MAA [3.1-3.3] 3D GEOMETRY – TRIANGLES

<u>SOLUTIONS</u>

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3D GEOMETRY

O. Practice questions

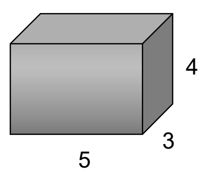
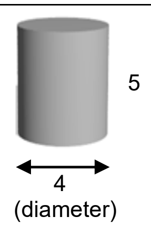
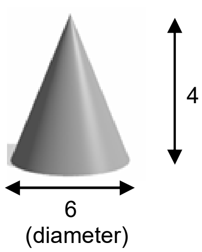
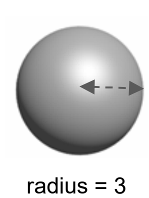
1. (a) $d_{AB}=\sqrt{3^2+4^2+0^2}=5$

(b) $d_{OB}=\sqrt{1^2+1^2+5^2}=\sqrt{27}=3\sqrt{3}$

(c) M(1/2, –1,5)

(d) C(–4,5,5)

2.

Solid	Volume	Surface area
	$V=5\times3\times4=60$	$S=2\times15+2\times12+2\times20=94$
	$r=2$ $h=5$ $V=\pi2^25=20\pi$	$S=2\pi(2)(5)+2\times(\pi2^2)=28\pi$
	$r=3$ $h=4$ $V=\frac{1}{3}\pi3^24=12\pi$	slant height: $l=5$ $S=\pi(3)(5)+\pi3^2=24\pi$
	$V=\frac{4}{3}\pi3^3=36\pi$	$S=4\pi3^2=36\pi$

3. (a) $V=\frac{1}{3}8^23=64$

(b) $AM^2=4^2+3^2\Rightarrow AM=5$

$S=8^2+4\times\left(\frac{1}{2}\times8\times5\right)=64+80=144$

TRIANGLES

4. (a)

$\sin\hat{B}$	$\frac{4}{5}$
$\cos\hat{B}$	$\frac{3}{5}$
$\tan\hat{B}$	$\frac{4}{3}$

$\sin\hat{C}$	$\frac{3}{5}$
$\cos\hat{C}$	$\frac{4}{5}$
$\tan\hat{C}$	$\frac{3}{4}$

(b)

$\frac{a}{\sin\hat{A}} = \frac{5}{1} = 5$ $\quad$ $\frac{b}{\sin\hat{B}} = \frac{4}{4/5} = 5$ $\quad$ $\frac{c}{\sin\hat{C}} = \frac{3}{3/5} = 5$

(c)

LHS	RHS
5^2	$3^2 + 4^2 - 2(3)(4)\cos\hat{A} = 9 + 16 - 0 = 25$
3^2	$4^2 + 5^2 - 2(4)(5)\cos\hat{C} = 16 + 25 - 32 = 9$
4^2	$3^2 + 5^2 - 2(3)(5)\cos\hat{B} = 9 + 25 - 18 = 16$

(d)

$Area = \frac{1}{2}\times 3\times 4\times \sin\hat{A} = 6$
$Area = \frac{1}{2}\times 4\times 5\times \sin\hat{C} = \frac{1}{2}\times 4\times 5\times \frac{3}{5} = 6$
$Area = \frac{1}{2}\times 3\times 5\times \sin\hat{B} = \frac{1}{2}\times 3\times 5\times \frac{4}{5} = 6$

5. $a^2 = 10^2 + 8^2 - 2(10)(8)\cos 60° \Rightarrow a = \sqrt{84} \quad (\cong 9.17)$

6. (a) $9^2 = 10^2 + 8^2 - 2(10)(8)\cos\hat{A} \Rightarrow \cos\hat{A} = \frac{83}{160}$

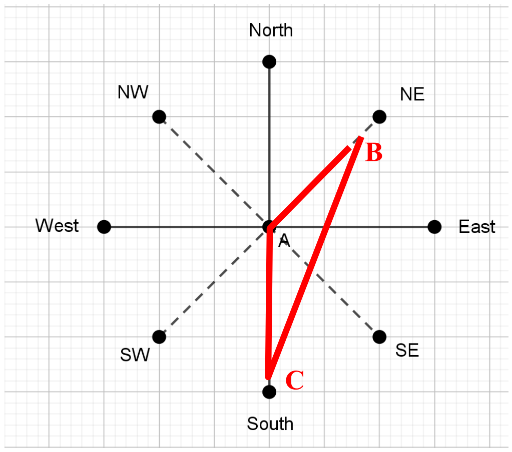
(b) $\hat{A} \cong 58.8°$

7. $\frac{a}{\sin 45°} = \frac{10}{\sin 30°} \Rightarrow a = \frac{10\sin 45°}{\sin 30°} \Rightarrow a = \frac{10\frac{\sqrt{2}}{2}}{\frac{1}{2}} \Rightarrow a = 10\sqrt{2}$

8. (a) $\frac{10}{\sin\hat{A}} = \frac{6}{\sin 30°} \Rightarrow \sin\hat{A} = \frac{10\sin 30°}{6} \Rightarrow \sin\hat{A} = \frac{5}{6}$

(b) $\hat{A} = 56.442... \cong 56.4°$ or $\hat{A} = 123.557... \cong 124°$

9.



