begin{cases} 2x - 3 & for \quad x < 2 \\ 2 & for \quad x = 2 \\ 3x - 5 & for \quad x > 2 \end{cases}.$$
   Is $f(x)$ continuous at $x = 2$? If not, how can $f(x)$ be made continuous at $x = 2$.
10. a) Find the derivative of $\sqrt{3 - 2x}$ from first principles.
    b) Find area between the curves $y^2 = 4ax$ and $x^2 = 4ay$.

Group 'C' [5 × 6 = 30]

11. Define domain and range of a function. Find the domain and the range of the $f(x) = \sqrt{21 - 4x - x^2}$.
12. If AM, GM and HM be the arithmetic, geometric and harmonic means between two unequal positive numbers prove that: i) $AM \times HM = GM^2$ ii) $AM > GM > HM$.
13. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to a line $x\cos\alpha + y\sin\alpha = p$. Also, find the distance between the parallel lines $5x - 12y + 8 = 0$ and $10x + 24y - 3 = 0$.

    OR

    Find the condition that the general equation of second degree may represent a line-pair. If $3x^2 + 5xy - 3y^2 + 3x + 2y = 0$ represents a line pair, show that the lines are perpendicular.
14. Define conjugate of a complex number. Find the square root of $\frac{2-36i}{2+3i}$.

15. Find the local maxima and minima, and also the point of inflection (if exists) of the function $f(x) = 4x^3 + 6x^2 - 9x + 1$ on the interval (-1, 2). Also, examine whether the function is increasing or decreasing at $x = 0$.

OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 8cm/sec and that of the outer circle at the rate of 5cm/sec. At a certain instant the radii of the inner and outer circles are respectively 24cm and 30cm. At what rate does the area between the two circles changes?

BEST OF LUCK

SET -2

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) Rewrite $-6 \leq x \leq 1$ by using absolute value sign.
   b) Prove that $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$.
   c) Define the even function. Examine whether $f(x) = \sin^3 x + x\cos x$ is even or odd or neither.
2. a) Express $\cos(2\cot^{-1} x)$ in terms of x.
   b) If a, b, c are in A.P. and b, c, a in H.P. prove that c, a, b are in G.P.
   c) Construct a 3×3 matrix whose elements a_{ij} are given by $a_{ij} = j^{i+1}$.
3. a) Solve the following system of linear equations by Cramers Rule:
   $2x + 5y = 24$; $2x + 3y = 12$.
   b) If $x - iy = \sqrt{\frac{1-i}{1+i}}$, prove that $x^2 + y^2 = 1$.
   c) When $x^3 + 2x^2 - px + 1$ is divided by $x - 1$, the remainder is 5, find the value of p.
4. a) Find the value of k so that the equation $2x^2 - 7xy + 3y^2 + 5x - 5y + k = 0$ many represent a pair of lines.
   b) Find the equation of the circle which touches x-axis and y-axis and whose radius is 3 units.
   c) Evaluate $\lim_{x \to 0} \frac{x\tan x}{1-\cos x}$.
5. a) Find the derivative of $\sqrt{3 + 2\sqrt{x+3}}$ with respect to x.
   b) Find the interval in which the curve $y = x^3 - 2x^2 + 10$ is increasing.
   c) Evaluate: $\int \frac{(1+\sqrt{x})^3}{2\sqrt{x}}\,dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) If A, B and C are any three non-empty sets, prove that $A - (B \cup C) = (A - B) \cap (A - C)$.

   OR

   Construct the truth table and hence show that $\sim((\sim p) \wedge q) \equiv p \vee (\sim q)$, where p and q are any two statements.

   b) Sketch the graph of $f(x) = (x-1)(x-3)(x-4)$ indicating its different characteristics.
7. a) Solve for θ: $\sin\left(\frac{k+1}{2}\right)\theta = \sin\left(\frac{k-1}{2}\right)\theta + \sin\theta$.

