INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 3.1-3.3]

3D GEOMETRY – TRIANGLES

Compiled by Christos Nikolaidis

O. Practice questions

3D GEOMETRY

1. [Maximum mark: 7] ***[without GDC]***

Let A(2,-3,5) and B(-1,1,5). Find

(a) the distance between A and B. [2]

(b) the distance between O and B. [1]

(c) the coordinates of the midpoint M of the line segment [AB]. [2]

(d) the coordinates of point C given that B is the midpoint of [AC]. [2]

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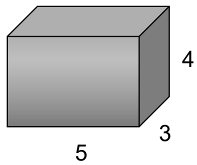
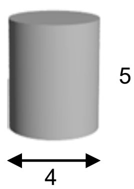
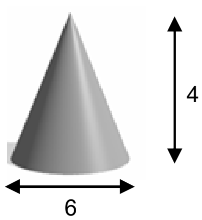
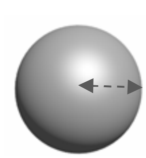
2. [Maximum mark: 16] ***[without GDC]***

Complete the table

Solid	Volume	Surface area
cuboid  		
cylinder  		
cone  		
sphere  radius = 3		

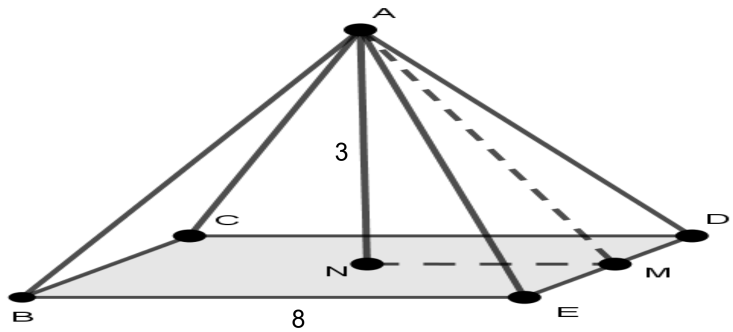
for each shape [1+3]

3. [Maximum mark: 7] ***[without GDC]***

For a right pyramid of square base of side 8 and vertical height 3 find

(a) the volume [2]

(b) the surface area [5]



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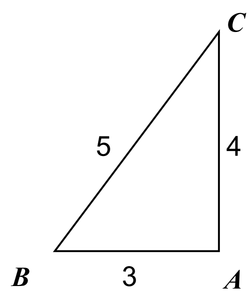
TRIANGLES

4. [Maximum mark: 14] ***[without GDC]***

Consider the following right-angled triangle, where $\hat{A}$ = 90°



(a) Complete the tables

$\sin\hat{B}$	
$\cos\hat{B}$	
$\tan\hat{B}$	

$\sin\hat{C}$	
$\cos\hat{C}$	
$\tan\hat{C}$	

[6]

(b) Confirm that the **sine rule** holds. (It is known that $\sin\hat{A}=1$)

$\dfrac{a}{\sin\hat{A}}=\dfrac{5}{1}=5$	$\dfrac{b}{\sin\hat{B}}=$	$\dfrac{c}{\sin\hat{C}}=$

[2]

(c) Confirm that all three versions of the **cosine rule** hold.
(the first version is given below; it is known that $\cos\hat{A}=0$)

LHS	RHS
5^2	$3^2+4^2-2(3)(4)\cos\hat{A}=9+16-0=25$
3^2	
4^2	

[4]

(d) Find the area of the triangle, by using all the three versions of the formula

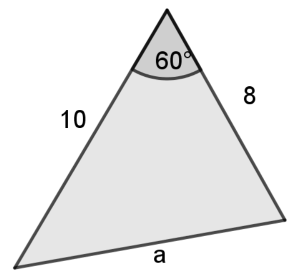
$$Area=\frac{1}{2}ab\sin\hat{C}$$ (the first version is given below)

$Area=\dfrac{1}{2}\times3\times4\times\sin\hat{A}=6$
$Area=$
$Area=$

[2]

5. [Maximum mark: 4] ***[with / without GDC]***

Use the **cosine rule** to find the size of the side a.



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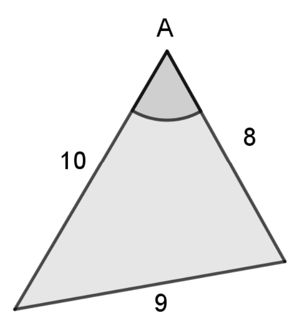
6. [Maximum mark: 5] ***[with GDC]***

(a) Use the **cosine rule** to find the cosine of the angle A. [4]

(b) Hence find the size of the angle A. [1]



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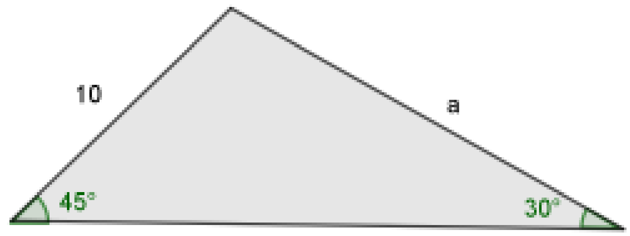
7. [Maximum mark: 4] ***[with / without GDC]***

Use the **sine rule** to find the size of the side a.



