<u>Transformations of the Sine Function (Vertical Stretch/Shrink)</u>

**Directions**: Open the "Transformations of Sine" GeoGebra activity and use it complete the the following questions.

**Intro:** Remember the definition of sine, $\boldsymbol{sin\theta} = \frac{\boldsymbol{y}}{\boldsymbol{r}}$ , where r is the radius of the circle, and y is the y coordinate of **the point on the circle AND the terminal side of $\boldsymbol{\theta}$.(The blue point).** Solving for y, we have $\boldsymbol{y} = \boldsymbol{rsin\theta}$ , which represents the y coordinate of the blue point. For a given **r**, we can now graph the relationship by plotting points **$(\boldsymbol{\theta}, \boldsymbol{y})$.** On the horizontal axis is the **measure** of **$\boldsymbol{\theta}$,** and the vertical axis is **y** (vertical position of blue point). The collection of all ordered pairs together forms the graph of the function $\boldsymbol{y} = \boldsymbol{rsin\theta}$**.**

**Directions:** Make sure the checkbox for $\boldsymbol{y} = \boldsymbol{rsin}(\boldsymbol{b}(\boldsymbol{\theta} - \boldsymbol{c}))$ is checked. Set **b = 1**, **c = 0** (**b** and **c** have no effect for these values. We will explore these later). Uncheck **plot.**

1. Set **r = 1.** So our function is $\boldsymbol{y} = \boldsymbol{sin\theta}$**.** Use the pink slider to adjust $\boldsymbol{\theta}$ to each of the values in the table below**.** Record in the table the y coordinate of the blue point for each $\boldsymbol{\theta}$. Then plot the ordered pairs from the table on the axes below.

| $\boldsymbol{\theta}$ | $\boldsymbol{y} = \boldsymbol{sin\theta}$ |
|---|---|
| 0 | |
| $\frac{\pi}{2}$ | |
| $\pi$ | |
| $\frac{3\pi}{2}$ | |
| $2\pi$ | |


2. Now check **plot** to and slide $\theta$ to $3\pi$ to graph $\boldsymbol{y} = \boldsymbol{sin\theta}$ for $-\pi \leq \theta \leq 3\pi$. Sketch the wave on the axes above. What are the max and min y values of the function?

3. Now adjust to **r** = **2.** Sketch the function on the axes above. Then do the same for **r = .5**. What is the relationship between the minimum and maximum values of the function and **r?**

# Transformations of Sine (Clockwise Rotation)

**Directions:** Make sure the checkbox for $\boldsymbol{y = rsin(b(\theta - c))}$ is unchecked. Then check the box for the red equation $\boldsymbol{y = -rsin(b(\theta - c))}$. Again set **b = 1**, **c = 0.** Uncheck plot.

**Note:** In the circle, the red angle is $\boldsymbol{-\theta}$, which is a clockwise rotation of the same amount as the counter clockwise rotation by the pink angle $\boldsymbol{\theta}$**.**

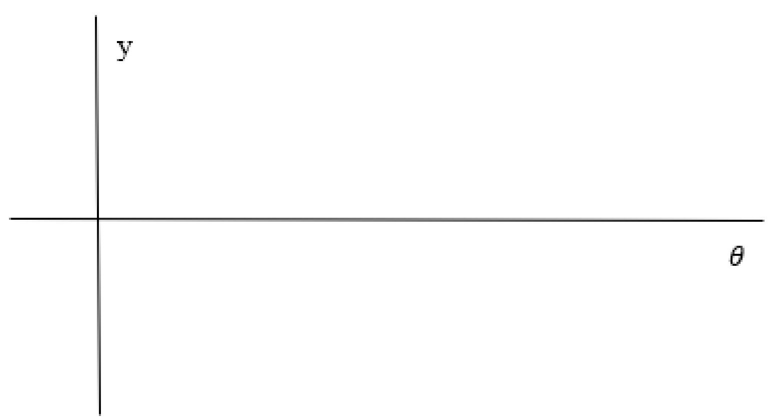
1. Set **r = 1**. Rotate $\boldsymbol{\theta}$ ***to*** $\boldsymbol{\frac{\pi}{4}}$**.** Draw the circle with $\boldsymbol{-\theta}$**,** $\boldsymbol{\theta}$**,** and $\boldsymbol{sin(-\theta)}$ as it appears on the screen