(a) $B\hat{A}C = 135°$

(b) Bearing of B from A = 45°
Bearing of C from A = 180°
Bearing of D from A = 270°

(c) Bearing of A from B = $180° + 45° = 225°$

(d) $BC^2 = 2^2 + 3^2 - 2(2)(3)\cos 135° \Rightarrow BC = 4.6352... \cong 4.64$

10. (a) $AC^2 = 5^2 + 3^2 \Rightarrow AC = \sqrt{34}$

(b) $AD^2 = \sqrt{34}^2 + 4^2 \Rightarrow AD = \sqrt{50}$

(c) $\tan E\hat{A}B = \frac{4}{5} \Rightarrow E\hat{A}B \cong 38.7°$

(d) $\tan D\hat{A}C = \frac{4}{\sqrt{34}} \Rightarrow D\hat{A}C \cong 34.4°$

(e) $\tan \hat{E} = \frac{4}{5} \Rightarrow \hat{E} \cong 38.7°$

11. (a) (i) $A = \frac{1}{2}(7)(5)\sin 40° \cong 11.2$

(ii) $BC^2 = 5^2 + 7^2 - 2(5)(7)\cos 40° \Rightarrow BC = 4.51$

(iii) $\frac{\sin 40}{4.51} = \frac{\sin \hat{B}}{5} \Rightarrow \hat{B} = 45.4°$ and so $\hat{C} = 180° - 40° - 45.4° = 94.6°$

(b) $\hat{B} = 27.3°$, $\hat{A} = 112.7°$

(c) $\hat{C} = 64.1°$, $\hat{A} = 75.9°$ or $\hat{C} = 115.9°$, $\hat{A} = 24.1°$

(d) (i) $7^2 = 6^2 + BC^2 - 2(6)(BC)\cos 40° \Rightarrow BC = 10.437... \cong 10.4$

(ii) Area = $\frac{1}{2}(6)(10.437..)\sin 40° = 20.126... \cong 20.1$

(e) (i) $6^2 = 7^2 + BC^2 - 2(7)(BC)\cos 40°$

$\Rightarrow BC = 1.3931... \cong 1.39$ or $BC = 9.3314... \cong 9.33$

(ii) Area 1 $= \frac{1}{2}(7)(1.3931...)\sin 40° \cong 3.13$, Area 2 $= \frac{1}{2}(7)(9.3314...)\sin 40° \cong 21.0$

A. Exam style questions (SHORT)

12. (a) $AC^2 = 7^2 + 9^2 - 2(7)(9)\cos 120°$

$AC = 13.9\ (= \sqrt{193}\,)$

(b) **METHOD 1**

sine rule $\frac{\sin \hat{A}}{9} = \frac{\sin 120}{13.9} \Rightarrow \hat{A} = 34.1°$

METHOD 2

cosine rule $\cos \hat{A} = \frac{7^2 + 13.9^2 - 9^2}{2(7)(13.9)} \Rightarrow \hat{A} = 34.1°$

13. ***Note:*** *largest angle* opposite largest side

$\cos \alpha = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5} = -\frac{1}{5} \Rightarrow \alpha = 101.5°$

14. (a) The smallest angle is opposite the smallest side.

$\cos \theta = \frac{8^2 + 7^2 - 5^2}{2 \times 8 \times 7} = \frac{88}{112} = \frac{11}{14} = 0.7857$

Therefore, $\theta = 38.2°$

(b) Area = $\frac{1}{2} \times 8 \times 7 \times \sin 38.2° = 17.3\ \text{cm}^2$

15. (a) $5^2 = 4^2 + 6^2 - 2 \times 4 \times 6\ \cos Q$

$P\hat{Q}R = 55.8°$ (0.973 radians)

(b) Area = $\frac{1}{2} \times 4 \times 6\ \sin 55.8 = 9.92\ (\text{cm}^2)$

16. (a) Angle $A = 80°$

$\frac{AB}{\sin 40°} = \frac{5}{\sin 80°} \Rightarrow AB = 3.26\ \text{cm}$

(b) Area $= \frac{1}{2}(5)(3.26)\sin 60° = 7.07\ \text{cm}^2$

17. (a) sine rule $\frac{\sin R}{7} = \frac{\sin 75°}{10}$

$P\hat{R}Q = 42.5°$

(b) $P = 180 - 75 - R = 62.5$

area Δ PQR = $\frac{1}{2} \times 7 \times 10 \times \sin P = 31.0\ (\text{cm}^2)$

