

INTERNATIONAL BACCALAUREATE  
Mathematics: analysis and approaches

**MAA**

**EXERCISES [MAA 2.7]**

**ASYMPTOTES**

Compiled by Christos Nikolaidis

**O. Practice questions**

1. [Maximum mark: 18] **[without GDC]**

Complete the following table by writing down the corresponding asymptotes

Function	Vertical asymptote	Horizontal asymptote
$f(x) = \frac{2x+7}{x-5}$	$x = 5$	$y = 2$
$f(x) = \frac{x+7}{2x-3}$		
$f(x) = \frac{-4x+1}{2x-6}$		
$f(x) = \frac{2}{x-5}$		
$f(x) = \frac{2}{x-5} + 1$		
$f(x) = \frac{2}{x-5} - 1$		
$f(x) = \frac{2x+7}{x-5} + 1$		
$f(x) = \frac{-4x+1}{2x-6} + 3$		
$f(x) = \frac{5-x}{5+x}$		
$f(x) = \frac{3x+134}{2x+5}$		

2. [Maximum mark: 12] **[with / without GDC]**

Let  $f(x) = \frac{3x - 6}{x + 2}$

- (a) Write down the equations of the asymptotes. [2]
- (b) Find the  $x$ - and  $y$ -intercepts. [2]
- (c) Sketch the graph of the function. Indicate the information found in (a) and (b). [4]
- (d) Write down the domain and the range of the function. [2]
- (e) Find the point of intersection between the graph of  $f$  and the line  $y = 1$ . [2]

.....

.....

.....

.....

.....

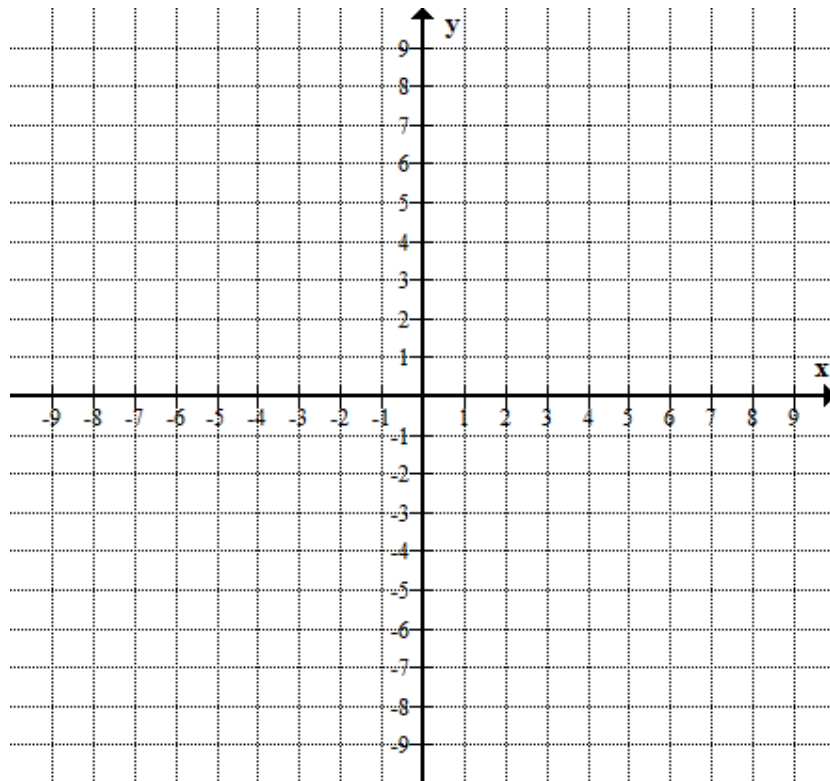
.....

.....

.....

.....

.....