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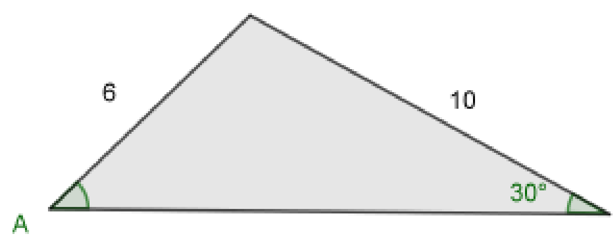
8. [Maximum mark: 6] ***[with GDC]***

(a) Use **the sine rule** to find the sine of the angle A. [4]

(b) Hence find the **two possible** values of the angle A. [2]



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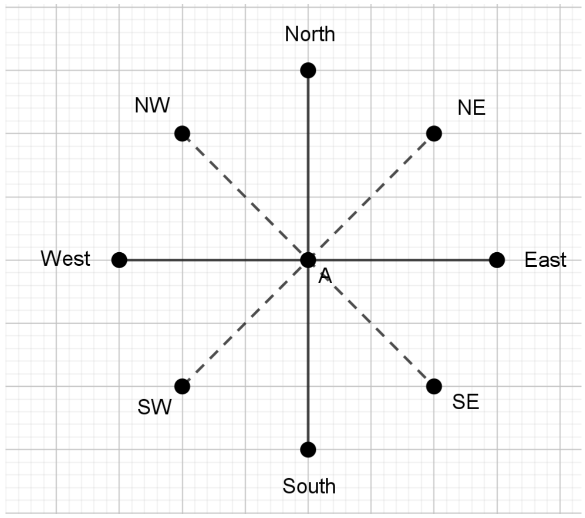
9. [Maximum mark: 10] ***[with GDC]***

Point A is at the center of the following diagram.



Bill and Chris and Dianna are located at point A and start moving,

Bill to the **NE** at point **B**

Chris to the **South** at point **C**

Dianna to the **West** at point **D**

(a) Write down the size of angle BAC. [1]

(b) Write down the bearing of the course of each person. [3]

(c) Find the bearing of the course from B to A. [2]

(d) Given that AB = 2 km and AC is 3km, Find the distance between B and C. [3]

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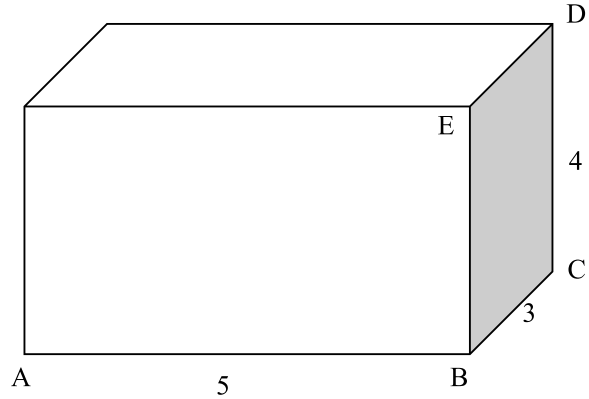
10. [Maximum mark: 14] ***[with GDC]***

Consider the following cuboid of dimensions 5×3×4, as shown.



(a) Find the length AC. [3]

(b) Find the length AD. [3]

(c) Find the angle of elevation from A to E. [3]

(d) Find the angle of elevation from A to D. [3]

(e) Find the angle of depression from E to A. [2]

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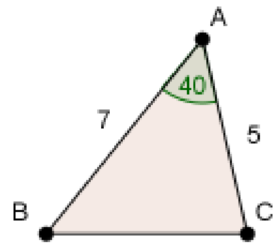
11*. [Maximum mark: 30] ***[with GDC]***

In each of the following triangles one of the angles has size 40°, two of the sides have lengths 5 and 7 respectively.

(a) For the following triangle



(i) Find the area of the triangle

(ii) Find BC

(iii) Find the size of $\hat{B}$ and **hence** the size of $\hat{C}$. [7]

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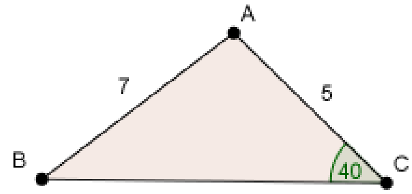
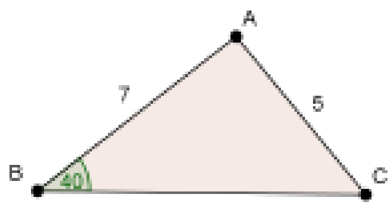
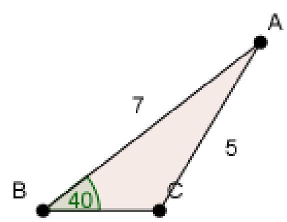
(b) For the following triangle



find the size of $\hat{B}$ and **hence** the size of $\hat{A}$. [5]

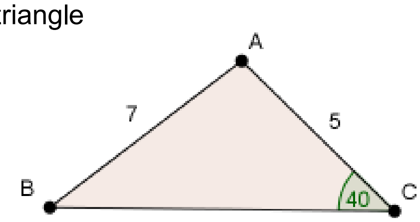
(c) For each of the following triangles (ambiguous case)





find the size of $\hat{C}$ and **hence** the size of $\hat{A}$. [6]

(d) For the following triangle



(i) Use the cosine rule to directly find the side BC.

