INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

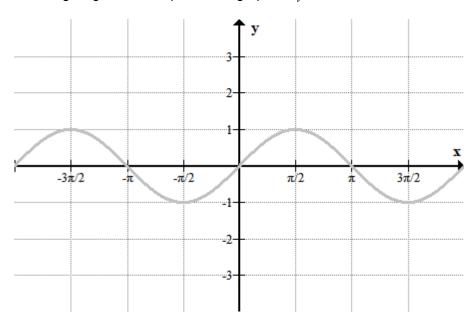
EXERCISES [MAA 3.7] TRIGONOMETRIC FUNCTIONS

Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 9] *[without GDC]*

The following diagram shows part of the graph of $y = \sin x$



(a) On the same diagram sketch the graphs of the functions

$$(i) \quad f(x) = \sin x + 2$$

(ii)
$$g(x) = \sin x + 2$$

[4]

(b) Complete the following table.

Function	Amplitude	Period	Central axis	Range
$y = \sin x$	1	2π	y = 0	$-1 \le y \le 1$
$f(x) = \sin x + 2$				
$g(x) = \sin x - 2$				

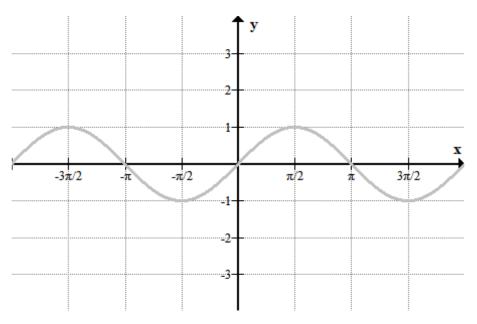
[4]

(c) Write down the equation of the **central axis** for the function $f(x) = \sin x + c$.

[1]

2. [Maximum mark: 8] [without GDC]

The following diagram shows part of the graph of $y = \sin x$



- (a) On the same diagram sketch the graph of the function $f(x) = 3\sin x$. [2]
- (b) Complete the following table.

Function	Amplitude	Period	Central axis	Range
$y = \sin x$	1	2π	y = 0	$-1 \le y \le 1$
$f(x) = 3\sin x$				

[2]

(c) Write down the **amplitude** for the function $f(x) = -3\sin x$.

[1]

(d) Write down the **amplitude** for the function $f(x) = a \sin x$.

[1]

(e) For the function $f(x) = a \sin x + c$, write down

(i) the amplitude

(ii) the equation of the central axis.

[2]

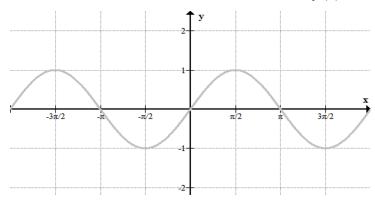
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3. [Maximum mark: 12] [without GDC]

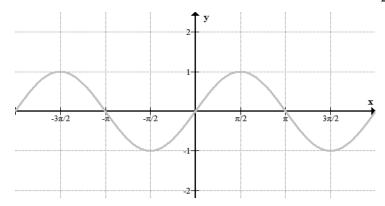
The following diagrams show part of the graph of $y = \sin x$.

(a) On the diagram below sketch the graph of the function $f(x) = \sin 2x$.



[2]

(b) On the diagram below sketch the graph of the function $f(x) = \sin \frac{x}{2}$.



[2]

(c) Complete the following table.

Function	Amplitude	Period	Central axis	Range
$y = \sin x$	1	2π	y = 0	$-1 \le y \le 1$
$f(x) = \sin 2x$				
$g(x) = \sin\frac{x}{2}$				

[4]

(d) Write down the **period** for the function $f(x) = \sin bx$.

[1]

(e) For the function $f(x) = a \sin bx + c$, write down

(i) the amplitude (i

(ii) the equation of the central axis.

(iii) the period.