3. [Maximum mark: 10] **[with / without GDC]**

Let  $f(x) = \frac{3}{x-2}$

- (a) Write down the equations of the asymptotes. [2]
- (b) Find any  $x$ - or  $y$ -intercepts. [2]
- (c) Sketch the graph of the function. Indicate the information found in (a) and (b). [4]
- (d) Write down the domain and the range of the function. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

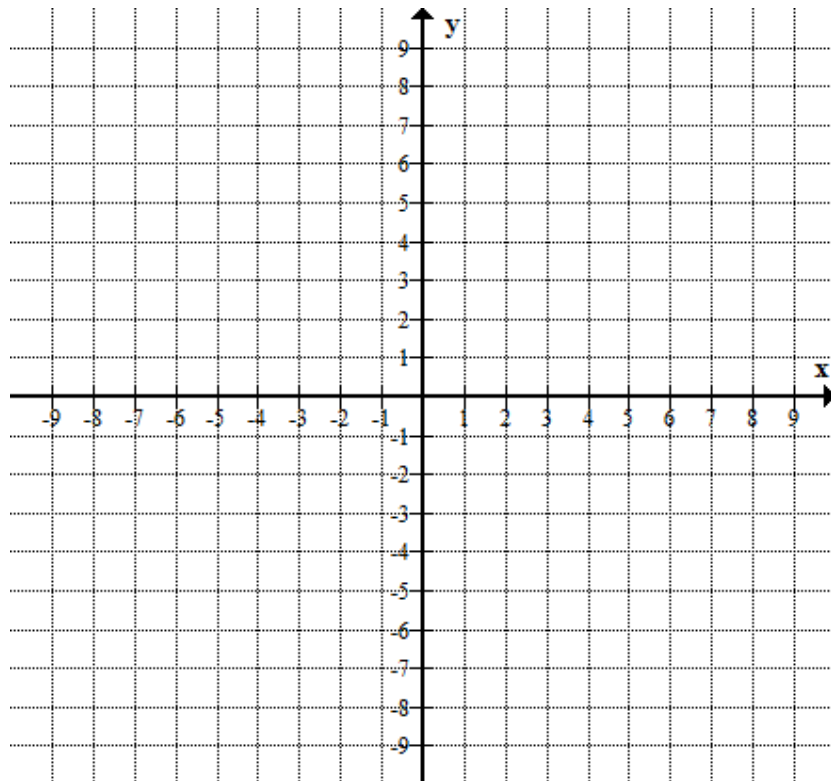
.....

.....

.....

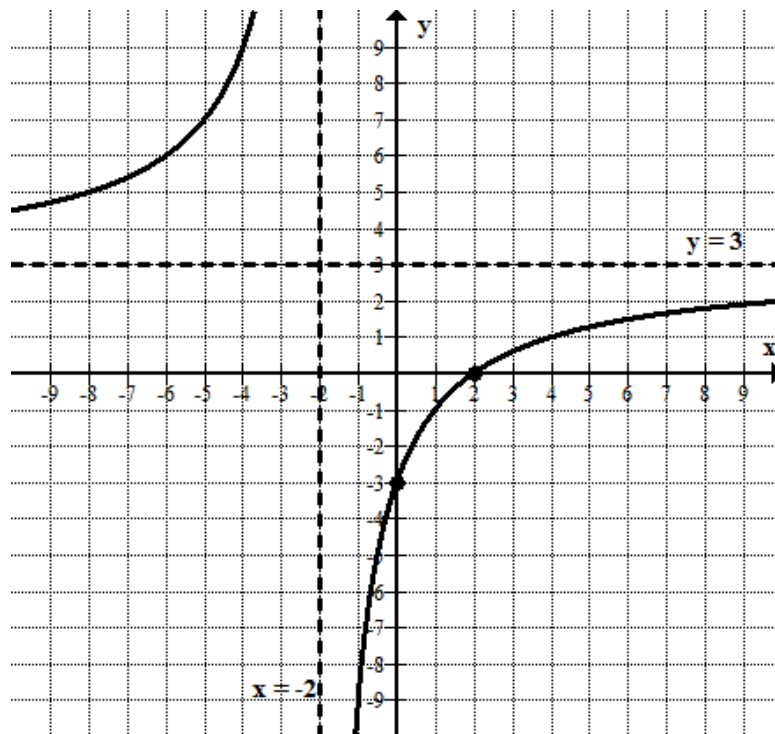
.....

.....



4. [Maximum mark: 8] **[without GDC]**

The graph of the function  $f(x) = \frac{ax+b}{x+c}$  is shown below.