18. Using sine rule: $\frac{\sin B}{5} = \frac{\sin 48°}{7} \Rightarrow \sin B = \frac{5}{7}\sin 48° = 0.5308\ldots \Rightarrow B = 32.06° = 32°$

19. $\frac{\sin(A\hat{C}B)}{20} = \frac{\sin 50°}{17} \Rightarrow \sin(A\hat{C}B) = \frac{20 \sin 50°}{17} = 0.901$

$A\hat{C}B = 64.3°$ or $A\hat{C}B = 180° - 64.3° = 115.7°$

(a) $A\hat{C}B > 90° \Rightarrow A\hat{C}B = 116$ (3 sf)

(b) In Triangle 1, $A\hat{C}B = 64.3°$

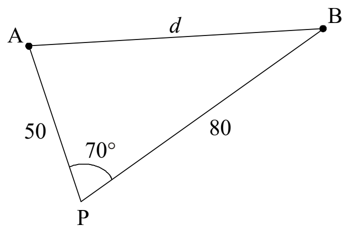
$\Rightarrow B\hat{A}C = 180° - (64.3° + 50°) = 65.7°$

Area = $\frac{1}{2}(20)(17)\sin 65.7° = 155\ (\text{cm}^2)$ (3 sf)

20. $\cos \hat{CAB} = \dfrac{48^2 + 32^2 - 56^2}{2(48)(32)}$

$\hat{CAB} \approx 86°$

21.



(2.5 × 20 = 50)   (2.5 × 32 = 80)

$d^2 = 50^2 + 80^2 - 2 \times 50 \times 80 \times \cos 70°$

$d = 78.5$ km

22. (a) $\dfrac{1}{2}(5)(13.6)\sin C = 20 = 20 \Rightarrow \sin C = 0.5882... \Rightarrow \ \hat{ACB} = 144°$ (2.51 radians)

(b) cosine rule $(AB)^2 = 5^2 + 13.6^2 - 2(5)(13.6)\cos 143.968...$

AB = 17.9

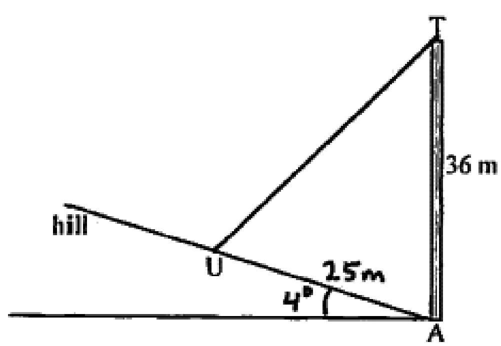
23. Area of a triangle = $\dfrac{1}{2} \times 3 \times 4 \sin A = 4.5 \Rightarrow \sin A = 0.75$

$A = 48.6°$ and $A = 131°$ (or 0.848, 2.29 radians)

24. $20 = \dfrac{1}{2}(10)(8)\sin \boldsymbol{Q} \Rightarrow \sin Q = 0.5$

$\hat{PQR} = 30°$ or $\dfrac{\pi}{6}$

25. (a)



(b) $\hat{TAU} = 86°$

$x^2 = 25^2 + 36^2 - 2(25)(36)\cos 86°$

$x = 42.4$

26. (a) **METHOD 1**

cosine rule $\cos A\hat{C}B = \frac{7^2 + 7^2 - 13^2}{2 \times 7 \times 7}$

$A\hat{C}B$ = 2.38 radians (= 136°)

METHOD 2

considering right-angled triangle

$\sin\left(\frac{1}{2} A\hat{C}B\right) = \frac{6.5}{7}$

$A\hat{C}B$ = 2.38 radians (= 136°)

(b) **METHOD 1**

$A\hat{C}D$ = 180 – 136.4 **OR** $A\hat{C}D$ = π – 2.381

sine rule in triangle ACD: . $\frac{6.5}{\sin 0.760...} = \frac{7}{\sin A\hat{D}C}$

$A\hat{D}C$ = 47.9...° **OR** $A\hat{D}C$ = 0.836...

$C\hat{A}D$ = 180 – (43.5... + 47.9...) = 88.5° **OR** $C\hat{A}D$ = π – (0.760... + 0.836...) = 1.54

METHOD 2

$A\hat{B}C = \frac{1}{2}(180 - 136.4)$ **OR** $A\hat{B}C = \frac{1}{2}(\pi - 2.381)$

sine rule in triangle ABD: $\frac{6.5}{\sin 0.380...} = \frac{13}{\sin A\hat{D}C}$

$A\hat{D}C$ = 47.9...° **OR** $A\hat{D}C$ = 0.836...