[3]

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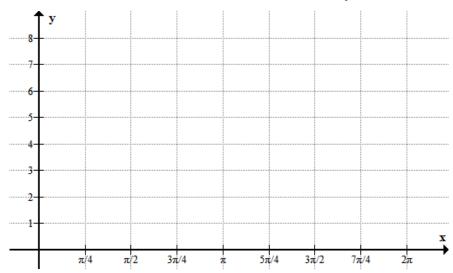
4. [Maximum mark: 8] *[without GDC]*

Let $f(x) = 3\sin 2x + 4$, $0 \le x \le 2\pi$

(a) Complete the table

Function	Amplitude	Period	Central axis	Range
f(x)				

(b) On the diagram below, draw the graph of the function f.



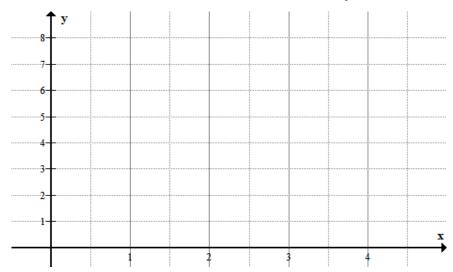
5. [Maximum mark: 8] *[without GDC]*

Let $f(x) = 3\sin \pi x + 4$, $0 \le x \le 4$

(a) Complete the table

	Function	Amplitude	Period	Central axis	Range
Ī	f(x)				

(b) On the diagram below, draw the graph of the function f.



[4]

[4]

[4]

[4]

6. [Maximum mark: 7] *[without GDC]*

(a) For the function $f(x) = a \cos bx + c$, where a > 0, complete the table below.

Function	Amplitude	Period	Central axis	Range
f(x)				

[4]

(b) For the function $f(x) = a \tan bx + c$, complete the table below.

Function	Period	Central axis	Range
f(x)			

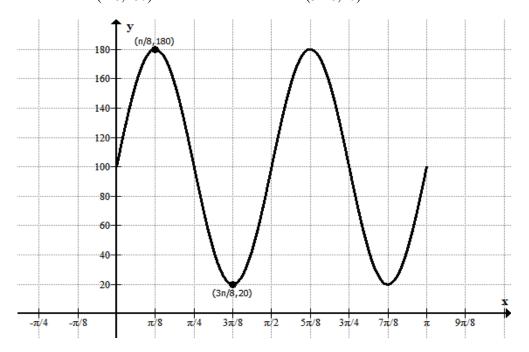
[3]

7. [Maximum mark: 14] *[without GDC]*Complete the following table

Function	Amplitude	Period	Central axis	Range
$f(x) = \sin x$	1	2π	y = 0	$-1 \le y \le 1$
$f(x) = \cos x$				
$f(x) = \sin x + 1$				
$f(x) = \sin x - 1$				
$f(x) = 5\sin x$				
$f(x) = -7\sin x$				
$f(x) = \sin 4x$				
$f(x) = -\cos 4x$				
$f(x) = 3\sin 4x$				
$f(x) = 3\sin 4x + 10$				
$f(x) = 3\sin 4x - 2$				
$f(x) = -5\sin 3x$				
$f(x) = -5\sin x + 10$				
$f(x) = \tan x$		π	y = 0	$y \in R$
$f(x) = \tan 4x$				
$f(x) = 5\tan 4x + 10$				

8*. [Maximum mark: 12] [without GDC]

Part of the graph of a trigonometric function f(x) is given below. There is a local maximum at $(\pi/8,180)$ and a local minimum at $(3\pi/8,20)$.



(a) Write down the values of

(i) the amplitude

- (ii) the central value
- (iii) the period
- [3]

[6]

(b) Express the function in the form $f(x) = A \sin Bx + C$

- [3]
- (c) Complete the following table by expressing f(x) in three alternative forms

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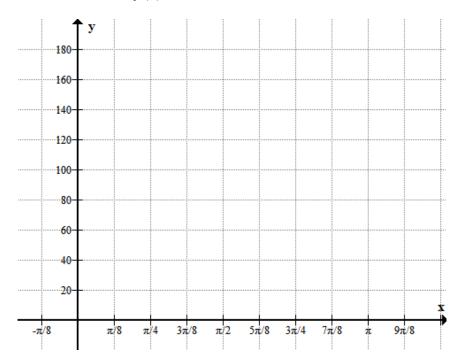
$f(x) = -80\sin[B(x-D)] + C$	
$f(x) = 80\cos[B(x-D)] + C$	

$$f(x) = -80\cos[B(x-D)] + C$$

9. [Maximum mark: 5] [without GDC]