(ii) **Hence** find the area of the triangle. [5]

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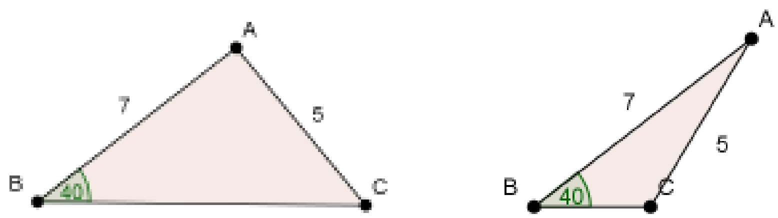
(e) For the following triangles (ambiguous case)



(i) Use the cosine rule to directly find the side BC of each triangle.

(ii) **Hence** find the area of each triangle. [7]

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A. Exam style questions (SHORT)

12. [Maximum mark: 6] ***[with GDC]***

The following diagram shows triangle ABC.

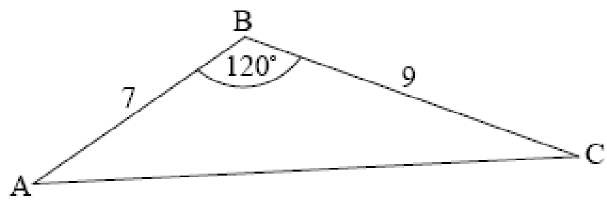

diagram not to scale

AB = 7 cm, BC = 9 cm and $A\hat{B}C$ = 120°.

(a) Find AC. [3]

(b) Find $B\hat{A}C$. [3]

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13. [Maximum mark: 4] ***[with GDC]***

A triangle has sides of length 4, 5, 7 units. Find, to the nearest tenth of a degree, the size of the largest angle.

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14. [Maximum mark: 4] ***[with GDC]***

The following diagram shows a triangle with sides 5 cm, 7 cm, 8 cm.

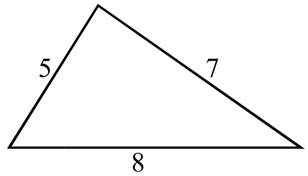

Diagram not to scale

(a) Find the size of the smallest angle, in degrees; [2]

(b) Find the area of the triangle. [2]

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15. [Maximum mark: 6] ***[with GDC]***

In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm. Calculate

(a) the size of $P\hat{Q}R$; [4]

(b) the area of triangle PQR. [2]

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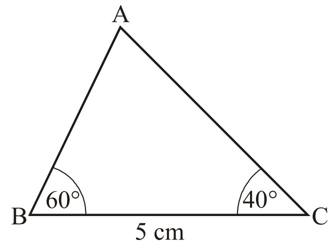
16. [Maximum mark: 6] ***[with GDC]***

The following diagram shows a triangle ABC, where BC = 5 cm, $\hat{B}$ = 60°, $\hat{C}$ = 40°.



(a) Calculate AB. [3]

(b) Find the area of the triangle. [3]

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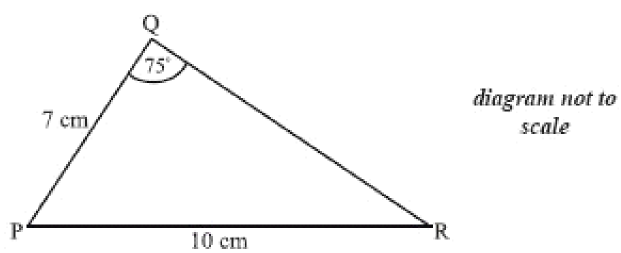
17. [Maximum mark: 6] ***[with GDC]***

The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $P\hat{Q}R$ is 75°.



(a) Find $P\hat{R}Q$ [3]

(b) Find the area of triangle PQR. [3]

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18. [Maximum mark: 6] ***[with GDC]***

In triangle ABC, AC = 5, BC = 7, $\hat{A}$ = 48°, as shown in the diagram.

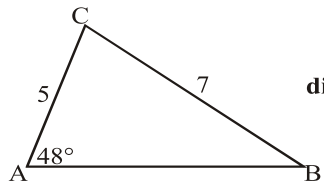

diagram not to scale

Find $\hat{B}$, giving your answer correct to the nearest degree.

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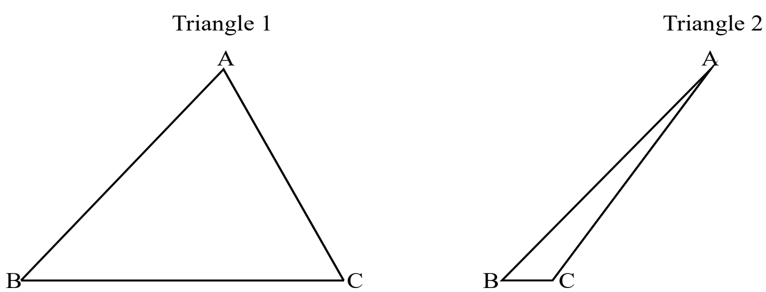
19*. [Maximum mark: 4] ***[with GDC]***

The diagrams below show two triangles both satisfying the conditions

AB = 20 cm, AC = 17 cm, $A\hat{B}C$ = 50°.