$C\hat{A}D$ = 180 – 47.9 – (180 – 136.4)) = 88.5° **OR** $C\hat{A}D$ = π – 0.836– (π – 2.381) = 1.54

Note: Two triangles are possible with the given information.

27. **METHOD 1**

$B\hat{A}C = 80°$

Using the sine rule to find AB

$\frac{105}{\sin 80°} = \frac{AB}{\sin 40°}$

AB = 68.533…

Area = $\hat{A} = \frac{1}{2}(105)(68.533..)\sin 60° = 3115.94... \cong 3116\,\text{m}$ (or 3120m to 3sf)

METHOD 2

$B\hat{A}C = 80°$

Using the sine rule to find AC

$\frac{105}{\sin 80°} = \frac{AC}{\sin 60°}$

$AC = \frac{105 \sin 60°}{\sin 80°}$ (= 92.335...)

Area = $\hat{A} = \frac{1}{2}(105)(92.335)\sin 40° = 3115.96... \cong 3116\,\text{m}$ (or 3120m to 3sf)

28. $\sin C = \dfrac{c\sin A}{a} = \dfrac{5\times 0.5}{3}$

$\hat{C} = 56.4°$ or $123.6°$

$\hat{B} = 93.6°$ or $26.4°$

29, **METHOD 1**

$$\frac{\sin 50^\circ}{8.71} = \frac{\sin \hat{B}}{10.9}$$

Finding the obtuse value of $\hat{B}$ from the range 106 to 107

Finding $\hat{C}$ from the range 23 to 24

$$\text{Area ABC} = \frac{1}{2}\times 10.9\times 8.71\times \sin\hat{C}$$
$$= 18.9\ (\text{cm}^2)$$

METHOD 2

$8.71^2 = \text{AB}^2 + 10.9^2 - 2\text{AB}\times 10.9\cos 50^\circ$

Solving a quadratic in AB

choosing $\text{AB} = 4.52(7\ldots)$

$$\text{Area triangle ABC} = \frac{1}{2}\times 10.9\times AB\sin 50^\circ$$
$$= 18.9\ (\text{cm}^2)$$

30.

(a) $1.19 = \dfrac{1}{2}(1.74)(\sin 42^\circ)\text{AC}$

$\text{AC} = 2.044\ldots$

$\text{AC} = 2.04\ (\text{cm})$

(b) $\text{AB}^2 = 2.044\ldots^2 + 1.74^2 - 2(1.74)(2.044\ldots)\cos 42^\circ$

$\text{AB}^2 = 1.919\ldots$ (or $1.913\ldots$)

$\text{AB} = 1.39\ (\text{cm})$ (accept 1.38 cm or 1.40 cm)

31.

$$\frac{\sin C}{7} = \frac{\sin 30^\circ}{5}$$

$\Rightarrow \sin C = \dfrac{7\sin 30^\circ}{5}$

$C = 44.4^\circ$

or $C = 135.6^\circ$

$\Rightarrow B = 105.6^\circ$ or 14.4°

$\Rightarrow$ Difference in area $\Delta\text{ABC} = \dfrac{1}{2}ac(\sin B_1 - \sin B_2)$

$= \dfrac{1}{2}(5)(7)(\sin 105.6^\circ - \sin 14.4^\circ)$

$= 12.5\ \text{cm}^2$

32. **METHOD A (using Sine rule)**

$$\frac{\sin 30}{3\sqrt{2}} = \frac{\sin C}{6} \Rightarrow \sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45^\circ \text{ or } C' = 135^\circ$$

For $C = 45^\circ$, $A = 105^\circ$, Area = $\frac{1}{2} 6 \times 3\sqrt{2} \times \sin 105^\circ = 12.3$

For $C'' = 135^\circ$, $A = 15^\circ$, Area = $\frac{1}{2} 6 \times 3\sqrt{2} \times \sin 15^\circ = 3.29$

METHOD B (using Cosine rule)

$$(3\sqrt{2})^2 = BC^2 + 6^2 - 2BC \times 6 \times \cos 30 \Rightarrow BC^2 - 6\sqrt{3}BC + 18 = 0$$

$$\Rightarrow BC = \pm 3 + 3\sqrt{3}$$

Area = $\frac{1}{2} 6 \times (\pm 3 + 3\sqrt{3} \times \sin 30 = \frac{\pm 9 + 9\sqrt{3}}{2} = 12.3$ or 3.29

33. **Method 1**:

Using the sine rule: $\frac{\sin \text{C}}{6} = \frac{\sin 30°}{3\sqrt{2}}$ $\quad$ $\sin \text{C} = \frac{1}{\sqrt{2}}$

C = 45°, 135°.

Again, $\frac{3\sqrt{2}}{\sin 30°} = \frac{\text{BC}}{\sin 105°}$ or $\frac{\text{BC}}{\sin 15°}$

Thus, BC = $6\sqrt{2}$ sin 105° or $6\sqrt{2}$ sin 15°

BC = 8.20 cm or BC = 2.20 cm.

