How Many Ways Can You Solve a Quadratic Equation Visually?
Worksheet

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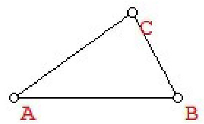
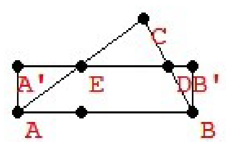
1. Quadrature Problem:
Given a region in the plane, find a root so that the square of this root has the same area.
   a. Quadrature of a Triangle: Given ΔABC, find a square ☐ DEFG with root = DE and area of ☐ DEFG = area of ΔABC
      i. Construct a rectangle equal in area to that of ΔABC



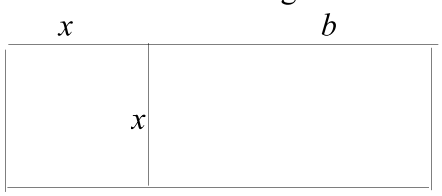
      ii. Construct a square equal in area to the rectangle.



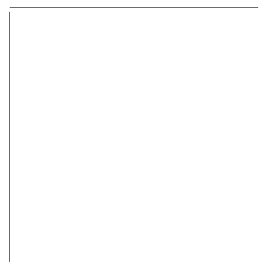
   b. Quadrature for Polygons:

Problem: Find the root of a square that has the same area as a given polygon.
Suggest the outline for a procedure to accomplish the solution of the problem..
Hint: Use triangles and the Pythagorean Theorem.

2. Example for completing the square problem:
[al'Khowarizmi ≈820 AD and al'Khayyam ≈1100 AD.]
Find the root of the square which when added to a rectangle with one side of the same length as the root gives a rectangle of area *c*.

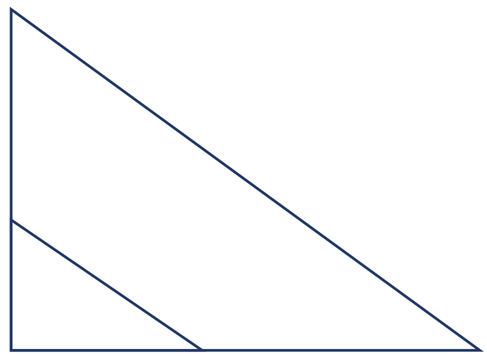


has area *c:*

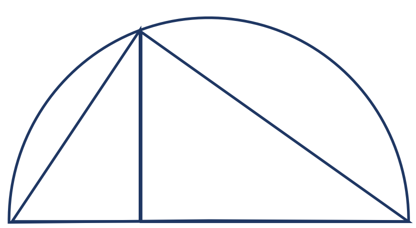


$x^2+bx=c$, has root $x=\sqrt{(b/2)^2+c}-b/2$

3. Descartes Arithmetic for Segments:
   a. Multiplication using a unit segment and proportional sides of similar triangles.
   Label the vertical and horizonal segments in the following figure with lengths $1, x, y$ and xy to illustrate how to construct using similar triangles a segment with length equal to the product of lengths x and y. Discuss how to execute the construction step by step.



   b. Square roots using a unit segment and right triangles in a semicircle.
   Label the vertical and horizonal segments in the following figure with lengths $1, x, \sqrt{x}$ to illustrate how to use a semicircle to construct a segment with that corresponds to the square root of x. Discuss how to execute the construction step by step.



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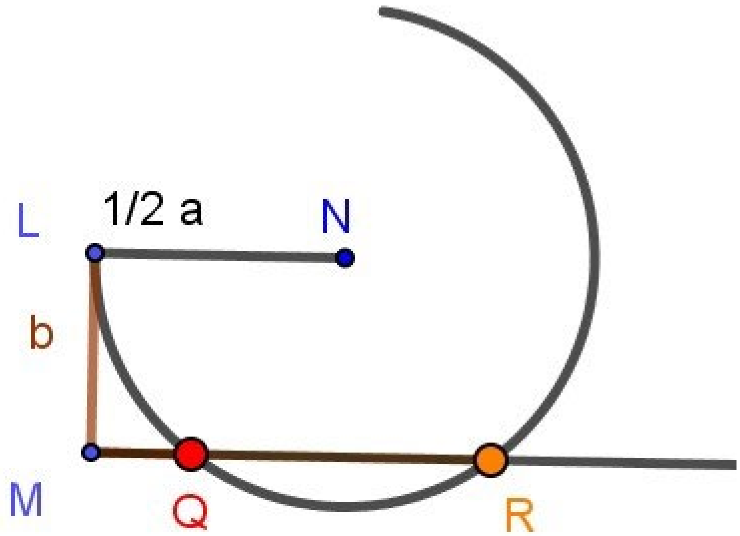
4. Descartes solves a quadratic equation for the arithmetic of segments.