Diagrams not to scale

Calculate

(a) the size of $A\hat{C}B$ in **Triangle 2.** [2]

(b) the area of **Triangle 1**. [2]

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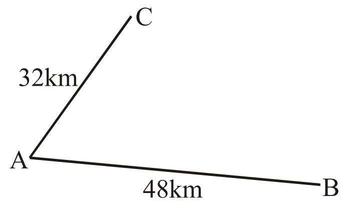
20. [Maximum mark: 4] ***[with GDC]***

Town A is 48 km from town B and 32 km from town C as shown in the diagram.



Given that town B is 56 km from town C, find the size of angle $\mathrm{C\hat{A}B}$ to the nearest degree.

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21*. [Maximum mark: 6] ***[with GDC]***

Two boats A and B start moving from the same point P. Boat A moves in a straight line at 20 km h^{-1} and boat B moves in a straight line at 32 km h^{-1}. The angle between their paths is 70°. Find the distance between the boats after 2.5 hours.

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22. [Maximum mark: 6] ***[with GDC]***

The following diagram shows the triangle ABC.

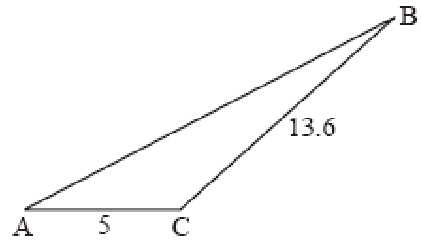

diagram not to scale

The angle at C is obtuse, AC = 5 cm, BC = 13.6 cm and the area is 20 cm^2.

(a) Find $\mathrm{A\hat{C}B}$. [3]

(b) Find AB. [3]

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23. [Maximum mark: 6] ***[with GDC]***

In a triangle ABC, AB = 4 cm, AC = 3 cm and the area of the triangle is 4.5 cm^2.

Find the **two** possible values of the angle $\mathrm{B\hat{A}C}$.

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24. [Maximum mark: 6] ***[without GDC]***

In triangle PQR, PQ is 10 cm, QR is 8 cm and angle PQR is acute. The area of the triangle is 20 cm^2. Find the size of angle $\mathrm{P\hat{Q}R}$.

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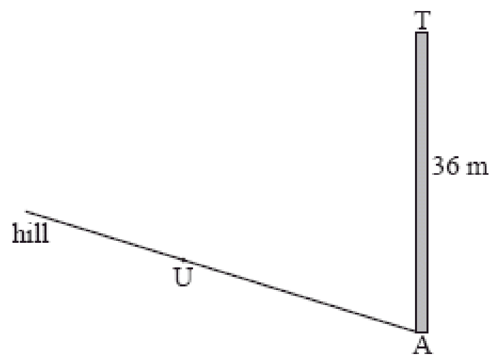
25. [Maximum mark: 7] ***[with GDC]***

There is a vertical tower TA of height 36 m at the base A of a hill. A straight path goes up the hill from A to a point U. This information is represented by the following diagram.



The path makes a 4° angle with the horizontal.

The point U on the path is 25 m away from the base of the tower.

The top of the tower is fixed to U by a wire of length x m.

(a) Complete the diagram, showing clearly all the information above. [3]

(b) Find x. [4]

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26. [Maximum mark: 8] ***[with GDC]***

The diagram below shows a triangle ABD with AB = 13 cm and AD = 6.5 cm.

Let C be a point on the line BD such that BC = AC = 7 cm.

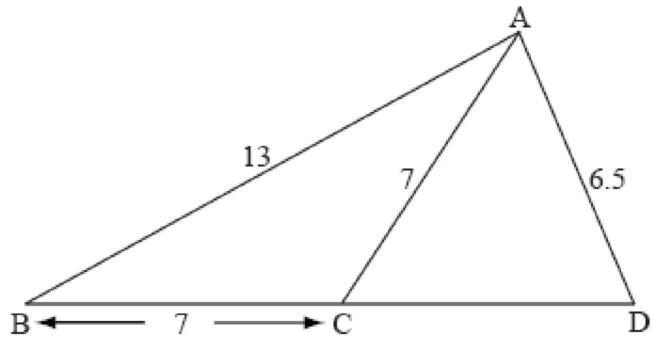


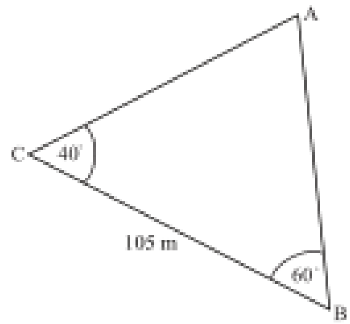
diagram not to scale

(a) Find the size of angle ACB. [3]

(b) Find the size of angle CAD. [5]

27. [Maximum mark: 6] ***[with GDC]***

The following diagram shows ΔABC, where $BC = 105$ m, $A\hat{C}B = 40°$, $A\hat{B}C = 60°$



Find the area of the triangle.