Method 2:

Using the cosine rule: $\text{AC}^2 = 6^2 + \text{BC}^2 - 2(6)(\text{BC})\cos 30°$

$18 = 36 + \text{BC}^2 - 6\sqrt{3}\ \text{BC}$

Therefore, $\text{BC}^2 - (6\sqrt{3})\text{BC} + 18 = 0$

Therefore, BC = $3\sqrt{3} \pm 3$, *i.e.* BC = 8.20 cm or BC = 2.20 cm.

34. Sine rule: $\frac{\sin 35}{4} = \frac{\sin \hat{B}}{6.5} \Rightarrow \sin \hat{B} = \frac{6.5 \sin 35}{4} \Rightarrow \sin \hat{B} = 0.932$

Hence $\hat{B} = 68.8$ or $\hat{B} = 180 - 68.8 = 111.2$

If $\hat{B} = 68.8$, then $\hat{C} = 180 - 35 - 68.8 = 76.2$

Sine rule again: $\frac{\sin 35}{4} = \frac{\sin 76.2}{AB} \Rightarrow AB = 6.77$

If $\hat{B} = 111.2$, then $\hat{C} = 180 - 35 - 111.2 = 33.8$

Sine rule again: $\frac{\sin 35}{4} = \frac{\sin 33.8}{AB} \Rightarrow AB = 3.88$

35. **METHOD 1**

$\frac{\sin C}{8} = \frac{\sin 20}{6} \Rightarrow \hat{C} = 152.9°$ (From diagram) smallest triangle when $\hat{C}$ is obtuse

$\Rightarrow C\hat{B}A = 7.13°$ (or 7.1°)

Area $\Delta ABC = \frac{1}{2}(8)(6)(\sin 7.13°) = 2.98(cm^2)$ (accept 2.97)

METHOD 2

Let AC = x

By the cosine rule $6^2 = 8^2 + x^2 - (2)(8)(x)\cos 20°$

$x = 2.178$

Area = $\frac{1}{2}$ AB × ACsin(20°) = $\frac{1}{2}(8)(2.178)\sin 20° = 2.98(cm^2)$

36. **METHOD 1**

$\frac{\sin 31}{3} = \frac{\sin BAC}{5}$

$\angle BAC = 59.137°$ or $120.863°$

$\angle ACB = 89.863°$ or $28.137°$

$\frac{3}{\sin 31} = \frac{AB}{\sin 89.863}$ $\frac{3}{\sin 31} = \frac{AB}{\sin 28.137}$

$AB = 5.82$ $AB = 2.75$

METHOD 2

$3^2 = 5^2 + AB^2 - 2 \times AB \times BC \times \cos 31°$

$AB = 5.82$ $AB = 2.75$

37. **METHOD 1 Using cosine rule**

$6^2 = 8.75^2 + AC^2 - 2 \times 8.75 \times AC \times \cos 37.8°$

$AC = 9.60$ or $AC = 4.22$

METHOD 2 Using sine rule

$\frac{BC}{\sin B\hat{A}C} = \frac{AB}{\sin A\hat{C}B}$

$\sin C = \frac{8.75 \sin 37.8°}{6}$ $(= 0.8938...)$

$C = 63.3576...°$

$C = 116.6423...°$

$B = 78.842...°$ or $B = 25.5576...°$

For **AC:**

EITHER

Attempting to solve $\frac{AC}{\sin 78.842...°} = \frac{6}{\sin 37.8°}$ or $\frac{AC}{\sin 25.5576...°} = \frac{6}{\sin 37.8°}$

OR

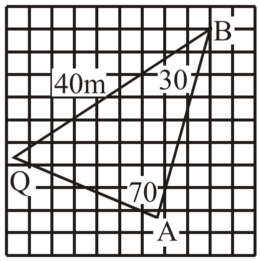
Attempting to solve $AC^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 25.5576...°$ or

$AC^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 78.842...°$

$AC = 9.60$ or $AC = 4.22$

38. (a) $\frac{PQ}{40} = \tan 36° \Rightarrow PQ \approx 29.1$ m (3 sf)

(b)



$A\hat{Q}B = 80°$

$\frac{AB}{\sin 80°} = \frac{40}{\sin 70°} \Rightarrow AB = 41\ 9.$ m (3 sf)

39. (a) $A\hat{B}C = 110°$

$AC^2 = 25^2 + 40^2 - 2(25)(40) \cos 110° \Rightarrow AC = 53.9$ (km)

(b) By using either sine rule or cosine rule:

