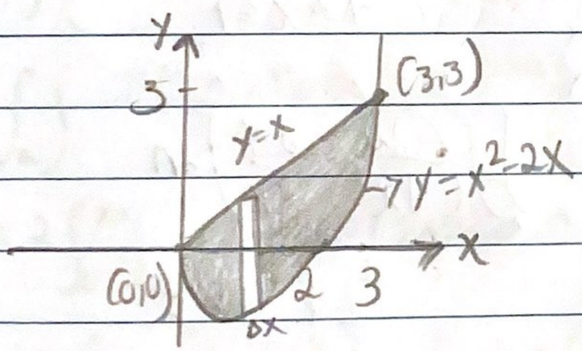
# Ejercicios de la unidad 15.4
## Teorema de Green.

En los ejercicios 7 a 10, utilizar el Teorema de Green para evaluar la integral

$\int_C \underbrace{(y-x)}_{M} dx + \underbrace{(2x-y)}_{N} dy$, sobre la trayectoria dada.

7) C: Frontera de la región comprendida entre las gráficas de $y = x$ y $y = x^2 - 2x$




$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$\frac{\partial N}{\partial x} = 2 \quad , \quad \frac{\partial M}{\partial y} = 1$$

$$\iint_R (2-1)\, dA = \iint_R 1\, dA$$

$R: x^2-2x \leq y \leq x$

$0 \leq x \leq 3$

$$\int_0^3 \int_{x^2-2x}^{x} 1\, dy\, dx = \quad I = \int_{x^2-2x}^{x} 1\, dy = y\Big|_{x^2-2x}^{x}$$

$$= x - (x^2 - 2x)$$

$$= x - x^2 + 2x = 3x - x^2$$

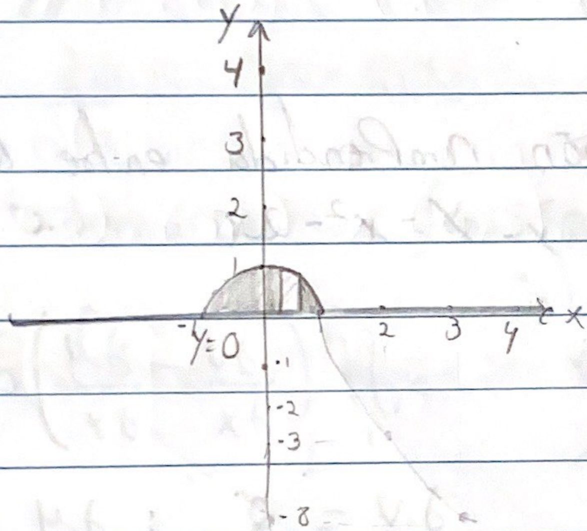
$$\int_0^3 3x - x^2\, dx = \frac{3x^2}{2} - \frac{x^3}{3}\Big|_0^3 = \frac{3}{2}(3)^2 - \frac{1}{3}(3)^3$$

$$= \frac{27}{2} - 9 = \boxed{\frac{9}{2}}$$

En los ejercicios 11 a 20, utilizar el Teorema de Green para evaluar la integral de línea

11) $\int_C \overbrace{2xy}^{M}\,dx + \overbrace{(x+y)}^{N}\,dy$

C: frontera de la región comprendida entre las gráficas de $y=0$ y $y=1-x^2$




$y=1-x^2$

$R: \ 0 \le y \le 1-x^2$

$-1 \le x \le 1$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$\frac{\partial N}{\partial x} = 1 \quad , \quad \frac{\partial M}{\partial y} = 2x$

$$\int_{-1}^{1}\int_{0}^{1-x^2} 1-2x\,dy\,dx = I = \int_0^{1-x^2} 1-2x\,dy = 1-2x\left[y\right]_0^{1-x^2}$$

$= (1-2x)(1-x^2)$

$= (1-x^2-2x+2x^3)$

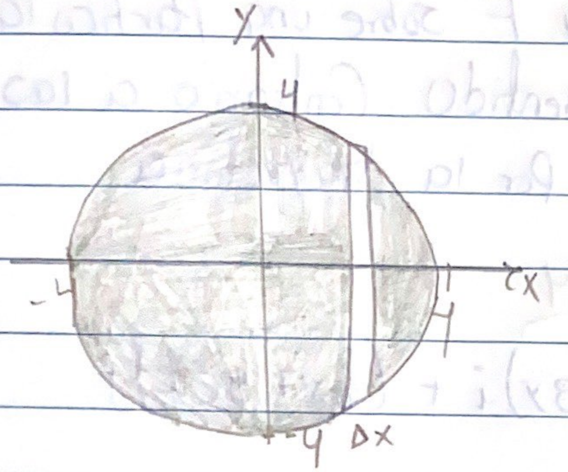
$$\int_{-1}^{1} 1-x^2-2x+2x^3\,dx = x-\frac{x^3}{3}-x^2+\frac{2}{4}x^4\Big|_{-1}^{1}$$

