

Cálculo vectorial

Ejercicios unidad 13.7 $\rightarrow$ Planos tangentes y rectas normales

En los ejercicios 5 a 16, hallar un vector unitario normal a la superficie en el punto dado.

[Sugerencia: Normalizar el vector gradiente $\nabla f(x, y, z)$].

7) Superficie = $x^2 + y^2 + z^2 = 6$

Punto = $(1, 1, 2)$

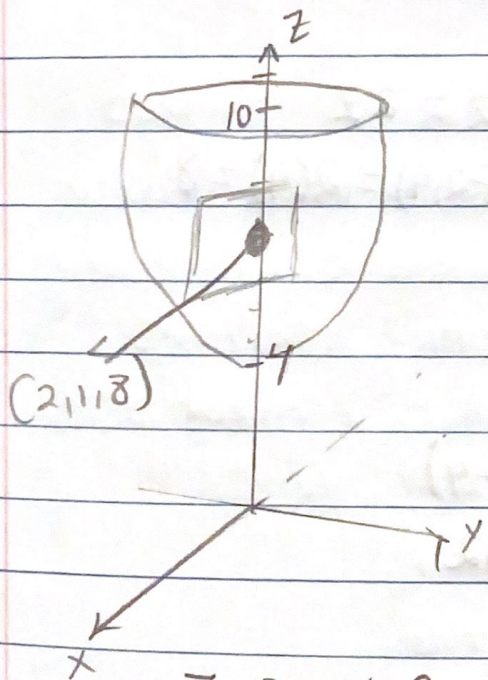
$$\nabla f(x, y, z) = \langle f_x, f_y, f_z \rangle$$
$$= \langle 2x, 2y, 2z \rangle$$

$$\nabla f(1, 1, 2) = \langle 2(1), 2(1), 2(2) \rangle$$
$$= \langle 2, 2, 4 \rangle$$

$$\vec{V} = \frac{\nabla f(1, 1, 2)}{\|\nabla f(1, 1, 2)\|} = \frac{\langle 2, 2, 4 \rangle}{\sqrt{2^2 + 2^2 + 4^2}} = \langle 2, 2, 4 \rangle \cdot \frac{1}{\sqrt{24}}$$
$$= \langle 2, 2, 4 \rangle \cdot \frac{\sqrt{24}}{24}$$

En los ejercicios 17 a 30, hallar una ecuación del plano tangente a la superficie en el punto dado.

17) $z = x^2 + y^2 + 3$, $(2, 1, 8)$



$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f_x = 2x$$

$$f_x(2, 1) = 2(2) = 4$$

$$f_y = 2y$$

$$f_y(2, 1) = 2(1) = 2$$

$$z - 8 = 4(x - 2) + 2(y - 1)$$

$$z - 8 = 4x - 8 + 2y - 2$$

$$z = 4x + 2y - 2$$

$$4x + 2y - z - 2 = 0$$

$$27) x^2 + 4y^2 + z^2 = 36 \quad (2, -2, 4)$$

$$Z - Z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \quad (F1)$$

$$f_x = 2x$$

$$f_z = 2z$$

$$f_x(2, -2) = 2(2) = 4$$

$$f_z(-2, 4) = 2(4) = 8$$

$$f_y = 8y$$

$$f_y(2, -2) = 8(-2) = -16$$

$$0 = 4(x - 2) - 16(y + 2) + 8(z - 4)$$

$$0 = 4x - 8 - 16y - 32 + 8z - 32$$

$$0 = 4x - 16y + 8z - 72$$

$$0 = x - 4y + 2z - 18$$

Hallar una ecuación del plano tangente y hallar ecuaciones simétricas para la recta normal a la superficie en el punto dado.

$$35) z = x^2 - y^2, \quad (3, 2, 5)$$

$$f_x = 2x$$

$$f_y = -2y$$

$$f_x(3, 2) = 2(3) = 6$$

$$f_y(3, 2) = -2(2) = -4$$

$$z - 5 = 6(x - 3) - 4(y - 2)$$

$$z = 6x - 18 - 4y + 8 + 5$$

$$0 = 6x - 4y - z - 5$$

$$\left\{ \begin{array}{l} \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \\ \frac{x - 3}{6} = \frac{y - 2}{-4} = \frac{z - 5}{-1} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{x - 3}{6} = \frac{y - 2}{-4} = \frac{z - 5}{-1} \end{array} \right.$$

$$37) \quad xyz = 10, \quad (1, 2, 5)$$

$$f_x = yz$$

$$f_x(1, 2, 5) = (2)(5) = 10$$

$$f_y = xz$$

$$f_y(1, 2, 5) = (1)(5) = 5$$

$$f_z = xy$$

$$f_z(1, 2, 5) = (1)(2) = 2$$

$$\begin{cases} x-1 = y-2 = z-5 \\ 10 \quad 5 \quad 2 \end{cases}$$

$$0 = 10(x-1) + 5(y-2) + 2(z-5)$$

$$0 = 10x - 10 + 5y - 10 + 2z - 10$$

$$0 = 10x + 5y + 2z - 30$$