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28. [Maximum mark: 6] ***[with GDC]***

In the triangle ABC, $\hat{A} = 30°$, BC = 3 and AB = 5. Find the two possible values of $\hat{B}$.

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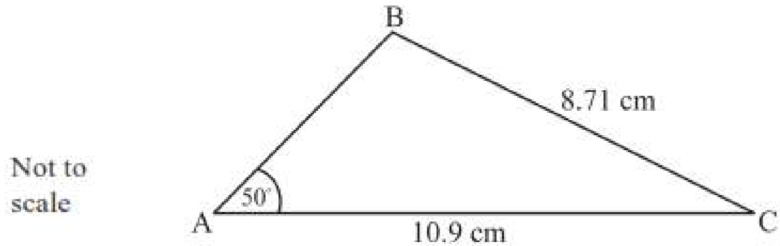
29. [Maximum mark: 6] ***[with GDC]***

In the **obtuse-angled** triangle ABC, $\mathrm{AC} = 10.9\,\mathrm{cm}$, $\mathrm{BC} = 8.71\,\mathrm{cm}$ and $\mathrm{B\hat{A}C} = 50°$.



Find the area of triangle ABC.

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30. [Maximum mark: 6] ***[with GDC]***

Triangle ABC has $\hat{C} = 42°$, $\mathrm{BC} = 1.74\,\mathrm{cm}$, and area $1.19\,\mathrm{cm}^2$.

(a) Find AC. [3]

(b) Find AB. [3]

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31*. [Maximum mark: 6] ***[with GDC]***

In the triangle ABC, $\hat{A}=30°$, $a=5$ and $c=7$. Find the difference in area between the two possible triangles for ABC.

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32*. [Maximum mark: 7] ***[with GDC]***

In a triangle *ABC*, $A\hat{B}C=30°$, $AB=6cm$, $AC=3\sqrt{2}\ cm$. Find the possible areas of the triangle.

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33*. [Maximum mark: 6] ***[with GDC]***

In a triangle ABC, $\text{A}\hat{\text{B}}\text{C} = 30°$, $\text{AB} = 6\,\text{cm}$, $\text{AC} = 3\sqrt{2}\,\text{cm}$. Find the possible lengths of $[\text{BC}]$.

METHOD A: Use Sine rule.

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METHOD B: Use Cosine rule.

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34*. [Maximum mark: 7] ***[with GDC]***

In a triangle ABC, $\hat{A}=35°$, $\mathrm{BC}=4\,\mathrm{cm}$ and $\mathrm{AC}=6.5\,\mathrm{cm}$. Find the possible values of $\hat{\mathrm{B}}$ and the corresponding values of AB.

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35*. [Maximum mark: 6] ***[with GDC]***

. Triangle ABC has $\mathrm{AB}=8\,\mathrm{cm}$, $\mathrm{BC}=6\,\mathrm{cm}$, $\mathrm{B\hat{A}C}=20°$. Find the smallest possible area of $\Delta\mathrm{ABC}$.

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36*. [Maximum mark: 6] ***[with GDC]***

In triangle ABC, $A\hat{B}C = 31°$, $AC = 3\,cm$, $BC = 5cm$. Calculate the possible lengths of $[AB]$.

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37*. [Maximum mark: 7] ***[with GDC]***

Consider triangle ABC with $B\hat{A}C = 37.8°$, $AB = 8.75$ and $BC = 6$. Find AC.

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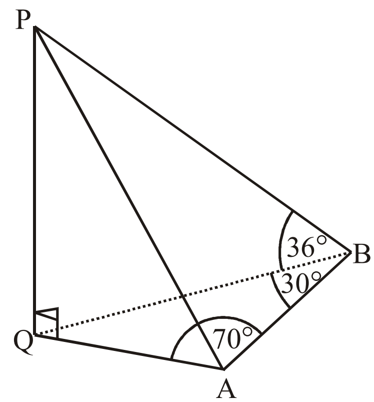
38*. [Maximum mark: 4] ***[with GDC]***
The diagram shows a vertical pole PQ, which is supported by two wires fixed to the horizontal ground at A and B.



BQ = 40 m
$\hat{PBQ}$ = 36°
$\hat{BAQ}$ = 70°
$\hat{ABQ}$ = 30°

Find

(a) the height of the pole, PQ; [2]

(b) the distance between A and B. [2]

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39*. [Maximum mark: 7] ***[with GDC]***

A ship leaves port A on a bearing of 030°. It sails a distance of 25 km to point B.

At B, the ship changes direction to a bearing of 100°. It sails a distance of 40 km to reach point C. This information is shown in the diagram below.

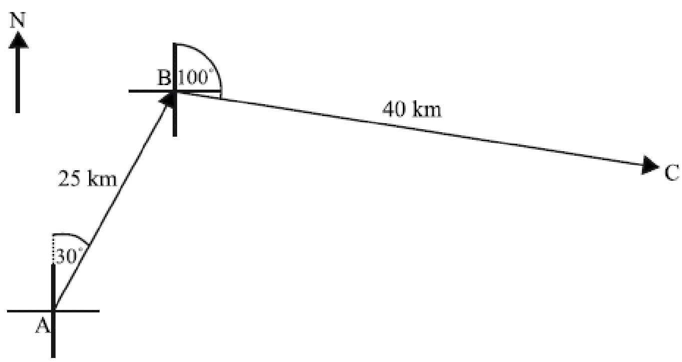

diagram not to scale

A second ship leaves port A and sails directly to C.