- (a) Find the values of  $a, b, c$ . [4]
- (b) Find  $f(4)$  and  $f^{-1}(1)$ . [2]
- (c) By observing the graph, solve the inequality  $f(x) \leq 1$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

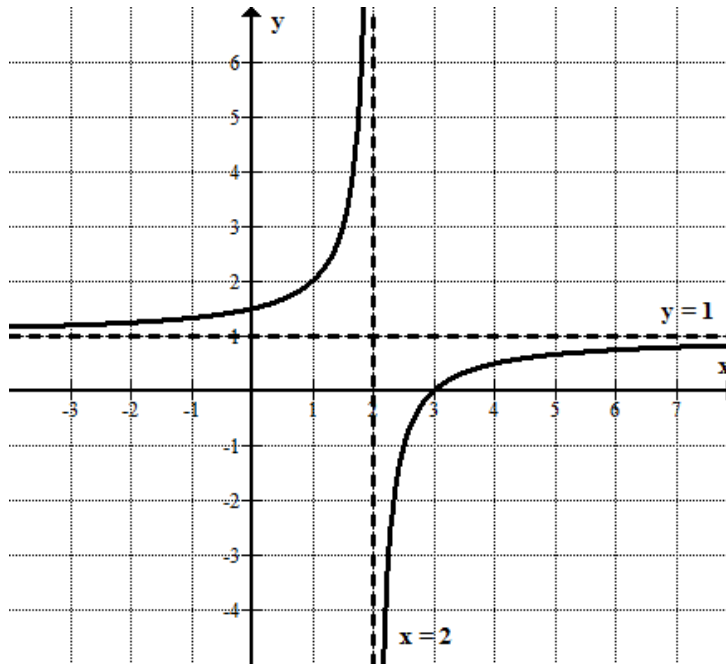
.....

.....

.....

5. [Maximum mark: 19] **[without GDC]**

The graph of the function  $f(x) = \frac{x-6}{x-2}$  is shown below.



- (a) Write down the images of the horizontal asymptote  $y = 1$  under the following transformations

[9]

$y = f(x) + 2$	$y = 3$	$y = f(x + 2)$	
$y = f(x) - 2$		$y = f(x - 2)$	
$y = 2f(x)$		$y = f(2x)$	
$y = f(x)/2$		$y = f(x/2)$	
$y = -f(x)$		$y = f(-x)$	

- (b) Write down the images of the vertical asymptote  $x = 2$  under the following transformations

[10]

$y = f(x) + 2$		$y = f(x + 2)$	
$y = f(x) - 2$		$y = f(x - 2)$	
$y = 2f(x)$		$y = f(2x)$	
$y = f(x)/2$		$y = f(x/2)$	
$y = -f(x)$		$y = f(-x)$	

6\*. [Maximum mark: 10] **[without GDC]**

(a) Express the function  $f(x) = 4 + \frac{5}{x+3}$  in the form  $f(x) = \frac{ax+b}{x+3}$ . [2]

(b) Express the function  $g(x) = \frac{x+10}{x+3}$  in the form  $g(x) = A + \frac{B}{x+3}$ . [2]

(c) Express the function  $h(x) = \frac{2x+10}{x+3}$  in the form  $h(x) = C + \frac{D}{x+3}$ . [2]

(d) Complete the following table

	$f$	$g$	$h$
Vertical asymptote			
Horizontal asymptote			

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7\*. [Maximum mark: 6] **[without GDC]**

Let  $g(x) = 3 + \frac{4}{x-2}$

(a) Write down the equations of the asymptotes of the graph of  $g$ . [2]

(b) Describe the sequence of transformations that map the graph of  $f(x) = \frac{1}{x}$  into the graph of  $g$ . [4]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

