# PROJECTILE MOTION *Circles*

This calculation shows that all points on trajectories with a given initial velocity that correspond to a given time will fall on a circle, regardless of the initial angle. First we have the parametric equations of motion:

$$x(t) = v_0\, t\, \cos(\theta) \qquad\qquad y(t) = v_0\, t\, \sin(\theta) - \frac{1}{2}\, g\, t^2$$

Note that the initial y is zero. Eliminate the angle from these:

$$\theta = \mathrm{acos}\left(\frac{x}{v_0\, t}\right) = \mathrm{asin}\left(\frac{y + \frac{1}{2}\, g\, t^2}{v_0\, t}\right)$$

Consider the triangle that corresponds to this angle. In standard position, the hypotenuse is $v_0$ t and the others are as expected, x and y. Therefore we have

$$x^2 + \left(y + \frac{1}{2}\, g\, t^2\right)^2 = \left(v_0\, t\right)^2$$

which is the equation of a circle with center at $\left(0, \frac{-1}{2}\, g\, t^2\right)$ and a radius of $v_0\, t$ So for some given time t, we will have a circle that corresponds to the position of the projectiles at that time, for any launch angle.