(a) Find the distance the second ship will travel. [4]

(b) Find the bearing of the course taken by the second ship. [3]

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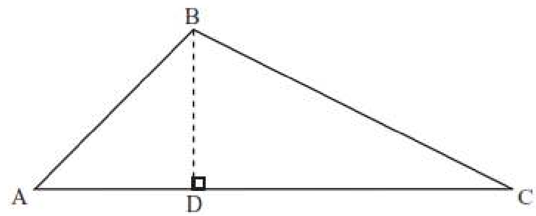
40*. [Maximum mark: 6] ***[without GDC]***

In triangle ABC, $\mathrm{BC}=a$, $\mathrm{AC}=b$, $\mathrm{AB}=c$ and $[\mathrm{BD}]$ is perpendicular to $[\mathrm{AC}]$.



(a) Show that $\mathrm{BD}=c\sin A$. [1]

(b) Show that $\mathrm{CD}=b-c\cos A$. [2]

(c) **Hence**, by using Pythagoras' Theorem in the triangle BCD, prove the cosine rule for the triangle ABC. [3]

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41.** [Maximum mark: 7] ***[without GDC]***

In triangle ABC, $\mathrm{BC}=a$, $\mathrm{AC}=b$, $\mathrm{AB}=c$ and $\mathrm{A\hat{B}C}=60°$.

Use the cosine rule to show that $c=\frac{1}{2}a\pm\sqrt{b^2-\frac{3}{4}a^2}$.

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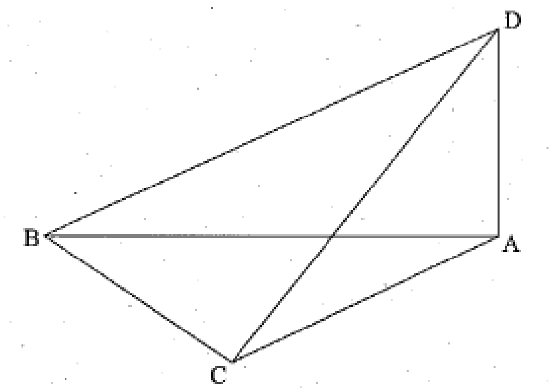
42.** [Maximum mark: 7] ***[with GDC]***

The following three dimensional diagram shows the four points A,B,C and D.

A,B,C are in the same horizontal plane and AD is vertical. The angle ABC is 45° , and BC = 50m. The angle of elevation from point B to point D is 30°, while the angle of elevation from point C to point D is 20°.



Using the cosine rule in the triangle ABC, or otherwise, find AD.

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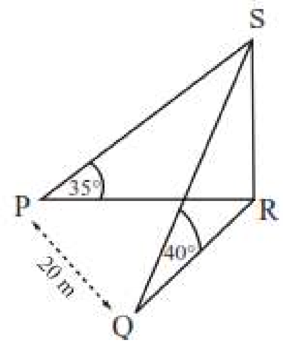
43.** [Maximum mark: 7] ***[with GDC]***



The above 3-dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively $35°$ and $40°$, and $\mathrm{PQ} = 20\,\mathrm{m}$.

Determine the height of the flagpole.

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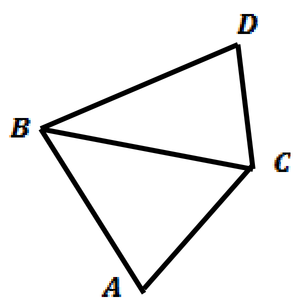
B. Past paper questions (*LONG*)

44. [Maximum mark: x] ***[with GDC]***

Consider the following diagram



$AB = 7$

$AC = 5$

$\hat{A} = 60^\circ$

$\hat{D} = 80^\circ$

$D\hat{B}C = 30^\circ$

Find

(a) Find the length of the side *BD*.

(b) Find the area of the quadrilateral *ABDC*.

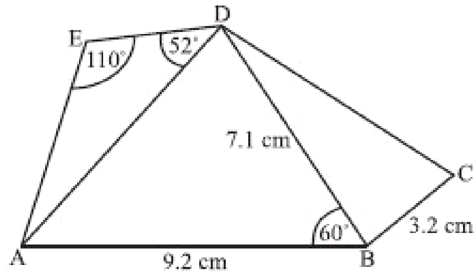
(c) Find the perimeter of the quadrilateral *ABDC*.

It is given that the bearing from *B* to *D* is 70º. Find

(d) Find (i) the bearing from *B* to *A*. (ii) Find the bearing from *A* to *B*.

45. [Maximum mark: 21] ***[with GDC]***

The following diagram shows a pentagon ABCDE, with $AB = 9.2$ cm, $BC = 3.2$ cm, $BD = 7.1$ cm, $A\hat{E}D = 110°$, $A\hat{D}E = 52°$ and $A\hat{B}D = 60°$.