$B\hat{A}C = 44.2° \Rightarrow$ bearing = 074°

40. (a) $\sin A = \frac{BD}{c} \Rightarrow BD = c \sin A$

(b) $\cos A = \frac{AD}{c} \Rightarrow AD = c \cos A$

$CD = AD - AD = b - c\cos A$

(c) $BC^2 = BD^2 + CD^2$

$$\begin{aligned} a^2 &= (c \sin A)^2 + (b - c\cos A)^2 \\ &= c^2 \sin^2 A + b^2 - 2bc\cos A + c^2 \cos^2 A \\ &= b^2 + c^2 - 2bc \cos A \end{aligned}$$

41.

$$b^2 = a^2 + c^2 - 2ac\cos 60° \Rightarrow b^2 = a^2 + c^2 - ac$$

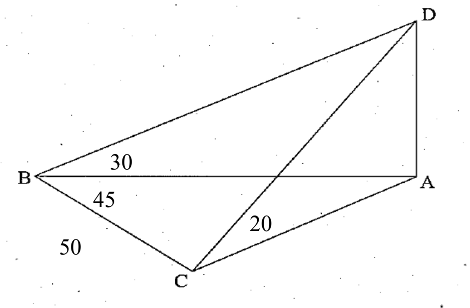
$$\Rightarrow c^2 - ac + a^2 - b^2 = 0$$

$$\Rightarrow c = \frac{a \pm \sqrt{(-a)^2 - 4(a^2 - b^2)}}{2}$$

$$= \frac{a \pm \sqrt{4b^2 - 3a^2}}{2} = \frac{a}{2} \pm \sqrt{\frac{4b^2 - 3a^2}{4}}$$

$$= \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$$

42.



Let $h = \text{AD}$. $\text{AB} = \dfrac{h}{\tan 30}$, $\text{AC} = \dfrac{h}{\tan 20}$

$$\left(\frac{h}{\tan 20}\right)^2 = \left(\frac{h}{\tan 30}\right)^2 + 50^2 - 100\left(\frac{h}{\tan 30}\right)\frac{\sqrt{2}}{2}$$

$\Rightarrow h = 13.6$

43.

$\text{PR} = h\tan 55^\circ$, $\text{QR} = h\tan 50^\circ$ where $\text{RS} = h$

Use the cosine rule in triangle PQR.

$$20^2 = h^2\tan^2 55^\circ + h^2\tan^2 50^\circ - 2h\tan 55^\circ\, h\tan 50^\circ \cos 45^\circ$$

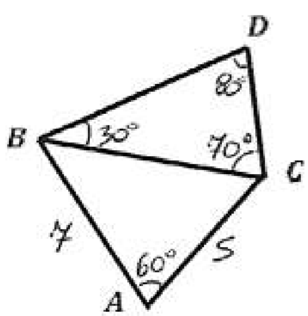
$$h^2 = \frac{400}{\tan^2 55^\circ + \tan^2 50^\circ - 2\tan 55^\circ \tan 50^\circ \cos 45^\circ}$$

$= 379.9...$

$h = 19.5$ (m)

B. Exam style questions (LONG)

44.



(a) $BC^2 = 5^2 + 7^2 - 2\cdot 5\cdot 7\cos 60 \Rightarrow BC = \sqrt{39}$

$\dfrac{BD}{\sin 70} = \dfrac{\sqrt{39}}{\sin 80} \Rightarrow BD = \sqrt{39}\,\dfrac{\sin 70}{\sin 80} \Rightarrow \boxed{BD = 5.96}$

(b) $A = A_{ABC} + A_{BDC} = \frac{1}{2}\cdot 5\cdot 7\sin 60^\circ + \frac{1}{2}(5.96)\sqrt{39}\sin 30^\circ$

$\Rightarrow A = 15.16 + 9.31 \Rightarrow \boxed{A \cong 24.5}$

(c) We need DC: $\dfrac{DC}{\sin 30^\circ} = \dfrac{\sqrt{39}}{\sin 80^\circ} \Rightarrow DC \cong 3.17$

Perimeter $\cong 5 + 7 + 5.96 + 3.17 \cong \boxed{21.1}$

(d) We need $A\hat{B}C$

$\dfrac{5}{\sin A\hat{B}C} = \dfrac{\sqrt{39}}{\sin 60} \Rightarrow \sin A\hat{B}C = \dfrac{5\sin 60}{\sqrt{39}} = 0.693$

$\Rightarrow A\hat{B}C = 43.9^\circ$

Bearing BA $= 70^\circ + 30^\circ + 43.9^\circ = \boxed{143.9^\circ}$

(e) Bearing AB $= 180^\circ + 143.9^\circ = \boxed{323.9}$

45. (a) $(AD)^2 = 7.1^2 + 9.2^2 - 2(7.1)(9.2)\cos 60° \Rightarrow AD = 8.35$ (cm)

(b) $180° - 162° = 18°$

$$\frac{DE}{\sin 18°} = \frac{8.35}{\sin 110°} \Rightarrow DE = 2.75 \text{ (cm)}$$