8\*\*. [Maximum mark: 7] **[without GDC]**

Let  $h(x) = \frac{3x+1}{x-2}$

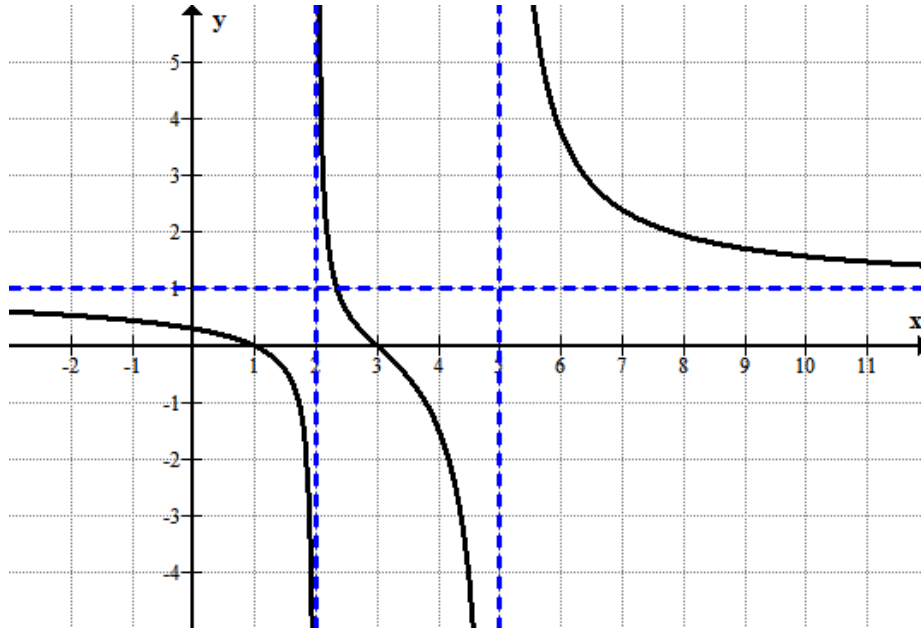
(a) Write down the equations of the asymptotes of the graph of  $h$ . [2]

(b) Describe the sequence of transformations that map the graph of  $f(x) = \frac{1}{x}$  into the graph of  $h$ . [5]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

9\*. [Maximum mark: 7] **[without GDC]**

The graph of a function  $f$  is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

(b) Write down the equations of all vertical asymptotes. [1]

(c) Complete the following table

(1) $\lim_{x \rightarrow +\infty} f(x) =$	(4) $\lim_{x \rightarrow -\infty} f(x) =$
(2) $\lim_{x \rightarrow 2^-} f(x) =$	(5) $\lim_{x \rightarrow 2^+} f(x) =$
(3) $\lim_{x \rightarrow 5^-} f(x) =$	(6) $\lim_{x \rightarrow 5^+} f(x) =$

[3]

(d) State the results from question (c) which justify

(i) the horizontal asymptote

(ii) the vertical asymptotes. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



**A. Exam style questions (SHORT)**

10. [Maximum mark: 6] **[without GDC]**

Consider the graph of  $f(x) = \frac{6x+1}{2x-4}$ .

- (a) Write down the domain and the range of  $f$ . [2]
- (b) Write down the equation of
  - (i) the horizontal asymptote
  - (ii) the vertical asymptote. [2]
- (c) Find the  $x$  – intercept and the  $y$  – intercept. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

11. [Maximum mark: 6] **[without GDC]**

Consider the graph of  $f(x) = \frac{6}{2x-4}$ .

- (a) Write down the domain and the range of  $f$ . [2]
- (b) Write down the equation of
  - (i) the horizontal asymptote
  - (ii) the vertical asymptote. [2]
- (c) Find the  $x$  – intercept and the  $y$  – intercept (if they exist). [2]

.....

.....

.....

.....

.....

.....