Sketch the graph of the function

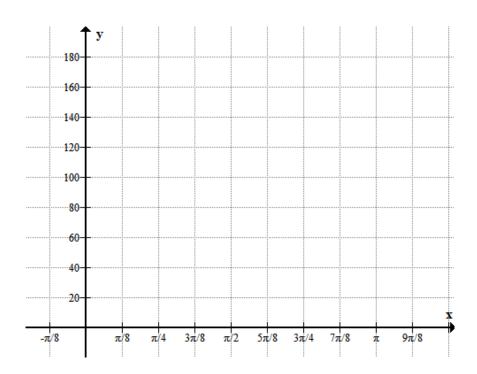
$$f(x) = 60\sin 4x + 100, \quad 0 \le x \le \pi$$



10. [Maximum mark: 5] **[without GDC]**

Sketch the graph of the function

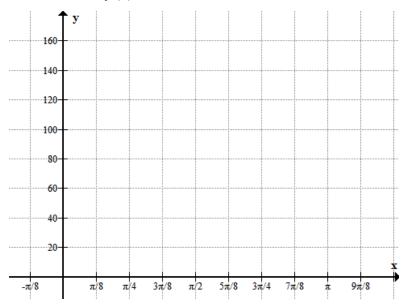
$$f(x) = 60\cos 4x + 100$$
, $0 \le x \le \pi$



11. [Maximum mark: 5] *[without GDC]*

Sketch the graph of the function

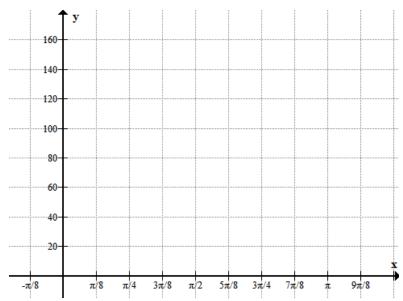
$$f(x) = -60\sin 4x + 80$$
, $0 \le x \le \pi$



12. [Maximum mark: 9] [without GDC]

(a) Sketch the graph of the function

$$f(x) = -60\cos 4x + 80$$
, $0 \le x \le \pi$



(b) Write down the possible values of k if the equation f(x) = k has

(i) exactly two solution

(ii) exactly three solutions

(iii) exactly four solutions

(iv) no solutions [4]

[5]

Α.	Exan	n style questions (SHORT)	
13.	_	ximum mark: 7]	
	The	function f is defined by $f(x) = 30 \sin 3x \cos 3x$, $0 \le x \le \frac{\pi}{3}$	
	(a)	Write down an expression for $f(x)$ in the form $a \sin 6x$, where a is an integer.	[2]
	(b)	Find the period of f .	[2]
	(c)	Solve $f(x) = 0$, giving your answers in terms of π .	[3]
14.	Let	ximum mark: 4] [with / without GDC] $f(x) = 4\sin\left(3x + \frac{\pi}{2}\right)$. For what values of k will the equation $f(x) = k$ have no	
	solu	tions?	

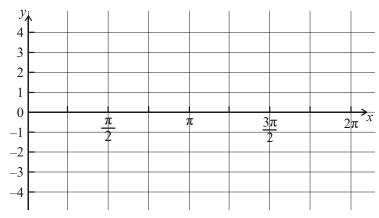
15. [Maximum mark: 6] *[without GDC]*

Consider $g(x) = 3\sin 2x$.

(a) Write down the period of g.

[1]

(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$.



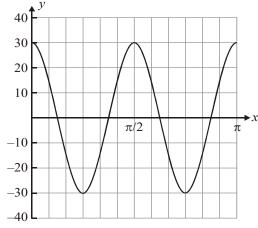
[3]

(c) Write down the number of solutions to the equation g(x) = 2, for $0 \le x \le 2\pi$. [2]

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16. [Maximum mark: 4] **[without GDC]**

The graph of a function of the form $y = p \cos qx$ is given in the diagram below.