(c) $5.68 = \frac{1}{2}(3.2)(7.1)\sin D\hat{B}C \Rightarrow \sin D\hat{B}C = 0.5$

$\Rightarrow D\hat{B}C\ 30°$ or $150°$

(d) Finding $A\hat{B}C$ $(60° + D\hat{B}C)$

$(AC)^2 = (AB)^2 + (BC)^2 \Rightarrow (AC)^2 = 9.2^2 + 3.2^2$

$\Rightarrow AC = 9.74$ (cm)

(e) Area $= \frac{1}{2} \times 9.2 \times 7.1 \sin 60° = 28.28...$

Area of ABCD = 28.28... + 5.68= 34.0 (cm^2)

46. (a) $BD = \sqrt{4^2 + 8^2 - 2\times 4\times 8\cos\theta} = \sqrt{16(5-4\cos\theta)} = 4\sqrt{5-4\cos\theta}$

(b) (i) $BD = 5.5653$ ...

$$\frac{\sin C\hat{B}D}{12} = \frac{\sin 25}{5.5653} \Rightarrow \sin C\hat{B}D = 0.911$$

(ii) $C\hat{B}D = 65.7°$ or $C\hat{B}D = 180 - 65.7° = 114°$

(iii) $B\hat{D}C = 89.3°$

$$\frac{BC}{\sin 89.3} = \frac{5.5653}{\sin 25} \text{ or } \frac{BC}{\sin 89.3} = \frac{12}{\sin 65.7}$$ (or cosine rule)

$\Rightarrow BC = 13.2$

Perimeter = 4 + 8 + 12 + 13.2 = 37.2

(c) Area $= \frac{1}{2} \times 4 \times 8 \times \sin 40° = 10.3$

47. (a) $AC^2 = 5^2 + 4^2 - 2 \times 4 \times 5\cos x \Rightarrow AC = \sqrt{41 - 40\cos x}$

(b) $$\frac{AC}{\sin x} = \frac{4}{\sin 30} \Rightarrow AC = 8\sin x$$

(c) (i) $8\sin x = \sqrt{41-40\cos x} \Rightarrow x = 111.32$ to 2 dp

(ii) $AC = 8\sin 111.32 = 7.45$

(d) (i) $7.45^2 = 32 - 32\cos y \Rightarrow y = 137$

(ii) Area $= \frac{1}{2} \times 4 \times 4 \times \sin 137 = 5.42$

48. (a) $\dfrac{6}{\sin A} = \dfrac{\frac{7\sqrt{2}}{2}}{\sin 45°} \Rightarrow \sin A = 6 \times \dfrac{\sqrt{2}}{2} \times \dfrac{2}{7\sqrt{2}} = \dfrac{6}{7}$

(b) (i) $B\hat{D}C + B\hat{A}C = 180°$

(ii) $\sin A = \dfrac{6}{7}$ => A = 59.0° or 121° (3 sf)

$\Rightarrow B\hat{C}D = 180° - (121° + 45°) = 14.0°$ (3 sf)

(iii) $\dfrac{BD}{\sin 14°} = \dfrac{\frac{7\sqrt{2}}{2}}{\sin 45°}$ =>BD = 1.69

(c) $\dfrac{\text{Area}\,\Delta BCD}{\text{Area}\,\Delta BAC} = \dfrac{\frac{1}{2} BD \times 6 \sin 45}{\frac{1}{2} BA \times 6 \sin 45} = \dfrac{BD}{BA}$

49. (a) $BC^2 = 65^2 + 104^2 - 2\,(65)\,(104)\cos 60° \Rightarrow BC = 91$ m

(b) area = $\frac{1}{2}(65)\,(104)\sin 60° = 1690\sqrt{3}$ ($p = 1690$)

(c) (i) $A_1 = \left(\frac{1}{2}\right)(65)\,(x)\sin 30° = \dfrac{65x}{4}$

(ii) $A_2 = \left(\frac{1}{2}\right)(104)\,(x)\sin 30° = 26x$

(iii) $A_1 + A_2 = A \Rightarrow \dfrac{65x}{4} + 26x = 1690\sqrt{3}$

$\Rightarrow \dfrac{169x}{4} = 1690\sqrt{3}$

$x = \dfrac{4 \times 1690\sqrt{3}}{169} \Rightarrow x = 40\sqrt{3}$ ($q = 40$)

(d) (i) supplementary angles have equal sines

(ii) using sin rule in ΔADB and ΔACD

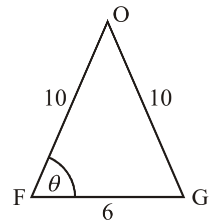
$\dfrac{BD}{\sin 30°} = \dfrac{65}{\sin A\hat{D}B} \Rightarrow \dfrac{BD}{65} = \dfrac{\sin 30°}{\sin A\hat{D}B}$

and $\dfrac{DC}{\sin 30°} = \dfrac{104}{\sin A\hat{D}B} \Rightarrow \dfrac{DC}{104} = \dfrac{\sin 30°}{\sin A\hat{D}C}$

since $\sin A\hat{D}B = \sin A\hat{D}C$

$\dfrac{BD}{65} = \dfrac{DC}{104} \Rightarrow \dfrac{BD}{DC} = \dfrac{65}{104} \Rightarrow \dfrac{BD}{DC} = \dfrac{5}{8}$

50.



