

# PRAVAC

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# PRAVAC - FORMULE

EXPLICITNI OBLIK JEDNADŽBE  
PRAVCA

$$y = kx + l \rightarrow \text{ODSJEČEK NA OSI } y$$

KOEFICIENT SMJERA

$$\rightarrow k = \operatorname{tg} \alpha$$

KUT KOSI PRAVAC ZATVARA  
S POZITIVNIM DJELOM  
OSI APSCISA

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

IMPLICITNI OBLIK JEDNADŽBE  
PRAVCA

$$Ax + By + C = 0$$

SEGMENTNI OBLIK JEDNADŽBE  
PRAVCA

$$\frac{x}{m} + \frac{y}{n} = 1 \rightarrow (m, 0) \text{ i } (0, n) \text{ TOČKE U KOSIMA  
PRAVAC SJEČE KOORDINATNE OSI}$$

KUT IZMEĐU PRAVACA  $\mu_1$  i  $\mu_2$

$$\operatorname{tg} \varphi = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$$

UVJET PARALELNOSTI

$$k_1 = k_2$$

UDALJENOST TOČKE OD PRAVCA

$$Ax + By + C = 0$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

UVJET OKOMITOSTI

$$k_1 k_2 = -1$$

1. KAKO GLASI JEDNAŽBA PRANCA KOJI PROLAZI TOČKOM  $D\left(-\frac{2}{3}, 1\right)$ , A PARALELAN JE S PRANCEM

$$3x - 4y - 5 = 0$$

$$y = kx + l$$

$$1 = \frac{-2}{3} \cdot \frac{-2}{3} + l$$

$$3x - 4y - 5 = 0$$

$$1 = \frac{-1}{2} + l$$

$$4y = 3x - 5$$

$$l = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

$$y = \frac{3}{4}x - \frac{5}{4}$$

$$\rightarrow k = \frac{3}{4}$$

$$y = \frac{3}{4}x + \frac{3}{2}$$

2. DVIJE STRANICE KVADRATA PRIPADAJU PRANCIIMA  $4x - 3y + 11 = 0$  I  $4x - 3y + 1 = 0$ . KOLIKA JE PLOŠTA KVADRATA.

$$4x - 3y + 11 = 0 \rightarrow 3y = 4x + 11 \rightarrow y = \frac{4}{3}x + \frac{11}{3}$$

$$4x - 3y + 1 = 0 \rightarrow 3y = 4x + 1 \rightarrow y = \frac{4}{3}x + \frac{1}{3}$$

$$T\left(0, \frac{1}{3}\right)$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|4 \cdot 0 - 1 + 11|}{\sqrt{16 + 9}}$$

$$= \frac{|-1 + 11|}{5} = \frac{10}{5} = 2$$

$$P = 2 \cdot 2$$

$$P = 4 \text{ kvj}$$

3. ODREDITE SVE PARAMETRE  $a \in \mathbb{R}$  TAKO DA POUŠINA  
 TROKUTA KOSEK PRANAC  $y = -3x + a$  ZATVARA S  
 KOORDINATNIM OSIMA BUDE JEDNAKA  $\frac{2}{3}$ .

$$P = \frac{|m \cdot m|}{2}$$

$$y = -3x + a$$

$$3x + y - a = 0$$

$$3x + y = a \quad /: a \neq 0$$

$$\frac{x}{\frac{a}{3}} + \frac{y}{a} = 1$$

$$\frac{2}{3} = \frac{\left| \frac{a}{3} \cdot a \right|}{2}$$

$$\frac{4}{3} = \left| \frac{a^2}{3} \right|$$

$$\frac{a^2}{3} = \frac{4}{3}$$

$$\frac{a^2}{3} = -\frac{4}{3}$$

$$a = \pm 2$$

4. ODREDITE SVE PARAMETRE  $m \in \mathbb{R}$  TAKO DA PRANCI

$3x + y = m$  I  $x + my = 2$  BUDU PARALELNI.