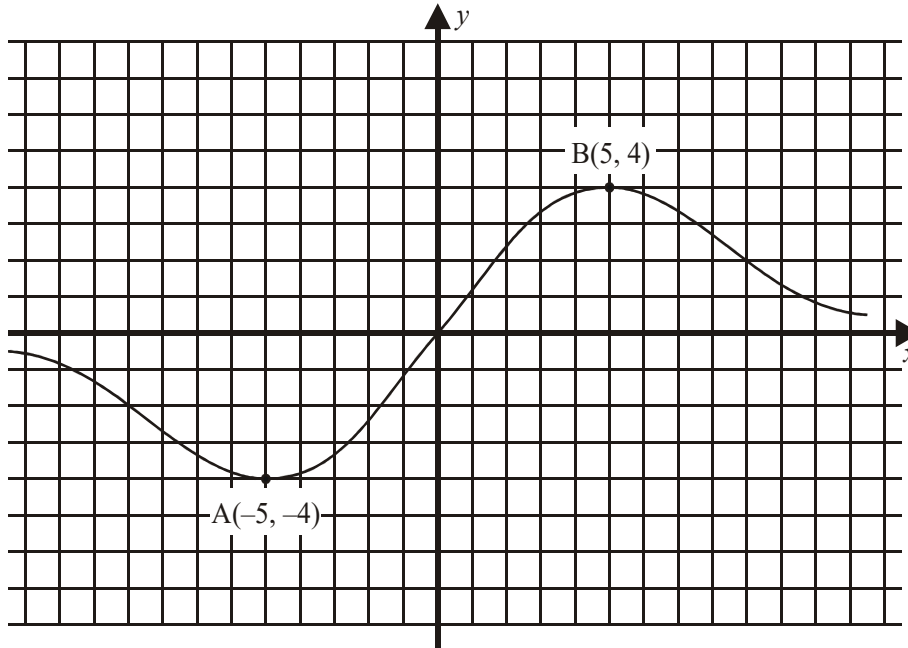
.....

.....

.....

12. [Maximum mark: 6] [without GDC]

The diagram shows the graph of  $y = f(x)$ , with the  $x$ -axis as an asymptote.



- (a) On the same axes, draw the graph of  $y = f(x + 2) - 3$ , indicating the coordinates of the images of the points A and B. [3]
- (b) Write down the equation of the asymptote
- (i) for the graph of  $y = f(x + 2)$ .
  - (ii) for the graph of  $y = f(x) - 3$ .
  - (iii) for the graph of  $y = f(x + 2) - 3$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

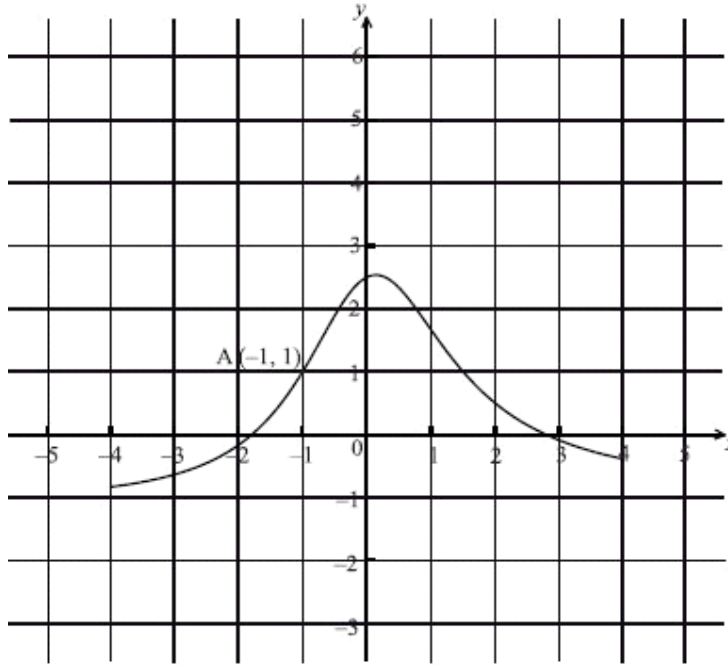
.....

.....

.....

13. [Maximum mark: 8] **[without GDC]**

The graph of a function  $f$  is shown in the diagram below. The point A  $(-1, 1)$  is on the graph, and  $y = -1$  is a horizontal asymptote.



- (a) Let  $g(x) = f(x - 1) + 2$ . On the diagram, sketch the graph of  $g$ . [3]
- (b) Write down the equation of the horizontal asymptote of  $g$ . [1]
- (c) Let  $A'$  be the point on the graph of  $g$  corresponding to point A. Write down the coordinates of  $A'$ . [2]
- (d) Write down the equation of the horizontal asymptote of
  - (i)  $y = f(2x)$ . [2]
  - (ii)  $y = 2f(x)$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