- (a) Write down the value of p. [2]
- (b) Calculate the value of q. [2]

17. [Maximum mark: 6] [with / without GDC]

Let $f(x) = \sin 2x$ and $g(x) = \sin(0.5x)$.

- (a) Write down
 - (i) the minimum value of the function f
 - (ii) the period of the function g.

[3]

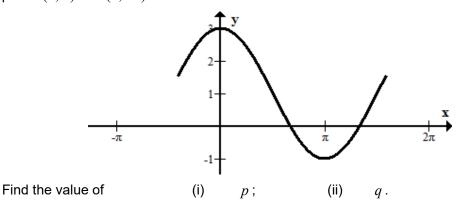
[3]

(b) Consider the equation f(x) = g(x). Find the number of solutions, for $0 \le x \le \frac{3\pi}{2}$

.....

18. [Maximum mark: 4] *[without GDC]*

Part of the graph of $y = p + q \cos x$ is shown below. The graph passes through the points (0, 3) and $(\pi, -1)$.



19.	[Maximum mark: 6]	[without GDC]
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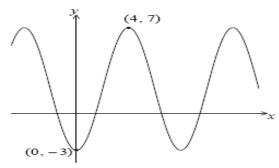
Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

- (a) Find an expression for h(x) [3]
- (b) Write down the period of h. [1]
- (c) Write down the range of h. [2]

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20. [Maximum mark: 7] *[without GDC]*

The graph of $y = p + \cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at (0, -3) and a maximum point at (4, 7).

- (a) Find the value of (i) p; (ii) q; (iii) r.
- (b) The equation y = k has exactly **two** solutions. Write down the value of k. [1]

[6]

21.	[Maximum mark: 4]	[without GDC]
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The depth, y metres, of sea water in a bay t hours after midnight may be represented by the function

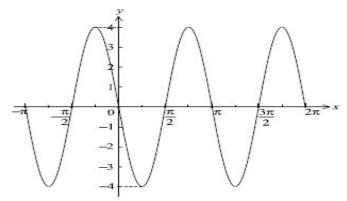
 $y=a+b\cos\left(\frac{2\pi}{k}t\right)$ where a, b and k are constants.

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of (I) a; (II) b; (III) k.

22*. [Maximum mark: 6] [without GDC]

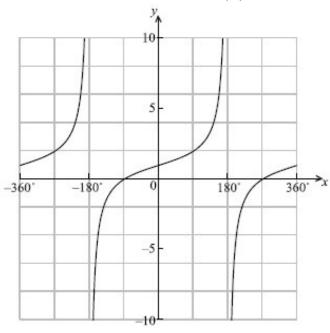
Let $f(x) = a \sin b(x-c)$. Part of the graph of f is given below.



Given that a , b and c are positive, find the value of a , of b and of c .

23. [Maximum mark: 6] [without GDC]

The diagram below shows the graph of $f(x) = 1 + \tan\left(\frac{x}{2}\right)$, for $-360^{\circ} \le x \le 360^{\circ}$.



(a) On the same diagram, draw the asymptotes.

[2]

- (b) Write down
 - (i) the period of the function;
 - (ii) the value of $f(90^\circ)$.

[2]

(c) Solve f(x) = 0 for $-360^{\circ} \le x \le 360^{\circ}$.

[2]

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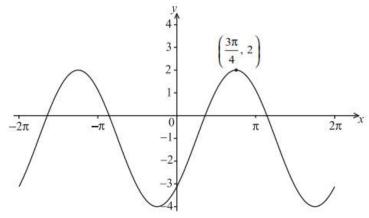
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24.	[Maximum	mark: 61	[without	GDCI
44.	[iviaxii iiui ii	mark. Oj	įwitiiout	GDCj

The graph of $y = \cos x$ is transformed into the graph of $y = 8 - 2\cos\frac{\pi x}{6}$. Find a sequence of simple geometric transformations that does this.

25*. [Maximum mark: 6] *[without GDC]*

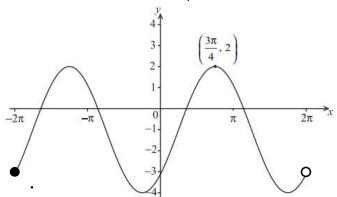
The graph below represents $f(x) = a \sin(x+b) + c$, where a,b, and c are constants.