   OR

   If $\frac{b+c}{a} = \cot\frac{A}{2}$, prove that ΔABC is a right angle triangle.

   b) Without expanding prove that
   $$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$
8. a) Applying Inverse Matrix or Row equivalent method, solve the system of linear equations : $9y - 5x = 3$; $x + z = 1$, $z + 2y = 2$.
   b) If $ax^2 + bx + c = 0$ has roots in the ratio 3 : 4, prove that $12b^2 = 49ac$.
9. a) Show that the four points (1, 0), (2, 7), (8, 1), (9, -6) are concyclic.

   OR

   Prove that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ will touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

   b) Evaluate: $\lim_{y \to \theta} \frac{(x+y)\sec(x+y) - x\sec x}{y}$.

   OR

   Discuss the continuity at $x = 7$ of the function $f(x) = \begin{cases} \frac{x^2-7x}{x-7} & for \quad x \neq 7 \\ 5 & for \quad x = 7 \end{cases}$.
10. a) Find $\frac{dy}{dx}$ if $x^y . y^x = 1$.
    b) Evaluate $\int e^{ax} \cos bx\, dx$

    OR

    Find area of ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

Group 'C' [5 × 6 = 30]

11. Let f, g: R→ R be any two functions. If $f(x) = x^3 - 1$ and $g(x) = 2x - 3$ find the formula for $f^{-1}(x)$ and $g^{-1}(x)$. Also find $(f^{-1} o g)(2)$ and $(f \circ g^{-1})(1)$

12. Define Harmonic series. If the sum of three consecutive numbers in H. S. is 37 and the sum of their reciprocals is $\frac{1}{4}$, find the numbers.

OR

By method of induction prove that $3^{2n+2} - 8n - 9$ is divisible by 64 for all $n \in Z^+$.

13. Find the separate equations represented by the equation $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$, Also find their points of intersections and angle between them.

14. State and prove De Moivre's theorem for positive integer. Also use De Moivre's theorem to evaluate $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{20}$

OR

Find the square root of $\frac{2-36i}{2+3i}$.

15. Find the local maxima and minima, and also the point of inflection (if exists) of the curve $f(x) = 4x^3 - 15x^2 + 12x + 7$.

OR

A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2cm/s. How fast is the height on the wall decreasing when the foot of the ladder is 4m away from the wall.

BEST OF LUCK

SET-3

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) Define irrational numbers. Represent $\sqrt{2}$ on the real number line.
   b) If $f : [-2, 2] \to R$ defined by $f(x) = x^2$, find the range of the function. Is the function onto?
   c) Sketch the graph of $\frac{1}{x-1}$.
2. a) Find the value of $\cos^2(\sin^{-1}\frac{1}{\sqrt{2}}) + \sec^2(\tan^{-1} 5)$.
   b) Define a geometric series. When an infinite geometric series has sum? Give an example to illustrate it.
   c) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then show that $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$
3. a) Define system of linear equations. When the system is consistent and independent, illustrate with an example.
   b) Define the complex number. Express $\sqrt{3} + i$ in the polar form.
   c) Find the quadratic equation whose one root is $\frac{1}{3-i}$.
4. a) Find the sides of the right angled isosceles triangle whose vertex is (-2, -3) and whose base is $x = 0$.
   b) Define circle. Find the equation of the circle which touches x-axis and y-axis and whose radius is 3 units.
   c) Show that the function $f(x) = \begin{cases} 2x+1 & for \quad x < 1 \\ 2 & for \quad x = 1 \\ 5x-2 & for \quad x > 1 \end{cases}$ is discontinuous at $x = 1$.
5. a) Find the derivative of $e^{ln\sqrt{x^2+1}}$.
   b) Determine whether the curve $y = 3x^2 - 2x$ is increasing or decreasing at $x = 1$.
   c) Evaluate: $\int x\sqrt{x+1}\,dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) Define statement and sentence. Construct a truth table for the statement $p \wedge (\sim q)$.

   OR

   Define the absolute value of a real number. Prove that $|x + y| \le |x| + |y|$.

   b) Sketch the graph of $f(x) = |x + 1|$.
7. a) In any triangle ABC prove that $a^2 \cot A + b^2 \cot B + c^2 \cot C = 4\Delta$.