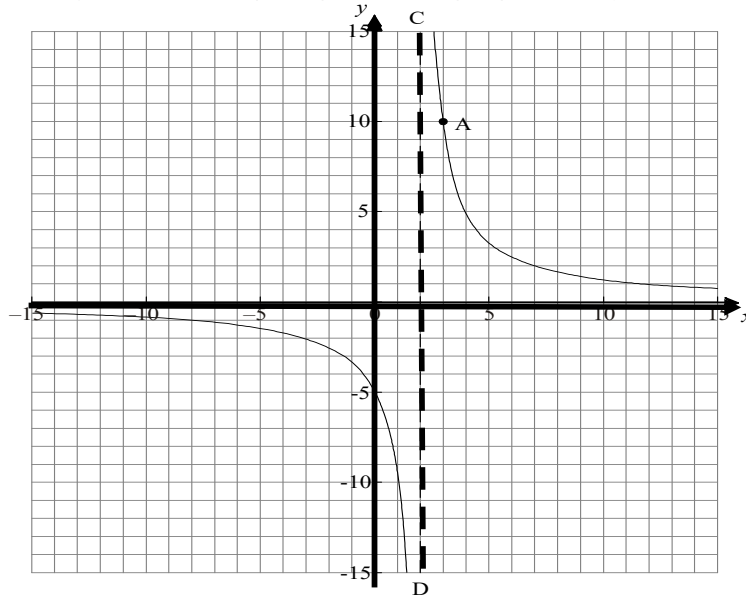
.....

.....

.....

14. [Maximum mark: 6] **[without GDC]**

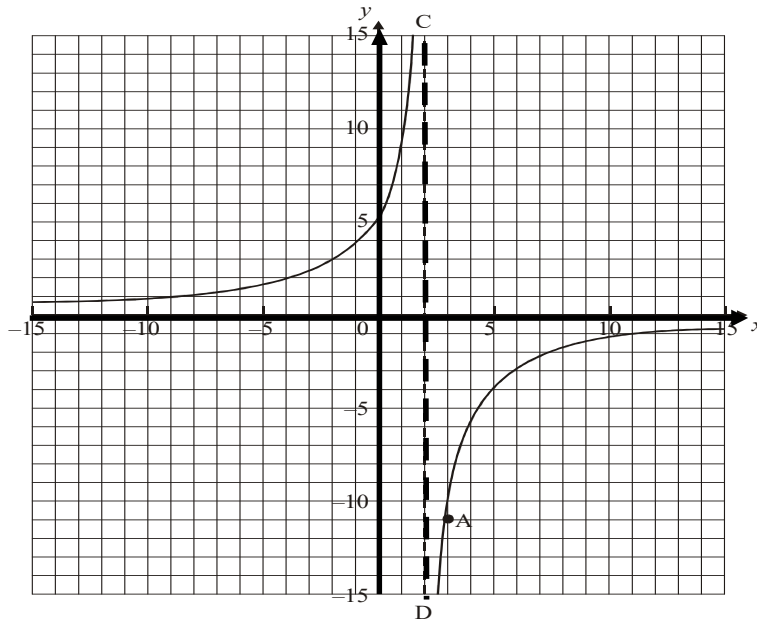
- (a) The diagram shows part of the graph of the function  $f(x) = \frac{q}{x-p}$ . The curve passes through the point A (3, 10). The line (CD) is an asymptote.



Find the value of (i)  $p$  (ii)  $q$

[4]

- (b) The graph of  $f(x)$  is transformed as shown in the following diagram. The point A is transformed to A' (3, -10).



Give a full geometric description of the transformation.

[2]

.....

.....

.....

.....

15. [Maximum mark: 7] **[with / without GDC]**

The function  $f(x)$  is defined as  $f(x) = 3 + \frac{1}{2x-5}$ ,  $x \neq \frac{5}{2}$ .