$$y = -3x + m$$

$$my = 2 - x$$

$$y = \frac{2 - x}{m}$$

$$y = \frac{-1}{m}x + \frac{2}{m}$$

$$k_1 = k_2$$

$$-3 = \frac{-1}{m}$$

$$-3m = -1$$

$$m = \frac{1}{3}$$

5. PRAVAC PROLAZI TOČKOM  $P(8,6)$  I S KOORDINATNIM OSIMA TVORI TROKUT PLOŠTINE 12. KAKO GLASI JEDNAČBA PRAVCA?

$$\frac{x}{m} + \frac{y}{m} = 1$$

$$P = \frac{|m \cdot m|}{2}$$

$$2h = \left| \frac{8m}{m-6} \cdot \frac{m}{1} \right|$$

$$2h = \left| \frac{8m^2}{m-6} \right|$$

$$\frac{8m^2}{m-6} = \frac{-24}{1}$$

$$8m^2 = 24m - 144$$

$$8m^2 - 24m + 144 = 0$$

~~$$m_1 = 6$$~~

~~$$m_2 = 12$$~~

$$\frac{8}{m} + \frac{6}{m} = 1$$

$$\frac{8}{m} = 1 - \frac{6}{m}$$

$$\frac{8}{m} = \frac{m-6}{m}$$

$$mm - 6m = 8m$$

$$m(m-6) = 8m$$

$$m = \frac{8m}{m-6}$$

$$m = \frac{8 \cdot 3}{3-6} = \frac{24}{-3}$$

$$m_1 = -8$$

$$m_2 = 4$$

$$8m^2 = -24m + 144$$

$$8m^2 + 24m - 144 = 0$$

$$m_1 = 3 \quad m_2 = -6$$

$$\frac{x}{-8} + \frac{y}{3} = 1$$

$$\frac{x}{4} + \frac{y}{-6} = 1$$

6. NAPIŠI JEDNADŽBU PRAVKA KOJI JE PARALELAN PRAVCU

$3x - 4y - 10 = 0$  I KOJI JE OD OVOG UDALJEN ZA 3.

$$4y = 3x - 10$$

$$y = \frac{3}{4}x - \frac{5}{2}$$

$$y = \frac{3}{4}x + c$$

$$T(0, -\frac{5}{2})$$

$$\frac{3}{4}x - y + c = 0 \quad | \cdot 4$$

$$3x - 4y + 4c = 0$$

$$k_1 = k_2$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$3 = \frac{|10 + 10 + 4c|}{\sqrt{3^2 + (-4)^2}}$$

$$3 = \frac{|10 + 4c|}{5}$$

$$15 = |10 + 4c|$$

$$10 + 4c = 15$$

$$4c = 5$$

$$c = \frac{5}{4}$$

$$3x - 4y - 25 = 0$$

$$10 + 4c = -15$$

$$4c = -25$$

$$c = -\frac{25}{4}$$

$$3x - 4y + 5 = 0$$

7. TOČKA  $A(x, y)$  PRIPADA PRAVCU  $3x - 2y + 5 = 0$ , DOK MU TOČKA  $B(-3, 8)$  NE PRIPADA. KOLIKI JE KOEFICIJENT SMJERA PRAVCA KROZ TOČKE A I B?