Find values for a,b, and c.

26. [Maximum mark: 6] *[without GDC]*

The graph below represents $f(x) = 3\sin(x - \frac{\pi}{4}) - 1$, $-2\pi \le x < 2\pi$



- (a) Write down the range of the equation f. [2]
- (b) Write down the number of solutions of the equation f(x) = -2. [1]

[3]

(c) Write down the values of k, for which the equation f(x) = k, for $-2\pi \le x < 2\pi$,

(i) has exactly 2 solutions;	(ii) has exactly 4 solutions;	(iii) has no solutions.

27*. [Maximum mark: 6] [with / without GDC]

The depth, h(t) metres, of water at the entrance to a harbor at t hours after midnight on a particular day is given by $h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right)$, $0 \le t \le 24$

- (a) Find the maximum depth and the minimum depth of the water . [2]
- (b) Find the values of t for which $h(t) \ge 8$. [4]

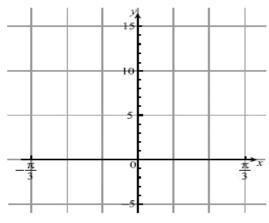
28.

liviax	Ilmum mark: 4/8] [with / without GDC]	
Cons	sider $y = \sin\left(x + \frac{\pi}{9}\right)$.	
(a)	The graph of y intersects the x -axis at point A. Find the x -coordinate of A,	
	where $0 \le x \le \pi$.	[2]
(b)	Solve the equation $\sin\left(x+\frac{\pi}{9}\right)=-\frac{1}{2}$, for $0 \le x \le 2\pi$.	[4]
	[with GDC] [4 marks]	
(**)	[without GDC] [8 marks]	
()		

29. [Maximum mark: 5] [with GDC]

Let
$$f(x) = 4 \tan^2 x - 4 \sin x$$
, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) On the grid below, sketch the graph of y = f(x).



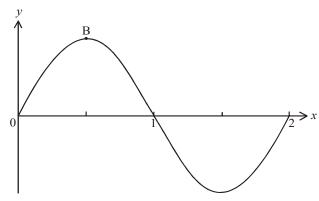
[3]

(b) Solve the equation f(x) = 1.

[2]

- **30.** [Maximum mark: 6] **[with GDC]**

Let $f(x) = 6\sin \pi x$ and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of f is shown on the diagram below. There is a maximum value at B (0.5, b).



(a) Write down the value of b.

[1]

(b) On the same diagram, sketch the graph of g.

[3]

(c) Solve $f(x) = g(x), 0.5 \le x \le 1.5$.

[2]

31.	[Max	kimum mark: 6]	
	Let	$f(x) = 2\sin(3x-1)$ and $g(x) = x^2$.	
	(a)	Describe the sequence of transformations from $y = \sin x$ to $f(x)$.	[4]
	(b)	Solve the inequality $f(x) \ge g(x)$.	[2]
32.	-	ximum mark: 5] [without GDC]	
		bserving the graph of the function $f(x) = \sin 3x + \sin 6x$, $0 < x < 2\pi$	
	(a)	Write down the range of the function f .	[2]
	(b)	Write down the number of solutions of the equation $f(x) = 0$.	[1]
	(c)	Write down the exact period of the function f .	[2]
33**.	[Max	kimum mark: 6] <i>[with GDC]</i>	
	Cons	sider the functions $f(x) = e^{2x}$ and $g(x) = \sin \frac{\pi x}{2}$ (x in radians).	
	(a)	Find the period of the function $f\circ g$.	[3]
	(b)	Find the intervals for which $(f \circ g)(x) > 4$.	[3]

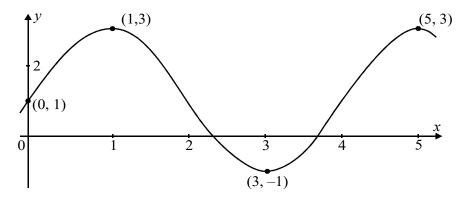
B. Exam style questions (LONG)

34. [Maximum mark: 18] [without GDC]

The diagram shows the graph of the function f given by

$$f(x) = A\sin\left(\frac{\pi}{2}x\right) + B$$
, for $0 \le x \le 5$,

where A and B are constants, and x is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

- (a) Write down the values of f(1) and f(5). [2]
- (b) Show that the period of f is 4. [2]

The point (3, -1) is a minimum point of the graph.