   OR

   Find the general values of x when $\sec x + \tan x = \sqrt{3}$.

   b) Prove that
   $$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2.$$
8. a) Solve $4x + 5y = 13$; $3x + y = -4$ by matrix method.
   b) If the equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, then either $a + b + c = 0$ or $a = b = c$.
9. a) Prove that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ will touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.
   b) Prove that the function $f(x) = \frac{x^2-16}{x-4}$ is discontinuous at $x = 4$. When a function can be made continuous?

   OR

   Evaluate: $\lim_{x \to 1} \frac{1 + \cos \pi x}{\tan \pi x}$.
10. a) Find the derivative of $\frac{1}{\sqrt{3x-2}}$ from first principles.
    b) Integrate $\int x \ln x \, dx$

OR

Find area of the circle $x^2 + y^2 = 25$ by using integration.

Group 'C' [5 × 6 = 30]

11. a) Show that the function $f : R \to R$ defined by $f(x) = 2x + 1$ is bijective

 b) Solve : $log_2(5x^2 - x - 2) - 2\ log_2\ x = 2$.

12. Sum to n terms the series 0.7 + 0.77 + 0.777 + ……………..

OR

Prove by mathematical induction that $1 + 2 + 3 + \ldots + n < (2n + 1)^2$.

13. Find the condition that line joining the origin and the points of intersections of the line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ are at right angle.

OR

Find the condition that the general equation of second degree may represent a line-pair.

14. Define conjugate of a complex number. Find the square root of $\frac{2-36i}{2+3i}$.

15. Find the local maxima and minima, and also the point of inflection (if exists) of the function $f(x) = 4x^3 + 6x^2 - 9x + 1$ on the interval (-1, 2).

OR

A 1.5m tall man walks at a uniform speed of 4km/hr. away from a lamp post 6 meters high. Find the rate at which the length of his shadow increases.

BEST OF LUCK

SET-4

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) What do you mean by interval? When does it becomes closed and open?

 b) If $a^2 + b^2 = 7ab$, prove that $\frac{a+b}{3} = \frac{1}{2}(log\ a + log\ b)$. .

 c) State whether the following function is symmetrical about x-axis, y-axis and origin or not $x^2 + y^2 = 25$.

2. a) Solve : $\sin 3\theta + \sin \theta = \sin 2\theta$.

 b) The AM and HM between two numbers are 25 and 16 respectively. Find the numbers.

 c) Define the transpose of a matrix. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, the prove that $A^T. A = I$.

3. a) Solve the following system of linear equations by inverse matrix method: $3x + 4y = 5$; $4x - y = -6$.

 b) If $a - ib = \sqrt{\frac{1-i}{1+i}}$, prove that $a^2 + b^2 = 1$.

 c) Distinguish between an identity and an equation. What are the roots of the quadratic equation $ax^2 + bx + c = 0$?

4. a) Find the angle between the lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$. Write the condition when the lines are perpendicular to each other?

 b) Find the equation of the circle which touches x-axis and center at (1, -2).

 c) Evaluate $\lim_{x \to 0} \frac{sin^{-1}x}{x}$.

5. a) Find the derivative of $log\left(\sqrt{x^2 + a^2} + b\right)$ with respect to x.

 b) The radius of the circle is increasing at the rate of 5 cm/s. Find the rate at which the area of the circle is increasing when the radius of circle is 4 cm.

 c) Evaluate: $\int_0^2 \frac{dx}{x^2+4}$.

Group 'B' [5 × 2 × 4 = 40]

6. a) Define contradiction. Construct the truth table for the following compound statement ~[$p \vee (\sim q)$], where p and q are any two statements.

OR

Define union and intersection of two sets. If A, B and C are any three non-empty subsets of universal set U, prove that $A - (B \cup C) = (A - B) \cap (A - C)$.

 b) What is periodic function. Sketch the graph of $f(x) = (x - 1)(x - 3)(x - 4)$

7. a) Solve for x : $tan^{-1}\frac{2x}{x^2-1} + cos^{-1}\frac{x^2-1}{x^2+1} = \frac{2\pi}{3}$.