$$3x - 2y + 5 = 0$$

$$2y = 3x + 5$$

$$y = \frac{3}{2}x + \frac{5}{2}$$

$$y = \frac{3}{2}x + \frac{5}{2}$$

$$\frac{3}{2}x = \frac{y}{1} - \frac{5}{2}$$

$$\frac{3}{2}x = \frac{3}{2}$$

$$x = 1$$

$$A(1, 4)$$

$$B(-3, 8)$$

$$y = kx + l$$

$$4 = k + l \quad | \cdot (-1)$$

$$8 = -3k + l$$

$$4 = -1 + l$$

$$l = 5$$

$$-4 = -k - l$$

$$8 = -3k + l$$

$$4 = -4k$$

$$-4k = 4$$

$$k = -1$$

$$y = -x + 5$$

8. NACRTAJ GRAF FUNKCIJE

$$y = |3 - 2x|$$

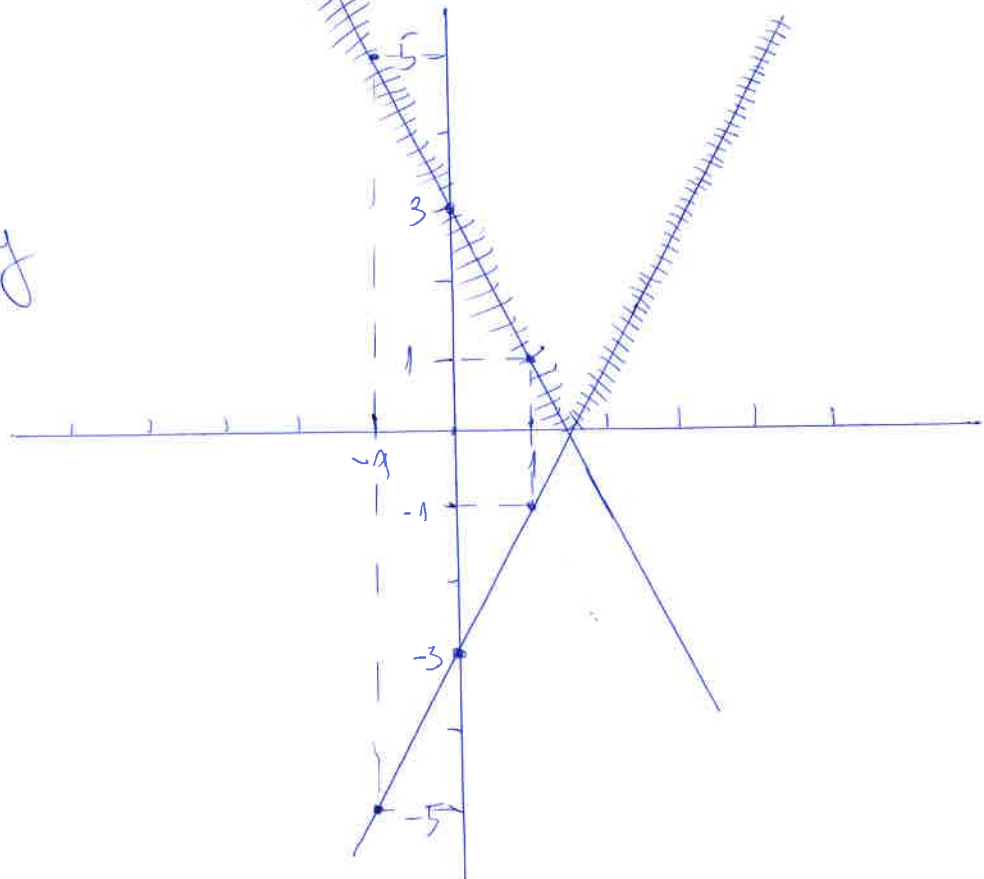
$$3 - 2x = y$$

$$3 - 2x = -y$$

| x  | y |
|----|---|
| 1  | 1 |
| -1 | 5 |
| 0  | 3 |

| x  | y  |
|----|----|
| 1  | -1 |
| -1 | -5 |
| 0  | -3 |

$$f(x) = |3 - 2x|$$



3. TOČKOU  $T(-2, 1)$  POLOŽÍ PRÁVICI  $K$  S PRAVKEM  $2x + 3y + 6 = 0$  ZATVĚRA KÚÍ OD  $45^\circ$ . ODŘEDI JEDNADŽBU TĚH PRÁVCE.

$$\tan \varphi = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$$

$$2x + 3y + 6 = 0$$

$$3y = -2x - 6$$

$$y = -\frac{2}{3}x - 2$$

$$1 = \left| \frac{-\frac{2}{3} - k_1}{1 - \frac{2}{3}k_1} \right|$$

$$1 = \left| \frac{-2 - 3k_1}{3 - 2k_1} \right|$$

$$1 = \left| \frac{-2 - 3k_1}{3 - 2k_1} \right|$$

$$\frac{-2 - 3k_1}{3 - 2k_1} = 1 \quad \frac{-2 - 3k_1}{3 - 2k_1} = -1$$

$$-2 - 3k_1 = 3 - 2k_1$$

$$-2 - 3k_1 = -3 + 2k_1$$

$$-k_1 = 5$$

$$-5k_1 = -1$$

$$k_1 = -5$$

$$5k_1 = 1$$

$$k_1 = \frac{1}{5}$$

$$k = 10 + e$$

$$Q = -9$$

$$y = -5x - 9$$

$$5x + y + 9 = 0$$

$$1 = \frac{-2}{5} + e$$

$$Q = \frac{1}{1} + \frac{2}{5} = \frac{7}{5}$$

$$y = \frac{1}{5}x + \frac{7}{5}$$

$$\frac{1}{5}x - y + \frac{7}{5} = 0 \quad / \cdot 5$$

$$x - 5y + 7 = 0$$



10. NULTOČKOM PRAVCA  $x - 2y + 6 = 0$  POLOŽI PRAVAC KOSI  
 ČE S OSI APSCISA ZATVARATI TROSTRUKO VEĆI KUT  
 NEGO ZADANI PRAVAC.

$$2y = x + 6$$

$$y = \frac{1}{2}x + 3$$

$$0 = \frac{1}{2}x + 3$$

$$\frac{1}{2}x = -3$$

$$x = -6$$

$T(-6, 0)$   
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 NULTOČKA

$$\operatorname{tg} p = k$$

$$p = 26^\circ 33' 54''$$

$$p' = 3p$$

$$p' = 79^\circ 41' 42.55''$$

$$k' = 5.5$$

$$0 = -33 + e$$

$$e = 33$$

$$y = 5.5x + 33$$

$$5.5x - y + 33 = 0 \quad | :2$$

$$11x - 2y + 66 = 0$$

11. ZA KOSI SE NAJMANJI KUT MORA ZAKRENUTI PRAVAC  
 $y = -2x + 4$  OKO SVOJEG SJECIŠTA S OSI ORDINATA  
 KAKO BI PROŠAO TOČKOM  $T(3, 3)$ ?