(a) (i) $\cos\theta = \left(\dfrac{10^2 + 6^2 - 10^2}{(2)(10)(6)}\right) \Rightarrow \theta = 72.5°$ (3 s.f.)

(ii) h = shortest distance from O to FG = 3 tanθ = 9.53939... = 9.54 m (3 s.f.)

(iii) Area of ΔOFG = $\frac{1}{2}(10)(6)(\sin\theta)$

total surface area of roof = $4 \times \dfrac{1}{2}(10)(6)(\sin\theta)$ = 114.4727... = 114 m^2 (3 s.f.)

(iv) Let φ = angle between slant height (line) and plane EFGH

$\cos\varphi = \left(\dfrac{3}{h}\right) \Rightarrow \varphi = 71.7°$ (3 s.f.)

(v) H = Height of tower from base to O = $40 + \sqrt{h^2 - 3^2}$ = 49.055... = 49.1 m (3 s.f.)

(b) Height (BP) = $\dfrac{6\sin 79°}{\sin(90° - 79°)}$ = 30.9 m (3 s.f.)

51. (a) (i) AP = $\sqrt{(x-8)^2 + (10-6)^2} = \sqrt{x^2 - 16x + 80}$

(ii) OP = $\sqrt{(x-0)^2 + (10-0)^2} = \sqrt{x^2 + 100}$

(b) $\cos O\hat{P}A = \dfrac{AP^2 + OP^2 - OA^2}{2AP \times OP} = \dfrac{(x^2 - 16x + 80) + (x^2 + 100) - (8^2 + 6^2)}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$

$= \dfrac{2x^2 - 16x + 80}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}} = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$

(c) For $x = 8$, $\cos O\hat{P}A = 0.780869 \Rightarrow O\hat{P}A = 38.7°$ (3 sf)

OR $\tan O\hat{P}A = \dfrac{8}{10} \Rightarrow O\hat{P}A = 38.7°$ (3 sf)

(d) $O\hat{P}A = 60° \Rightarrow \cos O\hat{P}A = 0.5$

$0.5 = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}} \quad \Rightarrow x = 5.63$

(e) (i) $f(x) = 1$ when $\cos O\hat{P}A = 1$, hence, when $O\hat{P}A = 0$.

This occurs when the points O, A, P are collinear.

(ii) The line (OA) has equation $y = \dfrac{3x}{4}$ When $y = 10$, $x = \dfrac{40}{3}$ $(= 13\frac{1}{3})$

OR directly $x = \dfrac{40}{3}$ $(= 13\frac{1}{3})$

52. (a) Sine rule $\frac{\text{PR}}{\sin 35} = \frac{9}{\sin 120} \Rightarrow \text{PR} = 5.96$ km

(b) **EITHER** Sine rule $\text{PQ} = \frac{9 \sin 25}{\sin 120} = 4.39$ km

OR Cosine rule: $\text{PQ}^2 = 5.96^2 + 9^2 - (2)(5.96)(9) \cos 25 = 19.29 \Rightarrow \text{PQ} = 4.39$ km

Time for Tom = $\frac{4.39}{8}$ Time for Alan = $\frac{5.96}{a}$

Then $\frac{4.39}{8} = \frac{5.96}{a} \Rightarrow a = 10.9$

(c) $\text{RS}^2 = 4\text{QS}^2$

$4\text{QS}^2 = \text{QS}^2 + 81 - 18 \times \text{QS} \times \cos 35 \Rightarrow 3\text{QS}^2 + 14.74\text{QS} - 81 = 0$

$\Rightarrow \text{QS} = -8.20$ **or** $\text{QS} = 3.29$

therefore QS = 3.29

OR

$\frac{\text{QS}}{\sin \text{S}\hat{\text{R}}\text{Q}} = \frac{2\text{QS}}{\sin 35} \Rightarrow \sin \text{S}\hat{\text{R}}\text{Q} = \frac{1}{2} \sin 35 \Rightarrow \text{S}\hat{\text{R}}\text{Q} = 16.7°$

Therefore, $\text{Q}\hat{\text{S}}\text{R} = 180 - (35 + 16.7) = 128.3°$

$\frac{9}{\sin 128.3} = \frac{\text{QS}}{\sin 16.7} \Rightarrow \text{QS} = 3.29$