Linear Transformations

$$\vec{v}' = A\vec{v}$$

Where A maps $\vec{v}$ onto $\vec{v}'$ as follows:

How can we multiply a vector and a matrix? A vector in the coordinate plane is analogous to a column matrix of the same order, making this a possible mathematical operation.

$$\langle x, y \rangle \equiv \begin{bmatrix} x \\ y \end{bmatrix} \qquad \langle x, y, z \rangle \equiv \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

<u>Reflections</u>

$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\vec{v}'$ is a reflection over the x-axis

$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ $\vec{v}'$ is a reflection over the y-axis

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\vec{v}'$ is a reflection over the line y=x

<u>Dilations</u>

$A = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$ $\vec{v}'$ is a horizontal expansion when k>1

$A = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$ $\vec{v}'$ is a vertical expansion when k>1

<u>Contractions</u>

$A = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$ $\vec{v}'$ is a horizontal contraction when 0<k<1

$A = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$ $\vec{v}'$ is a vertical contraction when 0<k<1

Shearing

$A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ $\vec{v}'$ is a horizontal shearing deformation

$A = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ $\vec{v}'$ is a vertical shearing deformation

Rotations

$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\vec{v}'$ is a CCW rotation about the origin θ angular units

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ $\vec{v}'$ is a CCW rotation about the x-axis θ angular units

$A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ $\vec{v}'$ is a CCW rotation about the y-axis θ angular units

$A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a CCW rotation about the z-axis θ angular units

Triangle Area and Convex Tetrahedral Volume

For a triangle with vertices at $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

$$Area = (\pm)\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

For a tetrahedron with vertices at $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4)$

$$Volume = (\pm)\frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$