OR

If $b - a = mc$ prove that $\cot\frac{B-A}{2} = \frac{1+m\cos B}{m\sin B}$.

 b) If $abc + 1 = 0$ then prove that $\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$.

8. a) Applying Cramer's Rule or Row equivalent method, solve the system of linear equations : $3x + 5z = 14$; $2x + y - 3z = 3$, $x + y + z = 4$.

 b) If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p - q = 0$ or $p + q + 1 = 0$.

9. a) Find the equation of the line through the point (1, -1) which cut of a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3x = 0$.

b) Find $\frac{dy}{dx}$ if $x^2 . y = \sec xy^2$.

OR

Find from the first principle, the derivative of $\frac{1}{\sqrt{2x-5}}$

10. a) Define the continuity of a function at a point. A function $f(x)$ is defined in (0, 3) as follows:

$$f(x) = \begin{cases} x^2 & for \quad 0 < x < 1 \\ x & for \quad 1 \le x < 2 \\ \frac{x^3}{4} & for \quad 2 \le x < 3 \end{cases} .$$

Show that $f(x)$ is continuous at $x = 1$ and $x = 2$.

b) Evaluate $\int x^2 e^x dx$.

OR

Find area between the curves $y^2 = 4ax$ and $x^2 = 4ay$

Group 'C' [5 × 6 = 30]

11. What do you mean by function? When does it becomes one to one and onto? Let Q be set of rational numbers, show that the function $f : Q \rightarrow Q$ such that $f(x) = 3x - 2$ for all $x \in Q$ is one to one and onto.

12. What is a sequence? Prove that the AM, GM, and HM between any two unequal positive numbers satisfy the following relations

i. $GM^2 = AM \times HM$

ii. AM > GM > HM

OR

Define mathematical induction with example. By method of induction prove that

$1.3 + 2.4 + 3.5 + \ldots\ldots\ldots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$.

13. Define a complex number. Express $z = \sqrt{3} - i$ in polar form. State and prove the De' Moivre's theorem.

14. Find the equation of bisector of the angles of the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$. Prove that the bisectors are right angles to each other. Also indentify the bisector of the angle between the lines containing the origin.

OR

Find the lines joining the origin to the points of intersection of $x + 2y = 3$ and $4x^2 + 16xy + 12y^2 - 8x + 12y - 3 = 0$ Also find the angle between these lines.

15. Define the stationary point and point of inflection. Water flows into an inverted conical tank at the rate of 24 cm^3/min. When the depth of water is 9 cm, how fast the level rising? Assume that the height of the tank is 15cm and the radius at the top is 5cm.

BEST OF LUCK

SET-5

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) What do you mean by logically equivalent? Write the truth table of $p \wedge q$.
   b) Define bijective function. Let f: R → R be defined by $f(x) = x^2 + 7$, $x \in$ R, show that $f(x)$ is not one to one function.
   c) Show that the odd function are symmetrical about the origin.
2. a) Solve : $\tan 5\theta \tan 3\theta = 1$.
   b) If G is the geometrical mean between two distinct numbers a and b show that $\frac{1}{G-a} + \frac{1}{G-b} = \frac{1}{G}$.
   c) Prove that A - A^T is skew symmetric matrix where $A = \begin{bmatrix} 3 & -2 \\ -5 & 1 \end{bmatrix}$
3. a) Solve the following system of linear equations by Cramers's rule: $2x + 3y - 16 = 0$; $3x - 4y - 7 = 0$.
   b) Find the value of real number x and y when $(3y - 2)i^{16} + (5 - 2x)i = 0$
   c) State the remainder theorem and use it to find the remainder of $f(x) = 4x^2 + 2x + 1$ when divided by $g(x) = x - 2$.
4. a) Let $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, show that $\frac{1}{a} + \frac{1}{b} = 1$.
   b) Find the equation of the circle which touches coordinate axis and whose center lies on the line $x - 2y = 3$.
   c) Evaluate $\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+1}$.
5. a) Define the point of inflection. Find the values of x of which $f(x) = -3x^2 + 12x + 8$ is increasing and decreasing.
   b) Find $\frac{dy}{dx}$ if $y = 5t^2 + 6t - 7$ and $t = x^3 - 2$.
   c) Evaluate: $\int_0^1 \frac{1-x}{1+x}\, dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) State and prove the De Morgan's Law for sets.

