Do We Need a Separate Philosophy of Geometry (PhoG)?

Dedicated to the Memory of Ruth Favro
Mathematical Friend and Cousin.

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Abstract

The philosophy of mathematics has long been focused primarily on topics such as the ontology of numbers and sets and the epistemology of results in the theory of numbers (arithmetic) and sets through issues of axioms and proofs for these theories. Though much of mathematics today seems to involve geometry in one form or another, the philosophical issues of geometry seem to receive little attention, treated as subservient to the general philosophy of mathematics or considered a part of the philosophy of physics.

The author will consider why the issues of geometry could use a distinct discussion of its philosophical issues for both intrinsic and pedagogical reasons.

Disclaimer

This only sketches an approach to the philosophy of geometry-
It is a sequel borrowing much from my 2010 paper-
The Articulation of Mathematics:
An Everyperson Pragmatic/Constructive Approach to The Philosophy of Mathematics.

It is presented as a continuation after a short period of contemplation- a little over a dozen years later.
As with most philosophy-

Expect more questions than answers. ☺

Outline

- Introduction and Personal History
- Math/Philosophy History (Abbreviated!)
- Teaching Geometry - GeT SLO's
- A (Dynamic) Philosophy of Geometry?
- Conclusion

Introduction: Confessions!

WHAT I AM AND AM NOT.

- I am a retired mathematics professor having taught over 40 years at the undergraduate level.
- I am not a research geometer.
- I do not claim to understand much of the current work in geometry of any of the many distinct and overlapping disciplines.
- I am not active in the research and current discourse of philosophy of mathematics, i.e. Post Quine, Lakatos, et alt
- I have taught undergraduate courses in geometry for math majors and prospective secondary level math teachers.
- I have taught undergraduate courses in the foundations of mathematics and philosophy of mathematics for mathematics majors.
- I have taught undergraduate courses in visual math as a general education courses.

What Is Geometry?

MY ANSWER IN A GEOMETRY COURSE

- **What are different aspects of geometry? How is the study of geometry organized?**
  - **Synthetic (axiomatic), : A geometry that focuses on connecting statements (theorems, constructions) to a foundation of "axioms" by using proofs. Euclid, Hilbert.**
  - **Analytic (numbers) , A geometry that focuses on connecting statements (theorems, constructions) to a foundation of number-based algebra. Descartes**
  - **Transformational. A geometry that focuses on tools (functions) that allow for changing figures:[the basis for studying different geometries in the Klein Erlangen Program.**
    - **Euclidean: translations, rotations, and reflections. Lengths are important**
    - **Similarity: magnifications, dilations. Shape is important**
    - **Affine: Preserve parallel lines. Parallel lines are important.**
    - **Projective: "linear projections"...line preserving. Shadows" are important. Poncelet**
    - **Differential: "smooth".. Curvature is important. Gauss, Riemann**
    - **Topology: continuous. General shape- (especially holes and connectedness)- is important Euler.**
  - **Structural . A geometry that focuses on connecting statements (theorems, constructions) to a foundation of structures (relations and operations) on sets by using proofs. Poincare?**

What Is Geometry?

PHILOSOPHICAL ANSWERS(?)

- **Geometry: An empirical science. Aristotle.**
- **Geometry: Synthetic a prior knowledge. Kant.**
- **Geometry: A formal system of information. Hilbert**
- **Geometry: A convention for scientific discourse. Poincare.**
- **Geometry: A specific kind of knowledge/discourse focused on special objects like triangles, or special qualities like convexity.**
- **Geometry has traditionally been interested in *both***

  •results- like the Pythagorean Theorem- and

  •foundations - using axioms to justify the result in some rigorous organization.

  •applications: primarily physics in the large and small.

What Is Geometry?

MY ANSWER IN A GEOMETRY COURSE

Key Examples for philosophical (ontological and epistemological) issues in geometry.