$$y = -2x + 4$$

$$y = kx + e$$

$$3 = 3k + 4$$

$$3k + 4 = 3$$

$$3k = -1$$

$$k = -\frac{1}{3}$$

$$\operatorname{tg} p = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$$

$$\operatorname{tg} p = \left| \frac{-\frac{2}{1} + \frac{1}{3}}{\frac{1}{1} + \frac{2}{3}} \right|$$

$$\operatorname{tg} p = \left| \frac{\frac{-8}{3}}{\frac{5}{3}} \right|$$

$$\operatorname{tg} p = 1$$

$$p = 45^\circ$$

12. PRAMCI  $2x+y+3=0$  I  $ax+3y+3=0$  NEĐUSOBNO SU OKOMITI. ODREDI NJIHOVO SRECIŠTE.

$$y = -2x - 3$$

$$3y = -ax - 3$$

$$y = \frac{-a}{3} - 1$$

$$k_1 = -\frac{1}{k_2}$$

$$-2 = \frac{3}{a}$$

$$-2a = 3$$

$$a = -\frac{3}{2}$$

$$y = \frac{1}{2} \cdot \frac{-4}{5} - \frac{1}{1}$$

$$y = \frac{-4}{10} - \frac{1}{1} = \frac{-4-10}{10} = \frac{-14}{10} = \frac{-7}{5}$$

$$y = \frac{-7}{5}$$

$$y = -2x - 3 \quad | \cdot (-1)$$

$$y = \frac{1}{2}x - 1$$


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$$-x = 2x + 3$$

$$y = \frac{1}{2}x - 1$$


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$$\frac{5}{2}x + 2 = 0$$

$$\frac{5}{2}x = -2$$

$$x = \frac{-2}{1} \cdot \frac{2}{5} = \frac{-4}{5}$$

13. NAĐI JEDNAŽBE SIMETRAZA KUTA PRI VERTU A TROKUTA ABC, AKO JE A(1,-2), B(5,4), C(-2,0)

$$y = kx + e$$

$$y = \frac{3}{2}x - \frac{7}{2}$$

$$\frac{3}{2}x - y - \frac{7}{2} = 0 \quad | \cdot 2$$

$$3x - 2y - 7 = 0$$

$$-2 = k + e \quad | \cdot (-1) \quad \boxed{3x - 2y - 7 = 0}$$

$$4 = 5k + e$$


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$$2 = -k - e$$

$$4 = 5k + e$$


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$$4k = 6$$

$$k = \frac{3}{2}$$

$$4 = \frac{15}{2} + e$$

$$e = \frac{-7}{2}$$

UNUTARNJEG I VANJSKOG KUTA, AKO JE A(1,-2),

$$-2 = k + e \quad | \cdot (-1)$$

$$0 = -2k + e \quad | \cdot 2 \quad 0 = \frac{4}{3} + e$$

$$2 = -k - e \quad | \cdot 2 \quad e = \frac{-4}{3}$$

$$0 = -2k + e$$


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$$2 = -3k$$

$$3k = -2$$

$$k = \frac{-2}{3}$$

$$y = \frac{-2}{3}x - \frac{4}{3}$$

$$\frac{2}{3}x + y + \frac{4}{3} = 0 \quad | \cdot 3$$

$$\boxed{2x + 3y + 4 = 0}$$

$$\frac{|A_1x + B_1y + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2x + B_2y + C_2|}{\sqrt{A_2^2 + B_2^2}}$$

$$\frac{|3x - 2y - 7|}{\sqrt{13}} = \frac{|2x + 3y + 4|}{\sqrt{13}}$$

$$|3x - 2y - 7| = |2x + 3y + 4|$$

$$3x - 2y - 7 = -2x - 3y - 4$$

$$5x + y - 3 = 0$$

$$3x - 2y - 7 = 2x + 3y + 4$$

$$x - 5y - 11 = 0$$

14. NAPIŠI JEDNAŽBU SIMETRALE KUTA  $\alpha$  ŠTO GA ČATVARAJU  
PRANCI  $3x - y - 4 = 0$  I  $2x + 6y + 3 = 0$ , AKO ISHODIŠTE  
KOORDINATNOG SUSTAVA PRIPADA KUTU  $\alpha$ .

$$\frac{|A_1x + B_1y + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2x + B_2y + C_2|}{\sqrt{A_2^2 + B_2^2}}$$

$$\frac{|3x - y - 4|}{\sqrt{10}} = \frac{|2x + 6y + 3|}{\sqrt{40}}$$

$$2(3x - y - 4) = 2x + 6y + 3$$

$$6x - 2y - 8 = 2x + 6y + 3$$

$$4x - 8y - 11 = 0$$

$$-2(3x - y - 4) = 2x + 6y + 3$$

$$-6x + 2y + 8 = 2x + 6y + 3$$

$$-8x - 4y + 5 = 0$$

$$\boxed{8x + 4y - 5 = 0}$$

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VIDJIMO IZ SKICE