- (c) Show that A = 2, and find the value of B. [5]
- (d) Solve the equation f(x) = 2 for $0 \le x \le 5$. [5]
- (e) Consider the equation f(x) = k, for $0 \le x \le 5$.

Write down the possible values of k if the equation has

- (i) exactly one solution
- (ii) exactly three solutions
- (iii) exactly two solutions
- (iv) no solutions [4]

35. [Maximum mark: 13] *[with GDC]*

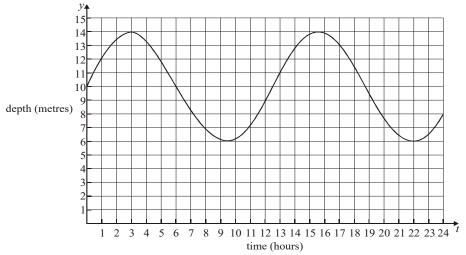
The depth y metres of water in a harbour is given by the equation

$$y = 10 + 4\sin\left(\frac{t}{2}\right).$$

where t is the number of hours after midnight.

(i) when
$$t = 2$$
;

The sketch below shows the depth y, of water, at time t, during one day (24 hours).



- (b) (i) Write down the maximum depth of water in the harbour.
 - (ii) Calculate the value of *t* when the water is first at its maximum depth during the day.

[3]

The harbour gates are closed when the depth of the water is less than seven metres. An alarm rings when the gates are opened or closed.

- (c) (i) How many times does the alarm sound during the day?
 - (ii) Find the value of t when the alarm sounds first.
 - (iii) Use the graph to find the length of time during the day when the harbour gates are closed. Give your answer in hours, to the nearest hour.

[7]

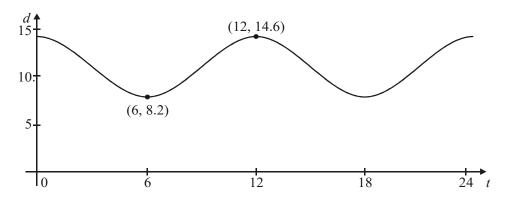
 	 	 	 	 	 	

36. [Maximum mark: 10] [with GDC]

A formula for the depth d metres of water in a harbour at a time t hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where P and Q are positive constants. In the following graph the point (6, 8.2) is a minimum point and the point (12, 14.6) is a maximum point.

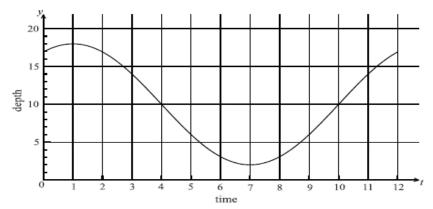


- (a) Find the value of (i) Q; (ii) P. [3]
- (b) Find the **first** time in the 24-hour period when the depth of the water is 10 metres. [3]
- (c) (i) Use symmetry of the graph to find the **next** time when the depth d is 10m.
 - (ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep. [4]

37. [Maximum mark: 11] *[with GDC]*

The following graph shows the depth of water, *y* metres, at a point P, during one day.

The time *t* is given in hours, from midnight to noon.



- (a) Use the graph to write down an estimate of the value of t when
 - (i) the depth of water is minimum;
 - (ii) the depth of water is maximum;
 - (iii) the depth of the water is increasing most rapidly.

[3]

- (b) The depth of water can be modelled by the function $y = A\cos(B(t-1)) + C$.
 - (i) Show that A = 8.
 - (ii) Write down the value of C.
 - (iii) Find the value of B.

[6]

[2]

(c) A sailor knows that he cannot sail past P when the depth of the water is less than12 m. Calculate the values of t between which he cannot sail past P.