- **The Pythagorean Theorem**

  •***The statement is about geometric objects (Ontology: right triangles and squares) and equality.***

  • ***What is needed to prove the statement?(Epsitemolgy)***

  - **In Euclid?**
  - **In other proofs?**

- **The construction of an equilateral triangle. Euclid: Book I Proposition I :**
  - **What is needed to accomplish the construction? (Ontology)**
    - **An error In Euclid?**
    - **Is the statement true (Epistemolgy)**
      - **In Euclid's Geometry?**
      - **In Hilbert's Geometry?**

Background-
A quick and incomplete look:
From Euclid to Kant and on to the 20th Century.

Euclid's Elements of Geometry encapsulated in an apparently tight logical web of definitions, axioms, and postulates in an Aristotelian science of space.

This was given almost religious status through many centuries of study, enhancement, and application. Its success allowed other sciences and engineering to flourish through the middle ages and the renaissance, culminating in some ways with the work of Newton.

It gave strong evidence to support Kant's treatment of Euclid's geometry as *a synthetic a priori intuition.*

Background
Philosophical Crises in Euclid's Geometry

1. **The existence of non commensurable line segments.**
   Resolved by the Eudoxian Theory of Proportions.

2. **The three hard problems of SEC constructible figures**
   Resolved as impossible in the 19^{th} century.
   a. ***Duplication of the cube.***
   b. ***Trisection of an angle.***
   c. ***Squaring the circle.***

3. **The development of *consistent geometries that denied the 5^{th} Postulate of Euclid.***

Background
Riemann's Geometric Revolution with Manifolds

Background
Felix Kline and Geometric Transformations

Background
Bertrand Russell and Projective Geometry as the Foundation

Background
David Hilbert and Axioms for “Euclidean” Geometry

Background
George Cantor and the (Geometric) Continuum Problem

Background
Grothendieck: Categorical Geometry and Its Decedents.

Background
David Corfield: Reviving the Philosophy of Geometry (2017)

Chapter 2 in Categories for the Working Philosopher EDITED BY Elaine Landry (Oxford University Press, 2017)

A review of why there is little PhoG, why PhoG is relevant, and what might be a way to revive it as a discipline as long as philosophers immerse themselves in some of the current mathematics of geometry.

> **What I have described in this chapter should suggest that there is a great deal of further work to be done in coming to understand extensions of homotopy type theory, certainly the cohesive variety so far as geometry goes. It should also be noted that with a linear logic variant of homotopy type theory it is possible to express synthetically many aspects of the quantization of higher gauge theory**

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Mathematics is to be understood by the fact that it constitutes a single tradition of intellectual enquiry. Ideas found at particular stages possess the seeds of later formulations, which retrospectively allow us to understand them better.

…it is sometimes revealed during and after moments of synthesis in mathematics that there is a reliance on aspects of cognition, perception, and language, which had possibly gone unnoticed.

Teaching Geometry - GeT SOL's

TEACHING GEOMETRY AT THE UNIVERSITY LEVEL:
IS THERE ANY PHILOSOPHY DISCUSSION IN THESE COURSES?
THE ANSWER: FOR THE MOST PART: NO

- **Differential Geometry**
- **Algebraic Geometry**
- **Projective Geometry (classical-modern)**
- **Topology**
- **Graph Theory**
- **Advanced Euclidean Geometry (Hilbert axioms) ?**
- **Geometry for Teachers (GeT) ?**

Teaching Geometry - GeT SLO's

- **GeT: A Pencil**

https://getapencil.org/

- **an inter-institutional faculty online learning community of**
- **instructors of geometry courses for teachers**
- GeT members share the interest of **improving the geometric preparation of secondary school teachers through stewarding the college geometry courses that future teachers take.** The website disseminates their work to the public and seeks to engage fellow travelers.
- **10 Student Learning Objectives for Geometry for Teachers courses**

Teaching Geometry - GeT SLO's

10 STUDENT LEARNING OBJECTIVES FOR GEOMETRY FOR TEACHERS COURSES

- **SLO 1: Derive and explain geometric arguments and proofs.**
- **SLO 2: Evaluate geometric arguments and approaches to solving problems.**
- **SLO 3: Understand the ideas underlying current secondary geometry content standards and use them to inform their own teaching.**
- ***SLO 4: <u>Understand the relationships between axioms, theorems, and different geometric models in which they hold.</u>**
- ***SLO 5: Understand the role of definitions in mathematical discourse.**

* INDICATES SLO MAY INVOLVE SOME PHILOSOPHICAL DISCUSSION.