2. Notice the y coordinate of the red point is $y = \boldsymbol{sin(-\theta)}$**.** Since sine is an odd function, we can also write

$$y = \boldsymbol{sin(-\theta)} = \_\_\_\_\_\_\_\_$$

3. Now we can plot ordered pairs $\boldsymbol{(\theta, y)}$ for $y = -\boldsymbol{sin(\theta)}$**.** Use the pink slider to adjust $\boldsymbol{\theta}$ to each of the values in the table below**.** Record in the table the y coordinate of the blue point for each $\boldsymbol{\theta}$. Then plot the ordered pairs from the table on the axes below

| $\boldsymbol{\theta}$ | $\boldsymbol{y = -sin\theta}$ |
|---|---|
| 0 | |
| $\frac{\pi}{2}$ | |
| $\pi$ | |
| $\frac{3\pi}{2}$ | |
| $2\pi$ | |




4. Now check **plot** and slide $\theta$ to $3\pi$ to graph $\boldsymbol{y = -sin\theta}$ for $-\pi \leq \theta \leq 3\pi$. Plot the full wave on the axes above. Now adjust to the following values and plot the waves you see for each value: **r = 1 , r = 2, r = .5.** How do these functions compare with the sketches from page 1? What transformation is applied?

<u>Transformations of Sine: (Frequency Change)</u>

**Directions:** Now uncheck $\boldsymbol{y = -rsin(b(\theta - c))}$ and again check the blue equation $\boldsymbol{y = rsin(b(\theta - c))}$. Set **c = 0** and **r = 1**.

So far we have explored with **b = 1**. In all sketches so far, notice that a full cycle, (**highlighted in yellow)** has a horizontal width of 2π. The horizontal width of the of one cycle is called the **period** of the function. Observing the circle, the **period** is also the value of θ for which the blue point completes a full rotation around the circle.

1. Set b = 2. Slide $\boldsymbol{\theta}$ to different values. Notice the the blue angle $\boldsymbol{2\theta}$ has a measure that is twice the pink angle. Set $\boldsymbol{\theta = \frac{\pi}{3}}$. Sketch the circle below showing $\boldsymbol{\theta, 2\theta, sin(2\theta)}$

2. Notice the y coordinate of the blue point is now is $y = \boldsymbol{sin(2\theta)}$. Use the pink slider to adjust $\boldsymbol{\theta}$ to each of the values in the table below. Record in the table y coordinate of the blue point for each $\boldsymbol{\theta}$. Then plot the ordered pairs from the table on the axes below

| $\boldsymbol{\theta}$ | $\boldsymbol{y = sin2\theta}$ |
|---|---|
| 0 | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{2}$ | |
| $\frac{3\pi}{4}$ | |
| $\pi$ | |


3. Now check **plot** to and slide $\theta$ to $3\pi$ to graph $\boldsymbol{y = sin2\theta}$ for $-\pi \le \theta \le 3\pi$. Sketch the wave on the axes above.

4. Starting at $\theta = 0$, to what value do you slide $\theta$ so that the blue point makes one rotation around the circle?

5. Remember to period is the horizontal width of a full cycle. What is the period of $y = \boldsymbol{sin(2\theta)}$? **Set b = 3** and answer 3 and 4 again. Then **b = .75.** What is the relationship between b and the period?