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38.	[Max	kimum mark: 13] <i>[with GDC]</i>	
	Let	$f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.	
	(a)	Sketch the graph of f .	[3]
	(b)	Write down	
		(i) the amplitude; (ii) the period; (iii) the x -intercept between $-\frac{\pi}{2}$ and 0.	[3]
	(c)	Hence write $f(x)$ in the form $p\sin(qx+r)$.	[3]
	(d)	Write down the x -coordinates of the points where f has a maximum.	[2]
	(e)	Write down the two values of k for which the equation $f(x) = k$ has exactly two	
		solutions.	[2]

39. [Maximum mark: 10] [with GDC]

A spring is suspended from the ceiling. It is pulled down and released, and then oscillates up and down. Its length, l centimetres, is modelled by the function

$$l = 33 + 5\cos\left((720t)^{\circ}\right),\,$$

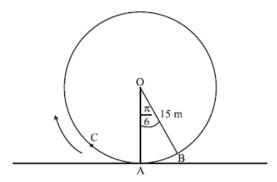
where t is time in seconds after release.

(a) (b) (c) (d)	Find the length of the spring after 1 second. Find the minimum length of the spring. Find the first time at which the length is 33 cm. What is the period of the motion?	[2] [3] [3] [2]

40*. [Maximum mark: 20] [with / without GDC]

A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram

below. Initially seat A is at ground level. The next seat is B, where $\hat{AOB} = \frac{\pi}{6}$.



- (a) Find the length of the arc AB. [2]
- (b) Find the area of the sector AOB. [2]
- (c) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$. Find the height of A above the ground.

[5]

The height, h metres, of seat A above the ground after t minutes, can be modelled by the function $h(t) = -a \cos 2t + c$.

- (d) (i) Find the values of a and c.
 - (ii) Find the time for a complete turn of the wheel.

The height, h metres, of seat C above the ground after t minutes, can be modelled by the function

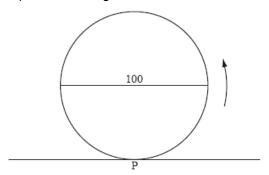
$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right)$$

- (e) (i) Find the height of seat C when $t = \frac{\pi}{4}$.
 - (ii) Find the initial height of seat C.

(iii)	Find the time at which seat C first reaches its highest point.	[8]

41*. [Maximum mark: 14] [without GDC]

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after (i) 10 min; (ii) 15 min.

Let h(t) metres be the height of P above ground level after t minutes. Some values of h(t) are given in the table below.

t	h(t)
0	0.0
1	2.4
2	9.5

t	h(t)
3	20.6
4	34.5
5	50.0

- (b) (i) Show that h(8) = 90.5. (ii) Find h(21). [4]
- (c) **Sketch** the graph of h, for $0 \le t \le 40$. [3]
- (d) Given that h can be expressed in the form $h(t) = a\cos bt + c$, find a, b and c. [5]

42.	[Max	rimum mark: 12] [with GDC]	
	Let	$f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.	
	(a)	On the same diagram, sketch the graphs of f and g .	[3]
	(b)	Consider the graph of f . Write down	
		(i) the x-intercept that lies between $x = 0$ and $x = 3$;	
		(ii) the period;	
		(iii) the amplitude.	[4]
	(c)	Consider the graph of g . Write down	
		(i) the two x -intercepts;	
		(ii) the equation of the axis of symmetry.	[3]
	(d)	Find the x -coordinates of the points of intersection between f and g .	[2]

3.	[Maximum mark: 11] [with GDC]		
	(a)	Consider the equation $x^2 + kx + 1 = 0$. For what values of k does this equation have two equal roots?	[3]
	Let	f be the function $f(\theta) = 2\cos 2\theta + 4\cos \theta + 3$, for $-360^{\circ} \le x \le 360^{\circ}$.	
	(b)	Show that this function may be written as $f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$.	[1]
	(c)	Consider the equation $f(\theta) = 0$, for $-360^{\circ} \le x \le 360^{\circ}$.	
		(i) How many distinct values of $\cos\theta$ satisfy this equation?	
		(ii) Find all values of $ heta$ which satisfy this equation.	[5]
	(d)	Given that $f(\theta) = c$ is satisfied by only three values of θ , find the value of c .	[2]