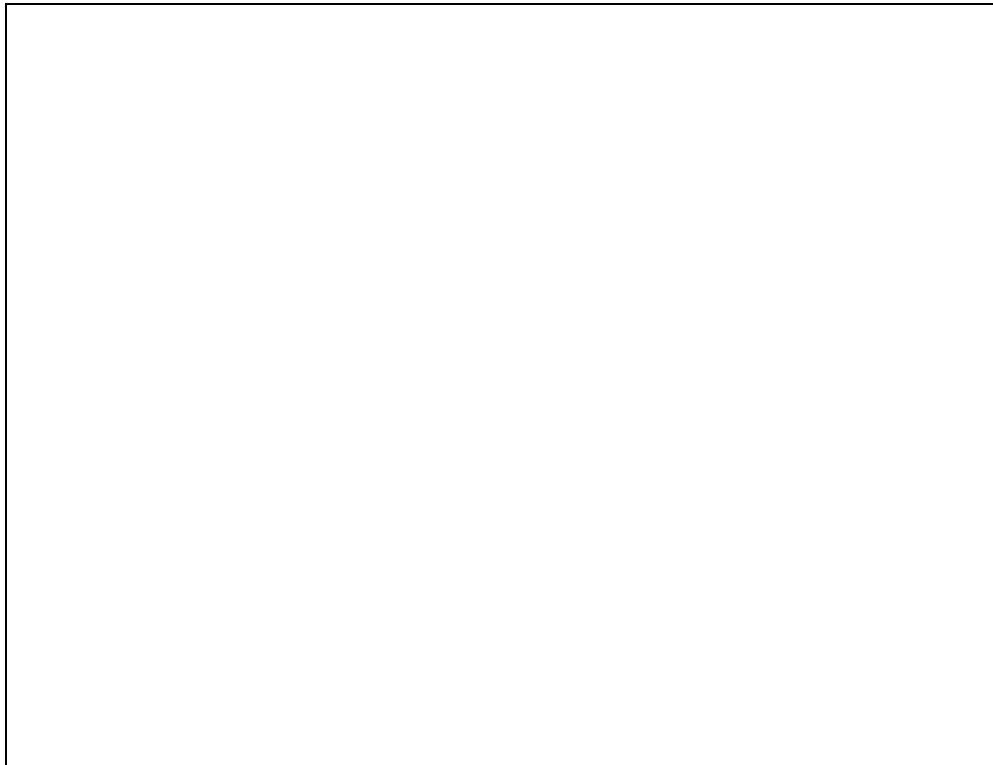
(a) Sketch the curve of  $f$  for  $-5 \leq x \leq 5$ , showing the asymptotes. [3]

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the  $x$ -intercept;

(iii) the value of the  $y$ -intercept. [4]



.....

.....

.....

.....

.....

.....

.....

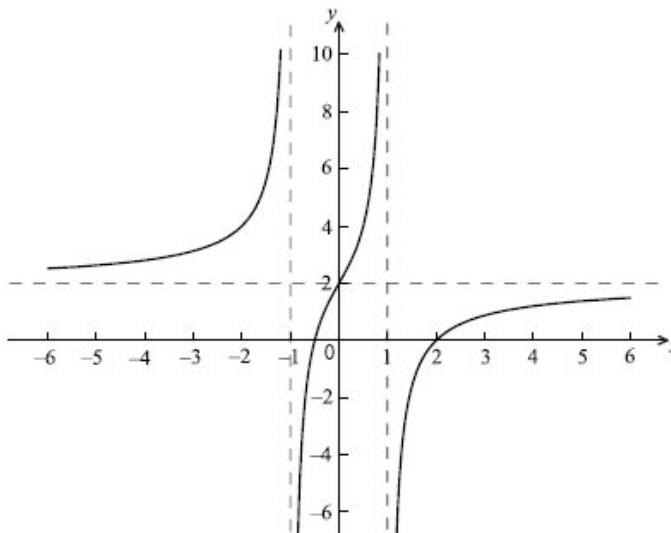
.....

.....

.....

16. [Maximum mark: 4] **[without GDC]**

Let  $f(x) = p - \frac{3x}{x^2 - q^2}$ , where  $p, q \in \mathbb{R}^+$ . Part of the graph of  $f$ , including the asymptotes, is shown below.



The equations of the asymptotes are  $x = 1$ ,  $x = -1$ ,  $y = 2$ .