   OR

   Define absolute value of real number. Prove that if a is any positive real number, then $|x| \leq a$ if and only if $-a \leq x \leq a$ for all $x \in$ R.
   b) Test the symmetry, intercepts and sketch the graph of $y = \frac{1}{x}$.
7. a) Prove that $Cos\ C = \frac{a^2+b^2-c^2}{2ab}$.

   OR

   What is domain of the inverse circular function $\tan^{-1} x$? Find the value of $\sin \cos^{-1} \frac{1}{3}$. Prove that $\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$.
   b) Show that $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.
8. a) Applying Row equivalent method, solve the system of linear equations : $x - 2y - 3z = -1$, $x + 3y - 2z = 13$, $2x + y + z = 6$.
   b) If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$ prove that $p^2m = l^2q$.
9. a) Find the condition that the line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$. Also find the equation of tangent when slope is given.
   b) Evaluate: $\lim_{y \to \theta} \frac{(x+y)\tan(x+y) - x\tan x}{y}$.

   OR

   A function $f(x)$ is defined as follows:

   $$f(x) = \begin{cases} 3x - 1 & for \quad x < 1 \\ 4 & for \quad x = 1 \\ 2x & for \quad x > 1 \end{cases}$$

   Verify that the limit of the function exits at $x = 1$. Is the function continuous at $x = 1$? If not why? State how can you make it continuous at $x = 1$.
10. a) Find from the first principle, the derivative of $\sec^2 x$.
    b) Evaluate $\int \frac{x}{(x^2+4)^{3/2}} dx$.

Group 'C' [5 × 6 = 30]

11. Define logarithmic and exponential functions. Prove that log (1 + 2 + 3) = log 1 + log 2 + log 3 and if $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$ show that $x^a y^b z^b = 1$.
12. Define arithmetic and geometric series. If $S_1, S_2, S_3,$ ………… are the sum of the infinite geometric series whose first terms are 1, 2, 3, ……….respectively and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ respectively, show that $S_1 + S_2 + S_3 + \ldots\ldots + S_n = \frac{n(n+3)}{2}$.
13. Define a complex number. If $x = \cos\theta + i\sin\theta$ and $\sqrt{1-c^2} = nc - 1$ then prove that $1 + c\cos\theta = \frac{c}{2n}(1 + nx)\left(1 + \frac{n}{x}\right)$.
14. What are the three standard forms equation of straight line. Find the equation of the straight line through the point of intersection of the line $x + y = 18$ and $3x - 3y + 1 = 1$ and perpendicular to the line joining the points (3, 4) and (5, -6).

    OR

    Find the condition for a general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represents a pair of straight lines. Also find the value of a such that the curve $x^2 + axy + 2y^2 + 3x + 5y + 2 = 0$ may represent a pair of straight line.

15. Define the monotoncity of a function. A cone is 10 inches in diameter deep. Water poured into it at the rate of 4 cubic inches per minute. At what rate is the water level rising at the instant when depth is 6 inches?