(a) Find AD. [4]

(b) Find DE. [4]

(c) The area of triangle BCD is $5.68\ \text{cm}^2$. Find $D\hat{B}C$. [4]

(d) Find AC. [4]

(e) Find the area of quadrilateral ABCD. [5]

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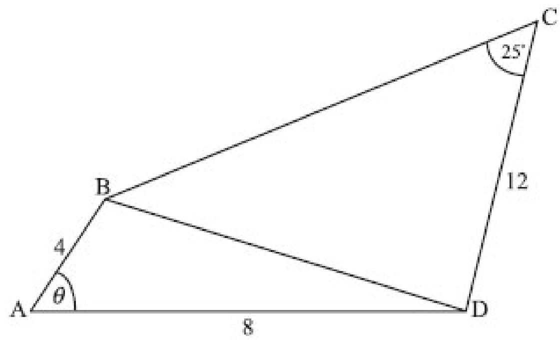
46. [Maximum mark: 16] ***[with GDC]***

The diagram below shows a quadrilateral ABCD. AB = 4, AD = 8, CD =12, $B\hat{C}D = 25°$, $B\hat{A}D = \theta$.



(a) Use the cosine rule to show that BD = $4\sqrt{5-4\cos\theta}$. [2]

Let $\theta = 40°$.

(b) (i) Find the value of $\sin C\hat{B}D$.

(ii) Find the two possible values for the size of $C\hat{B}D$.

(iii) Given that $C\hat{B}D$ is an acute angle, find the perimeter of ABCD. [12]

(c) Find the area of triangle ABD. [2]

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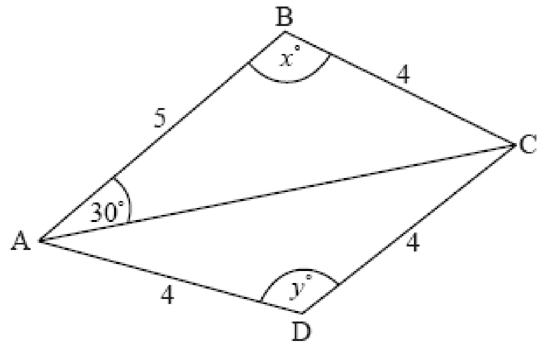
47. [Maximum mark: 14] ***[with GDC]***

The diagram below shows a quadrilateral ABCD with obtuse angles $A\hat{B}C$ and $A\hat{D}C$.



AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm, $B\hat{A}C = 30°$, $A\hat{B}C = x°$, $A\hat{D}C = y°$.

(a) Use the cosine rule to show that AC = $\sqrt{41-40\cos x}$. [1]

(b) Use the sine rule in triangle ABC to find another expression for AC. [2]

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC. [6]

(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD. [5]

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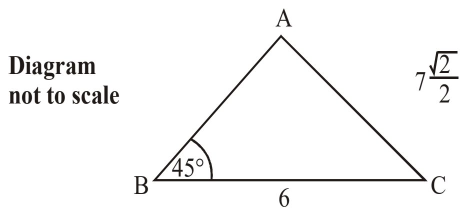
48. [Maximum mark: 10] ***[with GDC]***

The diagram shows a triangle ABC in which $AC = 7\dfrac{\sqrt{2}}{2}$, $BC = 6$, $A\hat{B}C = 45°$.



(a) Use the fact that $\sin 45° = \dfrac{\sqrt{2}}{2}$ to show that $\sin B\hat{A}C = \dfrac{6}{7}$. [2]

The point D is on (AB), between A and B, such that $\sin B\hat{D}C = \dfrac{6}{7}$.

(b) (i) Write down the value of $B\hat{D}C + B\hat{A}C$.

(ii) Calculate the angle BCD.

(iii) Find the length of [BD]. [6]

(c) Show that $\dfrac{\text{Area of } \Delta BDC}{\text{Area of } \Delta BAC} = \dfrac{BD}{BA}$. [2]

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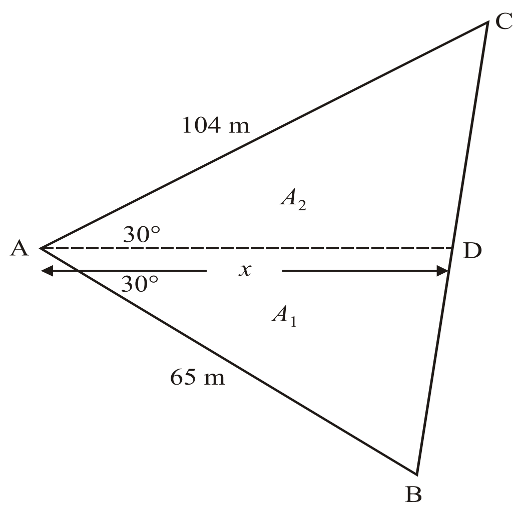
49. [Maximum mark: 18] ***[with GDC]***

A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

(a) Use the cosine rule to calculate the length of the third side of the field. [3]

(b) Given that $\sin 60° = \frac{\sqrt{3}}{2}$, find the area of the field in the form $p\sqrt{3}$ where $p \in Z$. [3]

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x metres, as shown on the diagram below.