$$z^2 = az - b^2 \ ; b < 1/2\ a$$

Solution:
NL = 1/2 *a*, LM = *b*, NL ⊥ LM, MQR | | LN.
Construct the circle with center N , through L, meeting MQR at Q and R.
Show that MQ and MR are solutions for *z* in the equation.
[Hint: Consider altitude for ΔQRN and the Pythagorean Theorem]



5. Suppose $f(x) = x^2 - 4x + 2$

a. Draw a sketch of the graph $g(x) = f(x) - 2$ by finding the roots of g.

b. Find the axis of symmetry for g and f.

c. Express f in the vertex form ("completing the square").

d. Solve the equation: $f(x) = x^2 - 4x + 2 = 0$.

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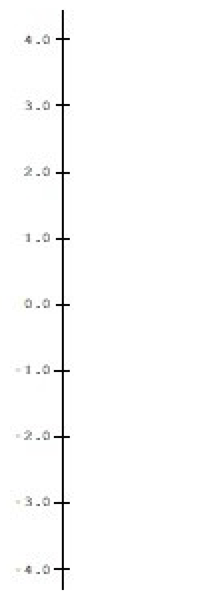
7.

a. Complete the following tables for $m(x) = 2x$ and $s(x) = x + 1$

x	$m(x) = 2x$	$s(x) = x + 1$
2		
1		
0		
-1		
-2		

b. Using the data from part a), on separate diagrams sketch mapping diagrams for $m(x) = 2x$ and $s(x) = x+1$

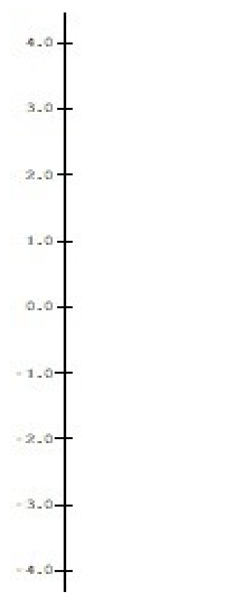


8. Let $q(x) = x^2$.

a. Complete the following table for $q(x) = x^2$.

x	$q(x) = x^2$
2	
1	
0	
-1	
-2	

b. Using the data from part a, sketch a mapping diagram for $q(x) = x^2$.



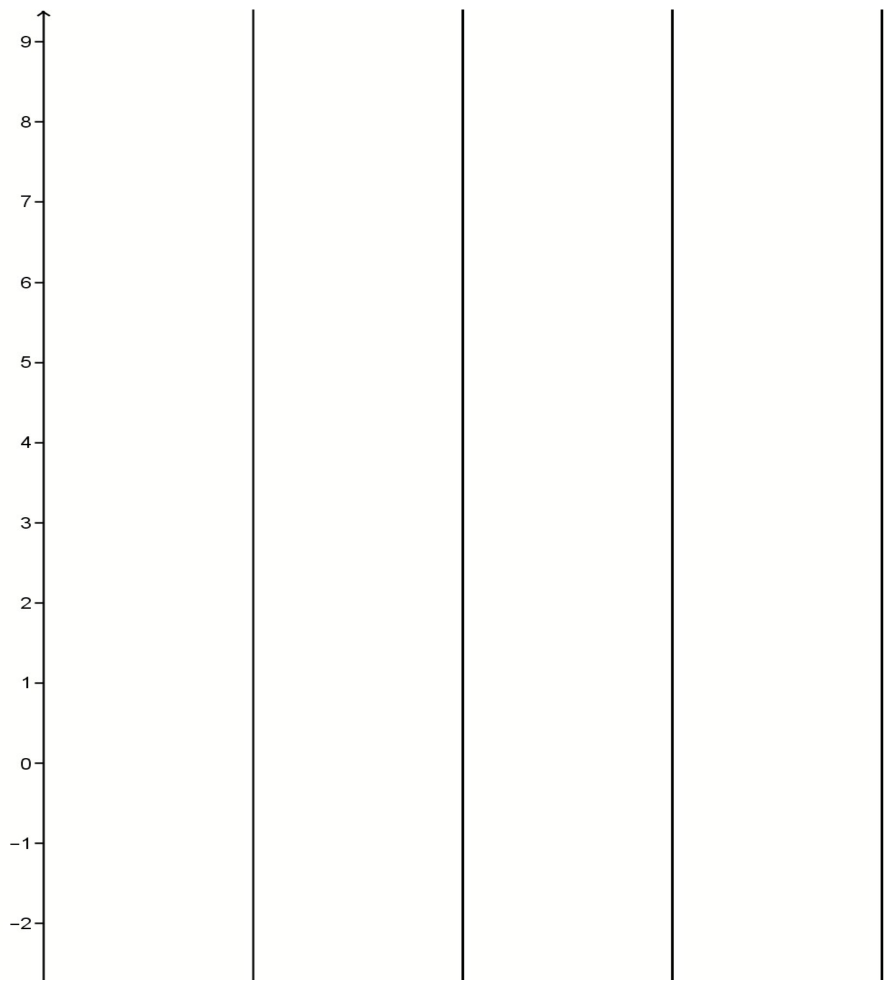
9. Solving $\mathbf{2(x-3)^2+\ 1\ =\ 9}$ **with a mapping diagram.**