BEST OF LUCK

SET-6

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) Is the following argument valid? All natural numbers are integers. x is not an integer. Therefore x is not a natural number.
   b) Prove that $log\frac{a+b}{3} = \frac{1}{2}(log\, a + log\, b), if\ a^2 + b^2 = 7ab.$
   c) Find the domain and range of $y = f(x) = x^2 - 6x + 6$.
2. a) State the domain and range of $\sin^{-1} x$ and $\cos^{-1} x$. Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
   b) Sum $1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \cdots\ to\ infinity.$
   c) If $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$, prove that $(A - 3I)(A + 2I) = 0$ where I is identity matrix.
3. a) Solve the following system of linear equations by Row equivalent matrix method: $3x + 2y = -9$; $2x - 3y = -6$.
   b) If $x - iy = \sqrt{\frac{1-i}{1+i}}$, prove that $x^2 + y^2 = 1$.
   c) Find the quadratic equation with rational coefficient one of whose root is $\frac{1}{5+3i}$.
4. a) What are the points on x-axis, whose perpendicular distance from the straight line $\frac{x}{a} + \frac{y}{b} = 1$.
   b) Find the equation of the tangents to the circle $x^2 + y^2 = 4$ which are parallel to $3x + 4y - 5 = 0$.
   c) Evaluate $\lim_{x \to \infty} \sqrt{x-a} - \sqrt{x-b}$.
5. a) Find the derivative of $\frac{1}{x+\sqrt{x^2-a^2}}$.
   b) Show that the function $f(x) = x - \frac{1}{x}$ is increasing for all $x \in R$ and $x \neq 0$.
   c) Evaluate: $\int \ln x\, dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) Define the difference and complement of sets. Prove that $A - (B\cup C) = (A - B)\cap(A - C)$.

   OR

   b) Sketch the graph of $y = x(x-1)(x-2)$.
7. a) If $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{a^2+b^2}$, prove that the triangle is either isosceles or right angled.

   OR

   Solve: $\tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 4$.

   b) Show that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$.
8. a) Solve the system of linear equations by Cramer's rule : $3x + 5y = 2$; $2x - 3z = -7$; $4x + 2z = 2$.
   b) Prove that the quadratic equation $ax^2 + bx + c = 0$ cannot have more than two roots.
9. a) Find the equation of the circle which touches the positive y-axis at a distance 4 from the origin and cuts off an intercept 6 from the axis of x.
   b) Evaluate: $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$.

   OR

   A function $f(x)$ is defined as follows: $f(x) = \begin{cases} 2x-3 & for\quad x < 2 \\ 2 & for\quad x = 2 \\ 3x-5 & for\quad x > 2 \end{cases}$

   Is the function continuous at $x = 2$? If not how can you make it continuous at $x = 2$.
10. a) Find from the first principle, the derivative of $\sqrt{\tan x}$.

    OR

    Find $\frac{dy}{dx}$ if $x^y . y^x = 1$.

    b) Evaluate $\int \sqrt{\frac{x}{a-x}}\, dx$.

    OR

    Obtain the are bounded by the curves $y = x^2$ and $y = 2x$.

Group 'C' [5 × 6 = 30]

11. Define one to one and onto function. Let f: A → B be a function defined by $f(x) = \frac{x-1}{x+2}$ with A = { -1, 0, 1, 2, 3, 4} and B =

$\left\{-2, 1, -\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{2}{5}\right\}$. Find the range of f. Is the function one to one and onto both? If not how can you make it one to one and onto both?

12. By mathematical induction prove that
$1.3 + 2.4 + 3.5 + \ldots\ldots\ldots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$

13. State De Moivre's theorem. Use it to find the cube root of unity. If $\omega = \frac{-1+i\sqrt{3}}{2}$ then prove that $1 + \omega + \omega^2 = 0$.

14. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to a line $x \cos \alpha + y \sin \alpha = p$. Also, find the distance between the parallel lines $y = 2x + 2$ and $6x - 3y = 5$.

OR

Find the condition that one of the lines given by $ax^2 + 2hxy + by^2 = 0$ may be perpendicular to one of the lines given by $a'x^2 + 2h'xy + b'y^2 = 0$.

15. A close cylindrical can is to be made so that it's volume is 52 cm^3. Find it's dimensions if the surface is to be minimum.

OR

A 5m ladder leans against a vertical wall. If the top slides downwards at the rate 12m/min, find the speed of the lower end when it is 4 m from the wall.