(c) (i) Show that the area of A_1 is given by $\frac{65x}{4}$.

(ii) Find a similar expression for the area of A_2.

(iii) **Hence**, find the value of x in the form $q\sqrt{3}$, where $q \in Z$. [7]

(d) (i) Explain why $\sin \text{A}\hat{\text{D}}\text{C} = \sin \text{A}\hat{\text{D}}\text{B}$.

(ii) Use the result of part (i) and the sine rule to show that $\frac{\text{BD}}{\text{DC}} = \frac{5}{8}$. [5]

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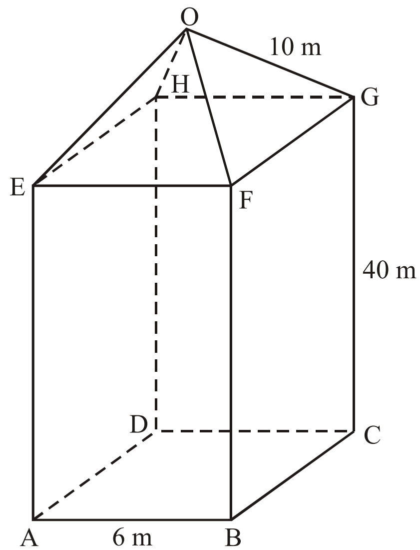
50. [Maximum mark: 14] ***[with GDC]***

An office tower is in the shape of a cuboid with a square base. The roof of the tower is in the shape of a square based right pyramid.

The diagram shows the tower and its roof with dimensions indicated. The diagram is **not** drawn to scale.



(a) Calculate, correct to three significant figures,

- (i) the size of the angle between OF and FG; [3]
- (ii) the shortest distance from O to FG; [2]
- (iii) the total surface area of the four triangular sections of the roof; [3]
- (iv) the size of the angle between the slant height of the roof and the plane EFGH; [2]
- (v) the height of the tower from the base to O. [2]

A parrot's nest is perched at a point, P, on the edge, BF, of the tower. A person at the point A, outside the building, measures the angle of elevation to point P to be 79°.

(b) Find, correct to three significant figures, the height of the nest from the base of the tower. [2]

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51*. [Maximum mark: 16] ***[with GDC]***

In the diagram below, the points O(0, 0) and A(8, 6) are fixed. The angle $\hat{OPA}$ varies as the point P(x ,10) moves along the horizontal line $y = 10$.

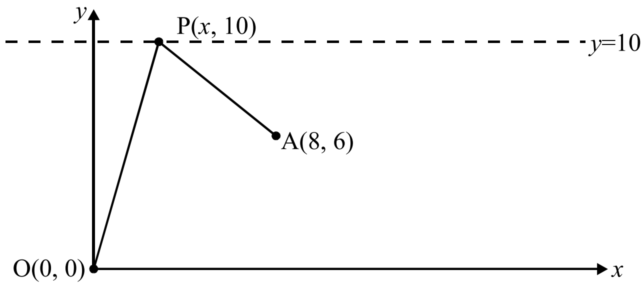

Diagram to scale

(a) (i) Show that $\text{AP} = \sqrt{x^2 - 16x + 80}.$

(ii) Write down a similar expression for OP in terms of x. [2]

(b) Hence, show that $\cos \hat{OPA} = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$, [3]

(c) Find, in degrees, the angle $\hat{OPA}$ when $x = 8$. [2]

(d) Find the positive value of x such that $\hat{OPA} = 60°$. [4]

Let the function f be defined by

$$f(x) = \cos \hat{OPA} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}, \ 0 \le x \le 15.$$ [4]

(e) Consider the equation $f(x) = 1$.

(i) Explain, in terms of the position of the points O, A, and P, why this equation has a solution.

(ii) Find the **exact** solution to the equation. [5]

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52*. [Maximum mark: 16] ***[with GDC]***

The points P, Q, R are three markers on level ground, joined by straight paths PQ, QR, PR as shown in the diagram. QR = 9 km, $P\hat{Q}R$ = 35°, $P\hat{R}Q$ = 25°.

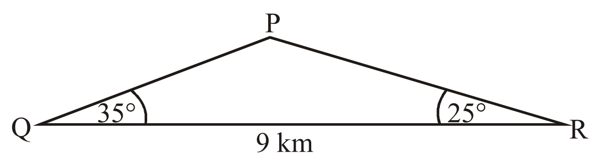
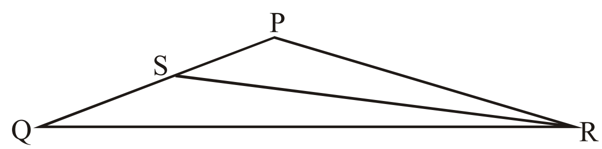


Diagram not to scale

(a) Find the length PR. [3]

(b) Tom sets out to walk from Q to P at a steady speed of 8 km h^{-1}. At the same time, Alan sets out to jog from R to P at a steady speed of a km h^{-1}. They reach P at the same time. Calculate the value of a. [7]

(c) The point S is on [PQ], such that RS = 2QS, as shown in the diagram.



Find the length QS. [6]

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