# Transformations of Sine: (Phase Shift)

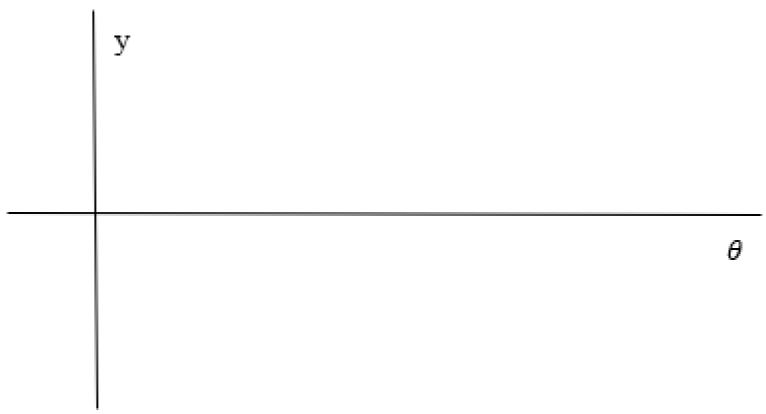
**Directions:** Make sure the checkbox for $\boldsymbol{y = r\sin(b(\theta - c))}$ is checked. Set **b = 1 and r=1**
Uncheck Plot.

**Note:** So far we have explored with **c = 0**. Slide $\boldsymbol{\theta}$ to 0 and now set $\boldsymbol{c} = \frac{\boldsymbol{\pi}}{\boldsymbol{6}}$. Now our equation is $\boldsymbol{y} = \sin\left(\boldsymbol{\theta}\text{-}\ \frac{\boldsymbol{\pi}}{\boldsymbol{6}}\right)$. Adjust $\boldsymbol{\theta}$ and notice that in the circle, the green angle is always $\frac{\boldsymbol{\pi}}{\boldsymbol{6}}$ radians behind $\boldsymbol{\theta}$. So the green angle has measure $\boldsymbol{\theta}\text{-}\ \frac{\boldsymbol{\pi}}{\boldsymbol{6}}$.

1. Set $\boldsymbol{\theta} = \frac{\pi}{3}$ and sketch the circle below showing $\boldsymbol{\theta}$, $\boldsymbol{\theta}\text{-}\ \frac{\boldsymbol{\pi}}{\boldsymbol{6}}$, and $\sin\left(\boldsymbol{\theta}\text{-}\ \frac{\boldsymbol{\pi}}{\boldsymbol{6}}\right)$.

2. Notice the y coordinate of the blue point is now is $y = \boldsymbol{\sin(\theta\text{-}\frac{\pi}{6})}$. Use the pink slider to adjust $\boldsymbol{\theta}$ to each of the values in the table below. Record in the table y coordinate of the blue point for each $\boldsymbol{\theta}$. Then plot the ordered pairs from the table on the axes below

| $\boldsymbol{\theta}$ | $\boldsymbol{y = \sin\left(\theta\text{-}\frac{\pi}{6}\right)}$ |
|---|---|
| 0 | |
| $\frac{\pi}{6}$ | |
| $\frac{2\pi}{3}$ | |
| $\frac{7\pi}{6}$ | |
| $\frac{5\pi}{3}$ | |
| $\frac{13\pi}{6}$ | |




3. Now check **plot** to and slide $\theta$ to $3\pi$ to graph $\boldsymbol{y = \sin\left(\theta\text{-}\frac{\pi}{6}\right)}$ for $-\pi \le \theta \le 3\pi$. Sketch the wave on the axes above.

4. Notice the full cycle highlighted in yellow. Slide c back and forth between 0 and $\frac{\boldsymbol{\pi}}{\boldsymbol{6}}$. What transformation is applied to the highlighted cycle?

5. Now slide c to $-\frac{\boldsymbol{\pi}}{\boldsymbol{6}}$. What is the relationship between the pink and green angle? What transformation is applied to the highlighted cycle now?

6. What is the relationship between c and the graph of $\boldsymbol{y = \sin\left(\theta\text{-}\frac{\pi}{6}\right)}$.