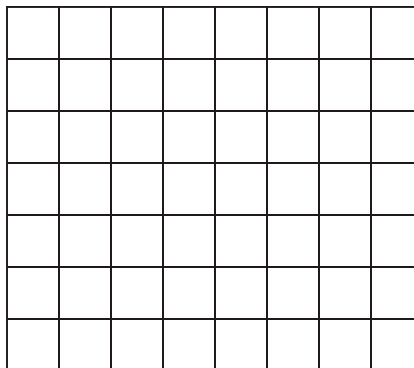
Write down the value of (i)  $p$  (ii)  $q$

.....  
 .....

17\*. [Maximum mark: 6] **[with GDC]**

The function  $f$  is defined by  $f(x) = \frac{3}{\sqrt{9-x^2}}$ , for  $-3 < x < 3$ .

(a) On the grid below, sketch the graph of  $f$ . [2]



(b) Write down the equation of each vertical asymptote. [2]

(c) Write down the range of the function  $f$ . [2]

.....  
 .....

**B. Exam style questions (LONG)**

18. [Maximum mark: 12] **[without GDC]**

The function  $f$  is given by

$$f(x) = \frac{2x+1}{x-3}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

- (a) (i) Show that  $y = 2$  is an asymptote of the graph of  $y = f(x)$ .
- (ii) Find the vertical asymptote of the graph.
- (iii) Write down the coordinates of the point  $P$  at which the asymptotes intersect. [4]
- (b) Find the points of intersection of the graph and the axes. [4]
- (c) Hence sketch the graph of  $y = f(x)$ , showing the asymptotes by dotted lines. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

19. [Maximum mark: 17] **[without GDC]**

Consider the functions  $f$  and  $g$  where  $f(x) = 3x - 5$  and  $g(x) = x - 2$ .

- (a) Find the inverse function,  $f^{-1}$ . [3]
- (b) Given that  $g^{-1}(x) = x + 2$ , find  $(g^{-1} \circ f)(x)$ . [2]
- (c) Given also that  $(f^{-1} \circ g)(x) = \frac{x+3}{3}$ , solve  $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$ . [2]

Let  $h(x) = \frac{f(x)}{g(x)}$ ,  $x \neq 2$ .

- (d) (i) **Sketch** the graph of  $h$  for  $-3 \leq x \leq 7$  and  $-2 \leq y \leq 8$ , including any asymptotes.
- (ii) Write down the **equations** of the asymptotes. [5]
- (e) (i) Find  $h^{-1}(x)$
- (ii) Write down the **equations** of the asymptotes of  $h^{-1}$  [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



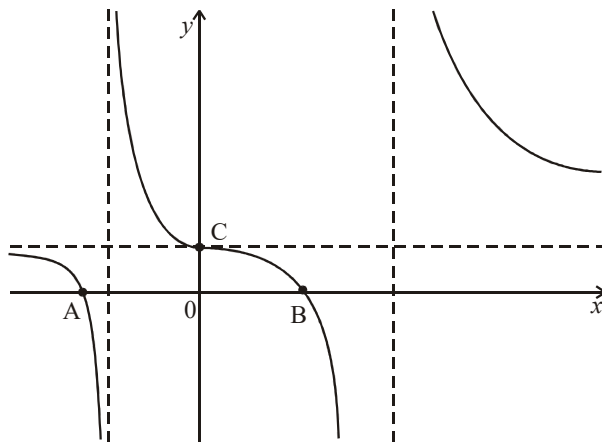
A series of horizontal dotted lines for writing, consisting of 20 lines.

20\*\*. [Maximum mark: 10] [with GDC]

Let  $g(x) = x^4 - 2x^3 + x^2 - 2$

(a) Solve  $g(x) = 0$ . [2]

Let  $f(x) = \frac{2x^3}{g(x)} + 1$ . A part of the graph of  $f(x)$  is shown below.



(b) Write down  
 (i) the domain of  $f$ ;      (ii) the range of  $f$ . [2]

(c) The graph has vertical asymptotes with equations  $x = a$  and  $x = b$  where  $a < b$ . Write down the value of  $a$  and of  $b$ . [2]

(d) The graph has a horizontal asymptote with equation  $y = 1$ . Explain why the value of  $f(x)$  approaches 1 as  $x$  becomes very large. [2]

(e) The graph intersects the  $x$ -axis at the points A and B. Write down the value of the  $x$ -coordinate at (i) A; (ii) B. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....