BEST OF LUCK

SET-7

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) Rewrite the following by using absolute sign $-6 \le x \le 1$.
   b) Let $f : R \to R$ be defined by $f(x) = x^2 + 1$ and $g(x) = x^5$, find $(fog)(x)$ and $(gof)(x)$.
   c) Define even and odd function. Examine whether the function $f(x) = x^3 + \sin x$ is odd or even.
2. a) Prove that $\tan^{-1}x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$.
   b) If a, b, c be in A.P. and b, c, a in H.P. prove that c, a, b are in G.P.
   c) If $A = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$, prove that $AA^T = A^TA = I$ where I is identity matrix of order 2.
3. a) Solve the following system of linear equations by Cramer's rule:
   $5x - 3y = 20$; $2x + 3y = 8$.
   b) Express $[3(\cos 120° + \sin 120°)]^3$ in the $a + ib$ form.
   c) For what value of p will the equation $5x^2 - px + 45 = 0$ have equal roots.
4. a) Find the equations of bisectors of the angle between the lines represented by $ax^2 + 2hxy + by^2 + k(x^2 + y^2) = 0$.
   b) Find the equation of the circle concentric with $x^2 + y^2 + 8x - 6y + 1 = 0$ and radius 3 units.
   c) Evaluate $\lim_{x \to 2} \frac{x-2}{|x-2|}$.
5. a) Find the derivative of x^x.
   b) Show that the graph of function $f(x) = 4x^2 + 5x + 1$ is concave upward for all $x \in R$.
   c) Evaluate: $\int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}}\, dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) Define the conjunction and disjunction. Let p and q be two statements, prove that $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
   b) Sketch the graph of $y = x^2 - 4x + 3$.
7. a) State and prove sine law.

   OR

   Solve: $\cos\theta - \sin 3\theta = \cos 2\theta$.
   b) Show that $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)(a-b)(b-c)(c-a)$.
8. a) Solve the system of linear equations by inverse matrix method : $x + 3y - 7z = 6$; $2x + 3y + z = 9$; $4x + y = 7$.
   b) Prove that if quadratic equation $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root, their other roots will satisfy $x^2 + ax + bc = 0$.
9. a) Prove that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ will touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.
   b) Evaluate: $\lim_{x \to 1} \frac{x-\sqrt{2-x^2}}{2x-\sqrt{2+2x^2}}$.

   OR

   A function $f(x)$ is defined as follows: $f(x) = \begin{cases} x^2 & for \quad 0 < x < 1 \\ x & for \quad 1 \le x < 2 \\ x^3 & for \quad 2 \le x < 3 \end{cases}$
   Show that the function is continuous at $x = 1$ and discontinuous at $x = 2$.
10. a) If $x^3y^6 = (x + y)^9$, then show that $\frac{dy}{dx} = \frac{y}{x}$.

OR

Find from the first principle, the derivative of a^x.

b) Evaluate $\int \frac{1}{(a^2+x^2)^{\frac{3}{2}}} dx$.

OR

Obtain the area of ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

Group 'C' [5 × 6 = 30]

11. What condition make a function to have its inverse? Let $f : R \to R$ be defined by $f(x) = 2x - 4$, $x \in R$. Show that f^{-1} exits. Find $f^{-1}(x)$ and prove that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$.
12. Find the n^{th} term and sum of the first n terms of the series 1 + 3 + 6 + 10 + ……

OR

State principle of mathematical induction and use it to prove that $2^n - 1$ is divisible by 7.

13. define n^{th} root of a complex number. Find the square roots of complex number $\frac{(8,-15)}{(0,\ 1)}$.
14. Determine the equation of the bisectors of the angle between the lines $3x - 2y + 1 = 0$ and $18x + y - 5 = 0$. Identify the bisector of the acute angle bisector.

OR

Define the homogenous equation of second degree. Find the equation of the pair of lines joining the origin to the intersection of the lines $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are right angles if $2c^2 = a^2(1 + m^2)$.

15. What are the criteria to be satisfied to have maximum and minimum value of the function? Find the maximum and minimum value of $f(x) = x^4 - 14x^2 - 24x + 1$.

OR

A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x-coordinates is increasing at the rate 2 cm/sec. Find the rate at which the distance of the point from the origin increasing when the point is at (2, 4).

BEST OF LUCK

SET-8

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL questions.