$$=\left(1-\frac{1}{3}-1+\frac{1}{2}\right)-\left(-1-\frac{1}{3}-1+\frac{1}{2}\right) = \frac{1}{6}-\left(\frac{-11}{6}\right) = \frac{1}{6}+\frac{7}{6}$$

$$=\frac{8}{6} = \boxed{\frac{4}{3}}$$

13) $\int_C (x^2-y^2)\,dx + 2xy\,dy$

$C: x^2+y^2=16$




$R = \begin{array}{l} 0 \leq r \leq 4 \\ 0 \leq \theta \leq 2\pi \end{array}$

$\frac{\partial M}{\partial y} = -2x$

$\frac{\partial N}{\partial x} = 2x$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R \left(2x-(-2x)\right) dA = \iint_R 2x+2y\,dA$$

$$\iint_R 2(x+y)\,dA = 2\int_0^{2\pi}\int_0^4 r\cos\theta + r\,\text{sen}\,\theta \; r\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\int_0^4 r^2(\cos\theta + \text{sen}\,\theta)\,dr\,d\theta = I = 2\int_0^4 r^2(\cos\theta + \text{sen}\,\theta)\,dr$$

$$= 2(\cos\theta + \text{sen}\,\theta)\int_0^4 r^2\,dr = 2(\cos\theta + \text{sen}\,\theta)\cdot \frac{r^3}{3}\Big|_0^4 = \frac{128}{3}(\cos\theta + \text{sen}\,\theta)$$

$$\frac{128}{3}\int_0^{2\pi} \cos\theta + \text{sen}\,\theta\,d\theta = \frac{128}{3}\left[\int_0^{2\pi}\cos\theta\,d\theta \int_0^{2\pi}\text{sen}\,\theta\,d\theta\right]$$

$$= \frac{128}{3}\left[\text{sen}\,\theta\Big|_0^{2\pi} + -\cos\theta\Big|_0^{2\pi}\right] = \frac{128}{3}\left[\text{sen}\,2\pi - \text{sen}\,0 + -\cos 2\pi + \cos 0\right]$$

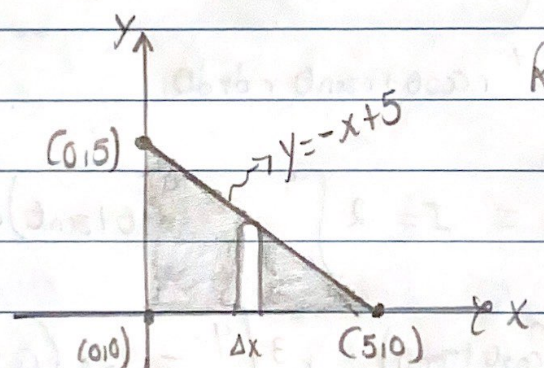
$$= \frac{128}{3}\left[(0-0)+(-1+1)\right] = \frac{128}{3}(0) = \boxed{0}$$

23) Trabajo. En los ejercicios 21 a 24, utilizar el Teorema de Green para calcular el Trabajo realizado por la fuerza F sobre una partícula que se mueve, en sentido contrario a las manecillas del reloj, por la trayectoria cerrada C.

23) $\vec{F}(x,y) = \overbrace{(x^{3/2} - 3y)}^{M} i + \overbrace{(6x + 5\sqrt{y})}^{N} J$

C: Contorno del triángulo cuyos vértices son $(0,0)$, $(5,0)$ y $(0,5)$.

$(5,0)$, $(0,5)$




$R = \begin{matrix} 0 \le y \le -x+5 \\ 0 \le x \le 5 \end{matrix}$

$\dfrac{\partial M}{\partial y} = -3$

$\dfrac{\partial N}{\partial x} = 6$

$\int_R\int \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) dA$

$\int_R\int (6 + 3)\, dA = \int_R\int 9\, dA$

$y - y_1 = \dfrac{y_1 - y_2}{x_1 - x_2}(x - x_1)$

$\int_0^5\int_0^{-x+5} 9\, dy\,dx = 9(-x+5) = -9x + 45$

$y - 0 = \dfrac{0-5}{5-0}(x-5)$

$y = \dfrac{-5}{5}(x-5)$

$\int_0^5 -9x + 45\, dx = -\dfrac{9}{2}x^2 + 45x \Big|_0^5$

$y = -(x-5)$

$= -225/2 + 225 = \boxed{\dfrac{225}{2}}$

$y = -x + 5$