Teaching Geometry - GeT SLO's

10 STUDENT LEARNING OBJECTIVES FOR GEOMETRY FOR TEACHERS COURSES

- **SLO 6: Effectively use technologies to explore geometry and develop understanding of geometric relationships.**
- ***SLO 7: Demonstrate knowledge of Euclidean geometry, including the history and basics of Euclid's *Elements* and its influence on math as a discipline.**
- **SLO 8: Be able to carry out basic Euclidean constructions and justify their correctness.**
- ***SLO 9: Compare Euclidean geometry to other geometries such as hyperbolic or spherical geometry.**
- ***SLO 10: Use transformations to explore definitions and theorems about congruence, similarity, and symmetry**

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Technology
20th Century Choices

- Practical computation using technology sets the tone for geometry [and arithmetic]:
- **I.<u>Foundations</u>.**
- <u>Definitions and Axioms</u> that identify the essence of objects by forms without reference to interpretations. (Formal symbolic logic as underlying language?)
- **II.The Web.**
- Propositions/theorems using quantification and logical connections that are proven based on formal logic and foundations set up in I.

Serenity for Working Geometers

- Serenity to accept what we cannot change.

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- ***Courage to change what we can.***

Serenity for Working Geometers

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- *Courage to change what we can.*
- **Wisdom to know the difference.**

"Philosophy of Acceptance" for a Working Geometer

- Some geometrical objects exist.

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- **Ontological commitments come from personal and common experiences.**

Enter-Epistemology

- The theory of knowledge, especially with regard to its methods for ascertaining validity, and their scope.
- Epistemology is the investigation of **what distinguishes justified belief from opinion.**

“Philosophy of Acceptance” for a Working Geometer

- Some geometrical objects exist.
- Ontological commitments come from personal and common experiences.
- **Methods for accessing geometrical knowledge are evolving, not static.**

"Philosophy of Acceptance" for a Working Geometer

- Some geometrical objects exist.
- Ontological commitments come from personal and common experiences.
- Methods for accessing geometrical knowledge are evolving, not static.
- **Epistemological commitments come from personal and common experiences**

“Philosophy of Acceptance” for a Working Geometer

- **Differences and conflicts in ontological and epistemological beliefs and viewpoints are evidence for not looking for a universal foundation for either.**

Response for Everyperson

Geometry **evolves** in a **dynamic** process of **articulation**.

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The result of work in geometry is **an inter-related web or fabric of information-data and concepts.**

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Geometry **evolves** in a **dynamic** process of **articulation**.

The result of work in geometry is **an inter-related web or fabric of information-data and concepts.**

What survives in geometry is a result of a **pragmatic standard founded on scientific empiricism and consistency.**

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Mathematics is to be understood by the fact that it constitutes a single tradition of intellectual enquiry. Ideas found at particular stages possess the seeds of later formulations, which retrospectively allow us to understand them better.

…it is sometimes revealed during and after moments of synthesis in mathematics that there is a reliance on aspects of cognition, perception, and language, which had possibly gone unnoticed

It is surely no accident that mathematicians speak of an 'atlas' to define a manifold, since an ordinary atlas provides a collection of maps which overlap. It seems likely we employ something like this in the cognitive maps by which we navigate our domain. Perhaps one of the invariants of geometry has been found here.

Thank you

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QUESTIONS?
Comments?

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