1. a) For any sets A and B prove that $A - (B \cup C) = (A - B) \cap (A - C)$.
   b) If $x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$, prove that $xyz + 1 = 2yz$.
   c) Define even and odd function with suitable examples.
2. a) If $\sin^{-1}x + \cos^{-1} y = \frac{\pi}{2}$. Prove that $x^2 + y^2 = 1$.
   b) Sum to n terms of the series: 1 + (1 + 2) + (1 + 2 + 3) + …..
   c) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, prove that $A^T.A = I$ where I is identity matrix of order 2.
3. a) Solve the equations by Cramer's rule: $\frac{2}{x} + \frac{3}{y} = 2$ and $\frac{8}{x} + \frac{9}{y} = 7$.
   b) If $a - ib = \sqrt{\frac{1-i}{1+i}}$, prove that $a^2 + b^2 = 1$.
   c) If one root of the equation $x^2 - px + q = 0$ be twice the other show that $2p^2 = 9q$.
4. a) Find the equation of the straight line which are parallel to the line $x + y + 6 = 0$ and passing through the point (1, 5).
   b) Find the equation of the circle whose ends of diameter are (1, 2) and (3, 4).
   c) Evaluate $\lim_{x \to a} \frac{x^{1/4}-a^{1/4}}{x^{1/3}-a^{1/3}}$.
5. a) Find $\frac{dy}{dx}$ from the relation $x^2 + y^2 = 2xy$.
   b) Prove that the function $f(x) = \sin x$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
   c) Evaluate: $\int \tan^3\ dx$.

Group 'B' [5 × 2 × 4 = 40]

6. a) What are converse, inverse and contrapositive? Construct the truth table for converse, inverse and contrapositvie.
   b) Sketch the graph of $y = \frac{x^2-4}{x-2}$.
7. a) State and prove cosine law in any triangle ABC.

   OR

   Solve for general values: $\sqrt{3}\sin x - \cos x = \sqrt{2}$.
   b) Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.
8. a) Solve by row-equivalent method: $x + y + z = 6$; $2x + y - z = 1$; $x - 2y + z = 0$.
   b) If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p - q = 0$ or $p + q + 1 = 0$.

9. a) Find the equation of the a circle which passes through the origin and cuts off intercepts of length 3 and 4 units on the positive parts of the line x- and y-axes.

OR

Derive the equation of the tangent at point (x_1, y_1) of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

b) Prove geometrically that $\lim_{x \to 0} \frac{\sin x}{x} = 1$, where x is measured in radians.

OR

A function $f(x)$ is defined as follows : $f(x) = \begin{cases} 4x+1 & for \quad x < 1 \\ 6x-1 & for \quad x > 1 \\ 11 & for \quad x = 1 \end{cases}$

Is the function continuous at $x = 1$ If not state how can it be made continuous at point $x = 1$?

10. a) Find derivative of sin $4x$ with respect to x from first principle.

b) Evaluate $\int cosec^3 x\, dx$.

OR

Obtain the area of ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ by using method of integration.

Group 'C' [5 × 6 = 30]

11. Define inverse and composite function with example. Let $f : R \to R$ and $g : R \to R$ be defined by $f = \{(1, 5), (2, 6), (3, 7), (4, 6)\}$ and $g = \{(5, 1), (6, 2), (7, 3)\}$. Find fog and gof. Is $fog = gof$?

12. Let A, G and H be the AM, GM and HM between two unequal positive numbers. Show that

i. A, G and H forms GP.

ii. A, G, H are in descending order in magnitude.

OR

State principle of mathematical induction and use it to prove that the sum of cube of three consecutive natural numbers is divisible by 9.

13. Find the length of the perpendicular drawn from (x_1, y_1) on the $Ax + By + C = 0$. Use it to find the distance between the parallel lines $2x - 5y = 6$ and $6x - 15y + 11 = 0$.

OR

Write the general equation of second degree in x and y. Find the condition for a general equation of second degree may represents a pair of straight lines.

14. State and prove the De' Mover's theorem and use it to find the square roots of $-i$.

15. What are the criteria to be satisfied to have maximum and minimum value of the function? Find the maximum and minimum value of $f(x) = x^4 - 14x^2 - 24x + 1$.

OR

A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x-coordinates is increasing at the rate 2 cm/sec. Find the rate at which the distance of the point from the origin increasing when the point is at (2, 4).

BEST OF LUCK