a. Express $\boldsymbol{f(x)} = \mathbf{2(x-3)^2+\ 1}$ as composition of core linear and quadratic functions.

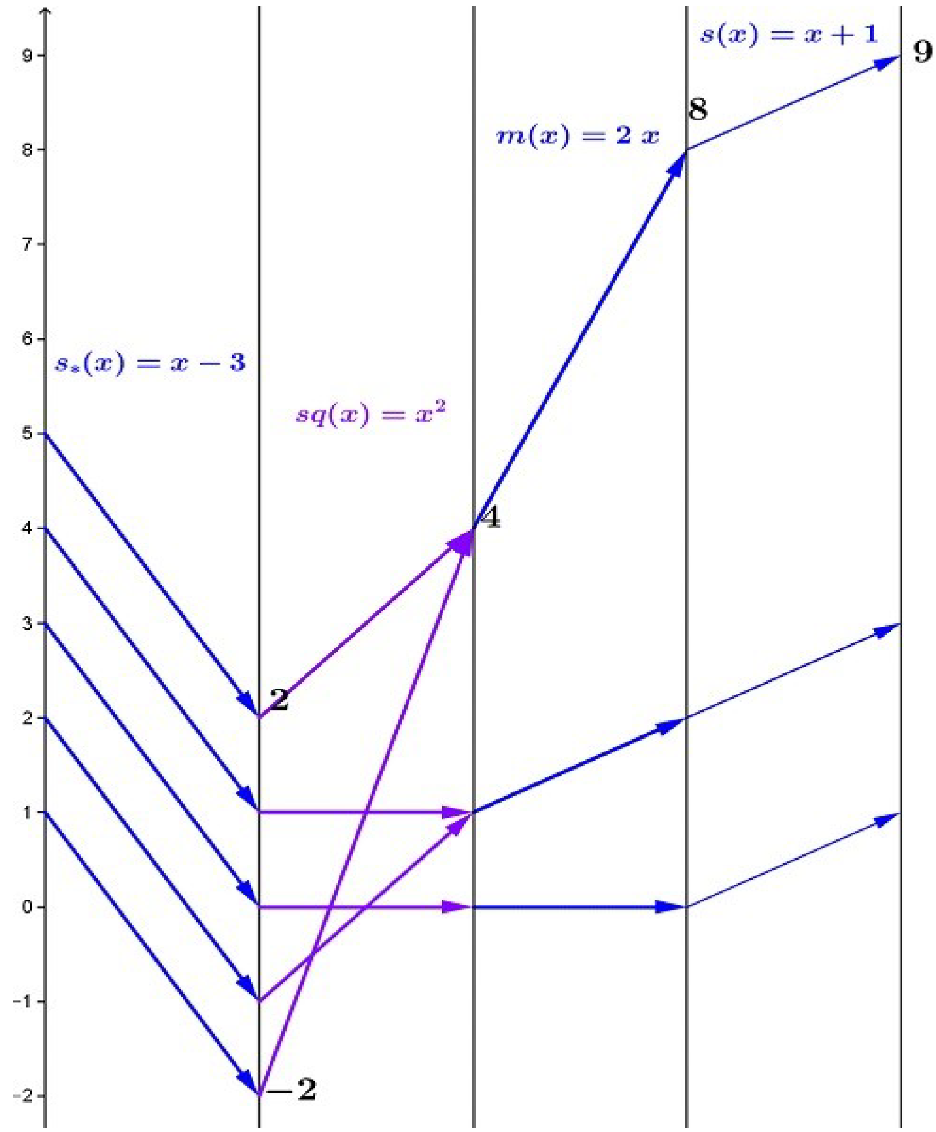
$f(x) = h\left(m\left(q\big(z(x)\big)\right)\right)$ where

$h\ (x) =$
$m\ (x) =$
$q\ (x) =$
z (x) =

b. Sketch a mapping diagram for f **as a composition with** $x =\ 1,2,3,4,5$.



c. **On the mapping diagram below indicate by circling numbers and arrows how the diagram visualizes the solution of $2(x-3)^2 + 1 = 9$. Check the solutions. Discuss how to execute the solution step by step.**



Check: