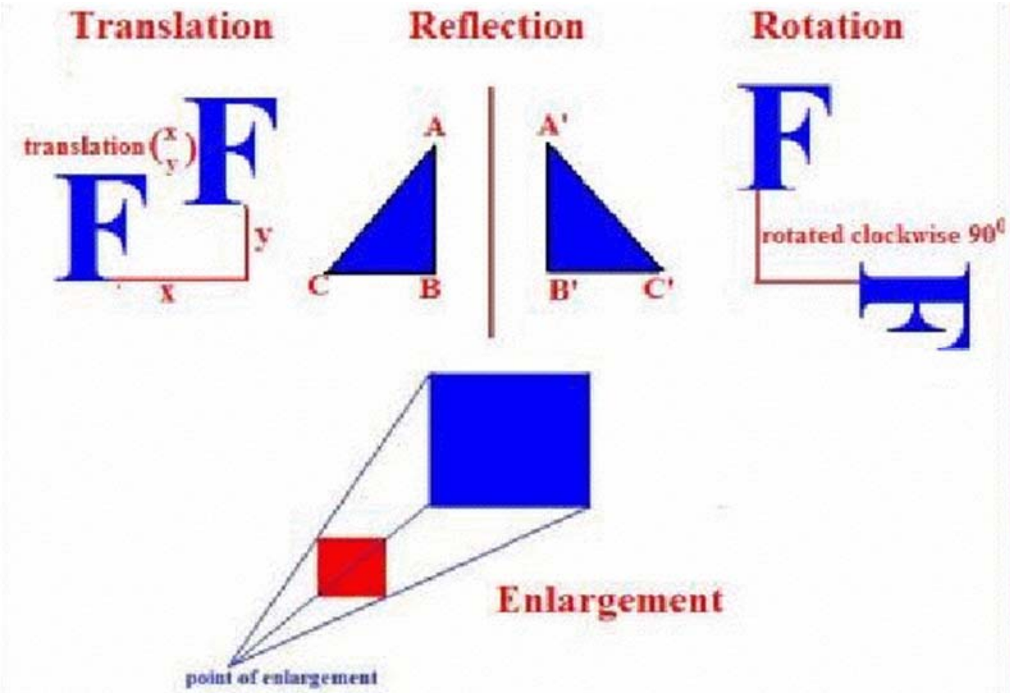
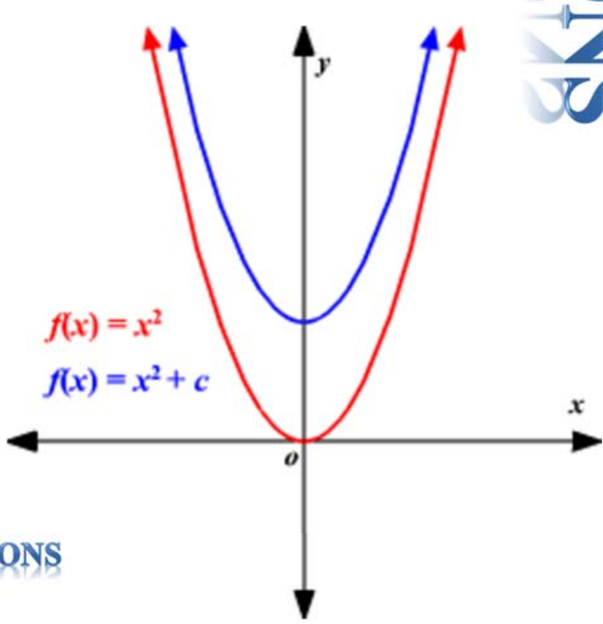
TRANSFORMATIONS



TRANSFORMATIONS



NAME:

TRANSFORMATIONS

Transformations: $f(x) \rightarrow f(x) = af(n(x-b)) + c$ **or** $(x, y) \rightarrow \left(\frac{x}{n} + b, ay + c\right)$

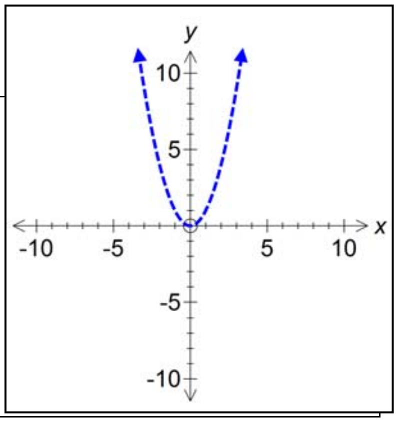
- Transformations of a function is one of the following:
  - Dilation (STRETCH) (from the *x*-axis or *y*-axis);
  - Reflection (FLIP) (in *x*-axis or *y*-axis);
  - Translation (SLIDE) (vertically and\or horizontally);
  - Rotation (we don't study these).

- The order to deal with the transformations is **DRT** (alphabetical)
- The **Cartesian Plane** is represented by the set $\boldsymbol{R}^2$ of all ordered pairs of real numbers.

Dilations

- This is a stretch or contraction of the graph from the *x*-axis or the *y*-axis
- a causes a dilation of factor a from the *x*-axis $(x, y) \rightarrow (x, ay)$
- n causes a dilation of factor $\frac{1}{n}$ from the *y*-axis $(x, y) \rightarrow (\frac{x}{n}, y)$
- We describe the dilations like:
  - The graph is dilated by a factor of a from the *x*-axis, or
  - The graph is dilated by a factor of a parallel to the *y*-axis
  - The graph is dilated by a factor of $\frac{1}{n}$ from the *y*-axis

Example: Sketch the graph of $f(x) = 3x^2$ by comparing it to $f(x) = x^2$



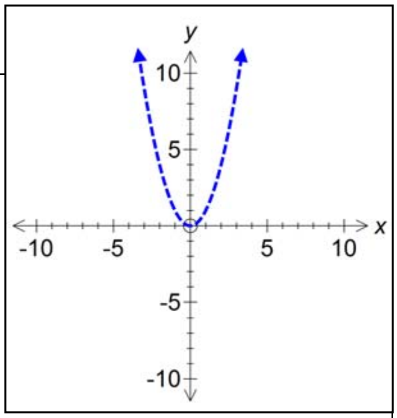
Here $a =$ ___

First sketch $f(x) = x^2$

Then multiply each *y* value by ____.

The graph is _______ by a factor of ___ from the _________.

Example: Sketch $f(x) = (2x)^2$



Here $n =$ _____

First sketch $f(x) = x^2$

Then multiply each x value by ______ .

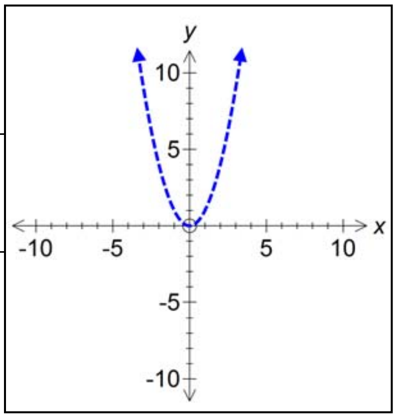
The graph is ________ by a factor of ____ from the __________.

Could also be a dilation of factor 4 from the *x*-axis. Why?

Reflections

- There are three types of reflections:
  - In the x-axis, $y = -f(x)$, $(x, y) \rightarrow (x, -y)$
  - In the y-axis, $y = f(-x)$, $(x, y) \rightarrow (-x, y)$
  - In the line $y = x$, which we dealt with in **Inverse functions**.

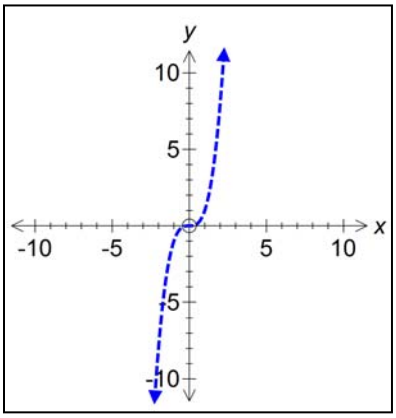
Reflections in the *x*-axis, $y = -f(x)$ or when $a < 0$



Example: Sketch $f(x) = \dfrac{-x^2}{2}$

Here $a =$ The graph is _______ in the ______ and _______ by a factor of__ from ______.

Reflections in the *y*-axis, $y = f(-x)$



Example: Sketch $f(x) = x^3 + 3$, $f(-x)$ and $-f(-x)$.

- $-f(-x)$ is a reflection in both x & y axes.

Reflections Worksheet #1

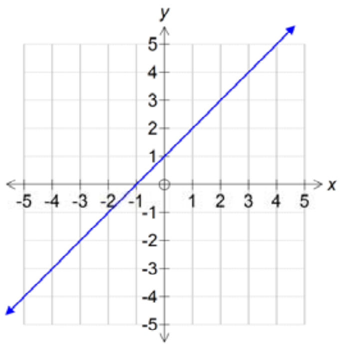
For each of the following graphs of $y = f(x)$, sketch:

(i) $y = -f(x)$

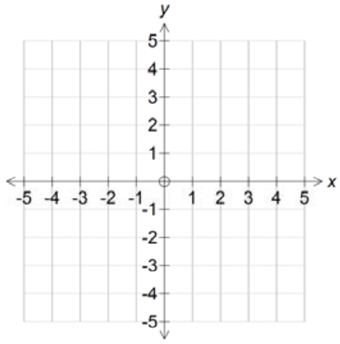
(ii) $y = f(-x)$

(iii) $y = -f(-x)$

(a)



(i)



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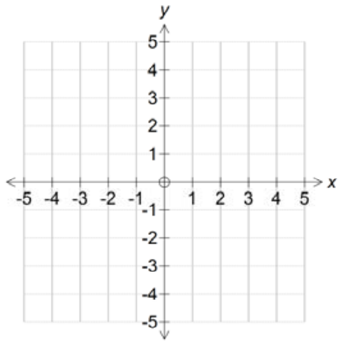

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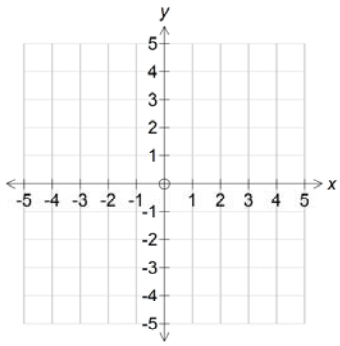

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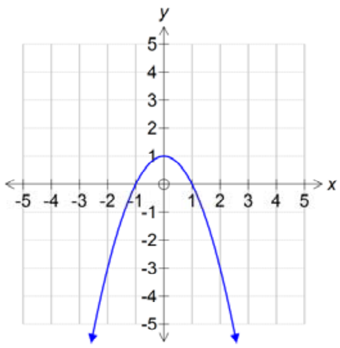
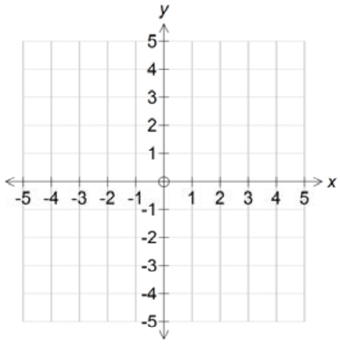


(i)



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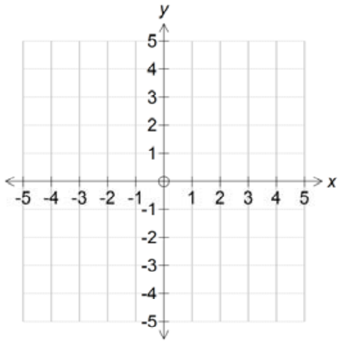

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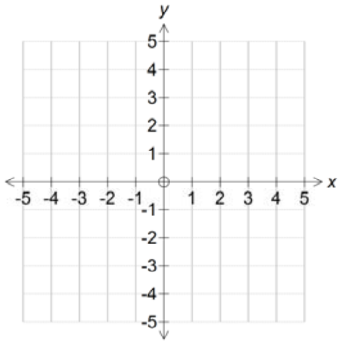
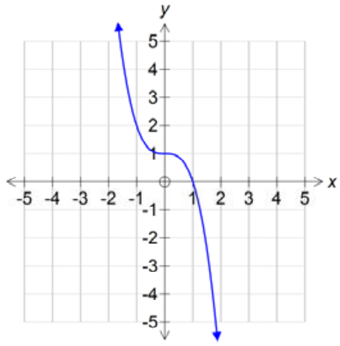
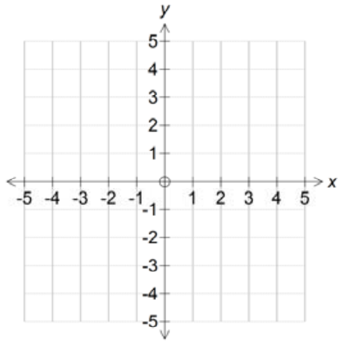


(c)



(i)



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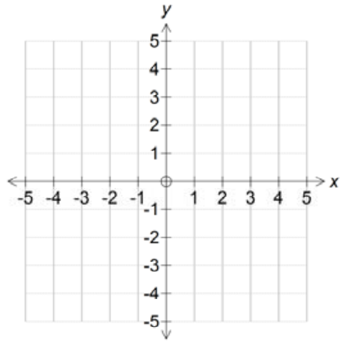

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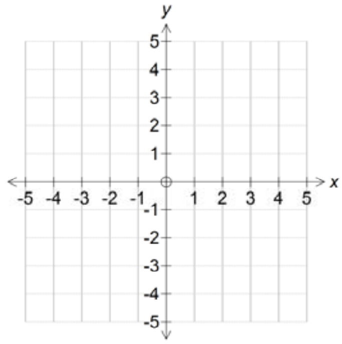


Reflections Worksheet #2

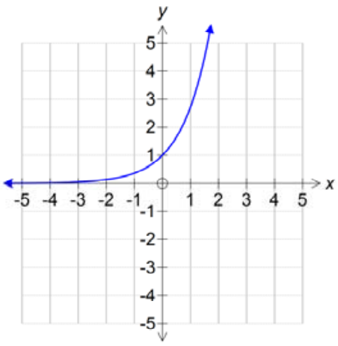
For each of the following graphs of $y = f(x)$, sketch:

(i) $y = -f(x)$

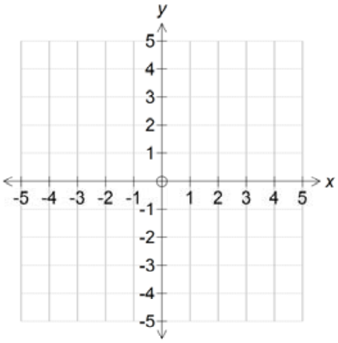
(ii) $y = f(-x)$

(iii) $y = -f(-x)$

(d)



(i)



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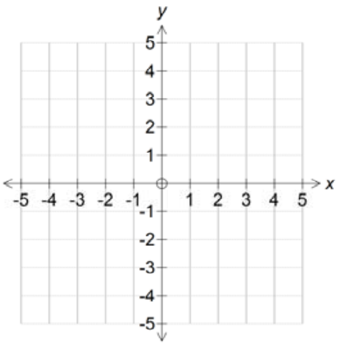

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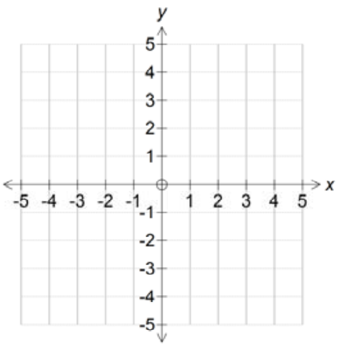
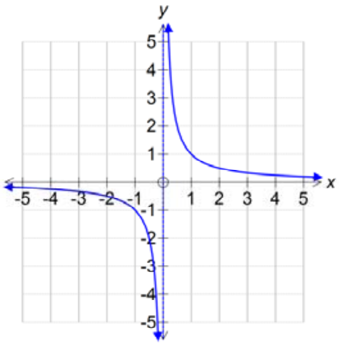
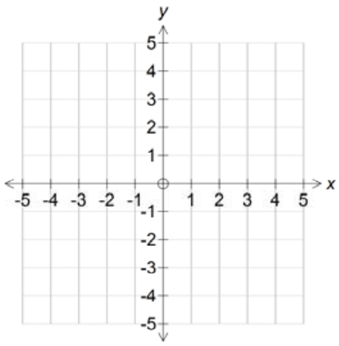


(e)



(i)



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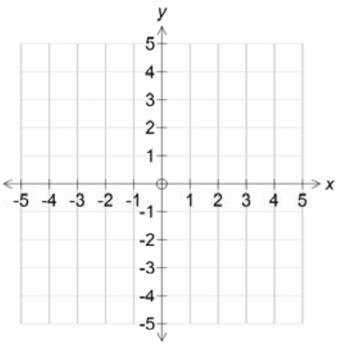

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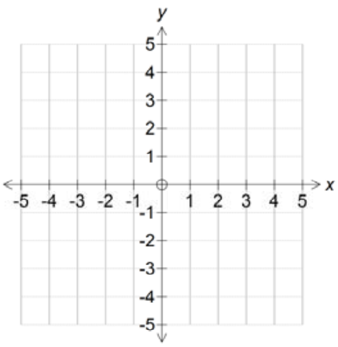

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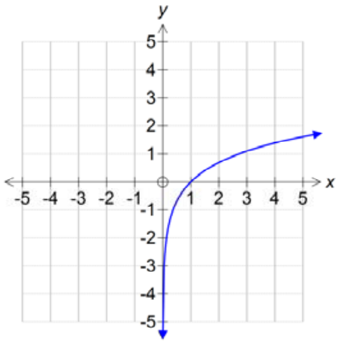
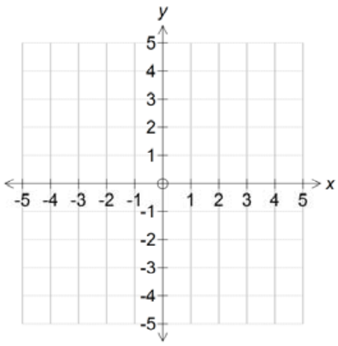


(i)



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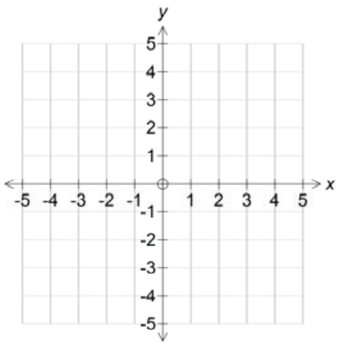

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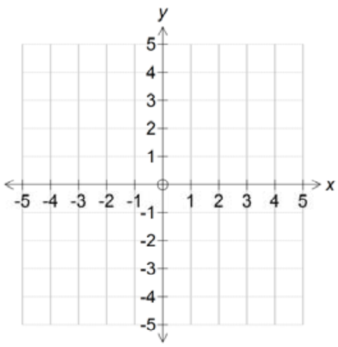
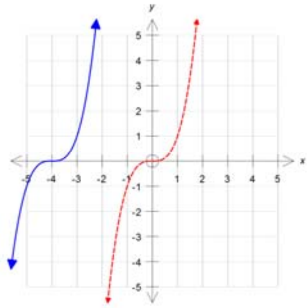


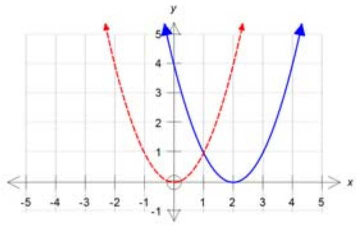
Translations

- There are two types of translations:
  - Along the direction of the x-axis : $f(x) = f(x-b)$; $(x, y) \longrightarrow (x + b, y)$
  - Along the direction of the y-axis: $f(x) = f(x)+c$.$(x, y) \longrightarrow (x, y + c)$

1. Along the direction of the *x*-axis : $f(x) = f(x-b)$

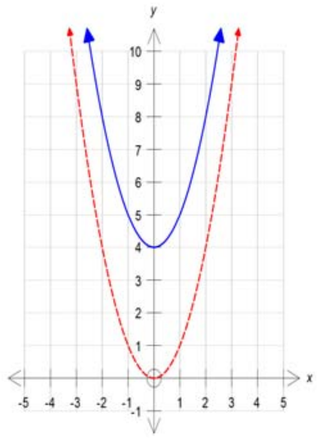


- Sketch the graph of $f(x) = (x+4)^3$.
- Translation of 4 units in the negative direction of the x-axis.



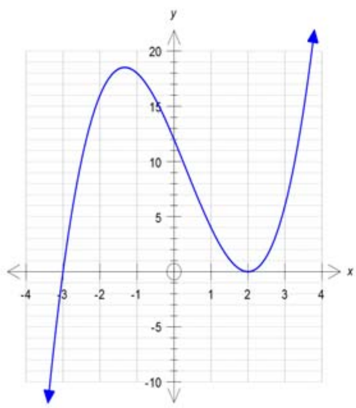
- Sketch the graph of $f(x) = (x-2)^2$
- __________ of __ units in the ________ direction of the _-axis.

2. Along the direction of the *y*-axis: $f(x) = f(x)+c$



- Sketch the graph of $f(x) = x^2 + 4$
- __________ of __ units in the ________ direction of the _-axis.

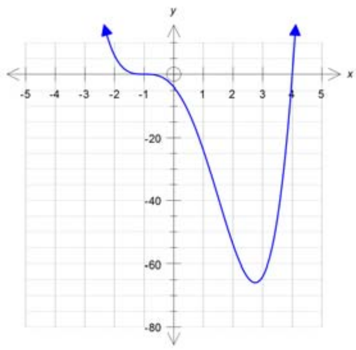
"Repeated Factor Squared"



- Consider the function $f(x) = (x+3)(x-2)^2$
- The X – intercepts are ___ and ___
- (2, 0) is also a _________________
- "A repeated factor squared is both an _____ and a _____________"

“Repeated Factor Cubed”

- Consider the function $f(x)=(x+1)^3(x-4)$
- The X – intercepts are ____ and ____ .
- (-1, 0) is also a _______ __ ____________.
- “A repeated factor cubed is both an _________and a ____________



- **Ex4A** Q 1, 3; **Ex 3A** Q 7 ab**;** **Ex 3B** Q 11a;
- **Ex 3C** Q 1; **Ex 3D** Q 1a; **Ex 3E** Q 2 abe, 3 bc

Transformations Summary $f(x)\to af(n(x-b))+c$ or $(x,y)\to\left(\frac{x}{n}+b, ay+c\right)$

Example 1: State the transformations from $f(x)$ to $y=-2f(3(x+4))-1$.

Example 2: Describe the transformations undergone by $y=\log_e x$ to $y=1-3\log_e(2x-8)$.

Example 3: Write the equation of the rule when $y=x^2$ is transformed by:

- a translation of 1 unit in the positive direction of the x – axis and 2 units in the positive direction of the y – axis, followed by,
- a dilation of factor of 2 from the y – axis, followed by,
- a reflection in the x – axis.

Exercise on Sequence of Transformations

1. State the sequence of transformations that each of the following functions have undergone from $y=f(x)$**.**

(a) $y=3f(-2(x+3))+4$.

(b) $y=0.5f(3(x-2))+1$

(c) $y=2f(-0.4(x+3))-0.2$

(d) $y=2-3f(2x+1)$

2. Describe the transformations undergone by each of the following functions to produce the second function.

(a) $y=\log_e x$ to $y=4\log_e 2(x+3)-5$

(b) $y=\sqrt{x}$ to $y=2\sqrt{3x+4}+5$

(c) $y=\cos x$ to $y=-3\cos\left(2x+\frac{\pi}{4}\right)+1$

(d) $y=x^6$ to $y=3(2x+5)^6-2$

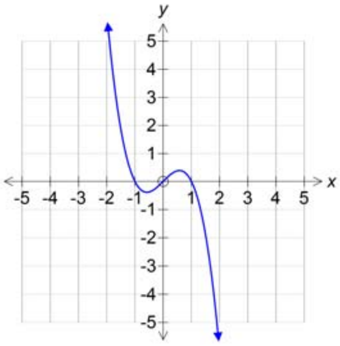
(e) $y=\sin x$ to $y=2\sin\pi(3x-4)$

- **Ex4E** Q 1,2, 3, 4; **Ex 3A** Q 7 d**;** **Ex 3B** Q 4; **Ex 3C** Q 2b, 4a; **Ex 3D** Q 4d; **Ex 3E** Q 1a; **Ex4F** Q 1, 2, 3, 4, 5, 6

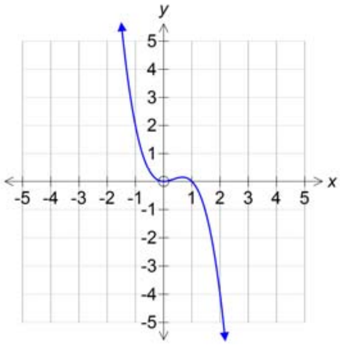
Determining a Rule for a Function from a Graph

- **Worksheet** – Matching Graphs to their rules
  Match the following graphs with the correct equation:

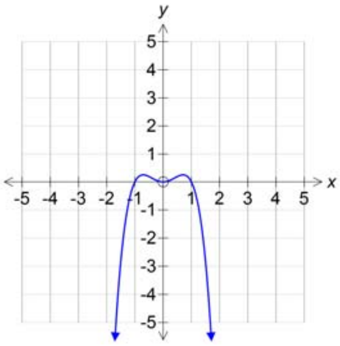
(a)



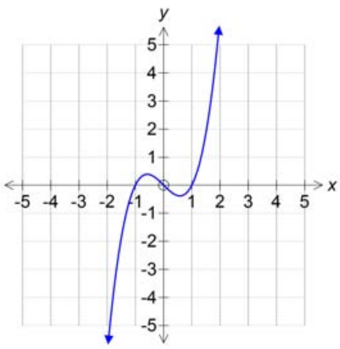
(b)



(c)



(d)



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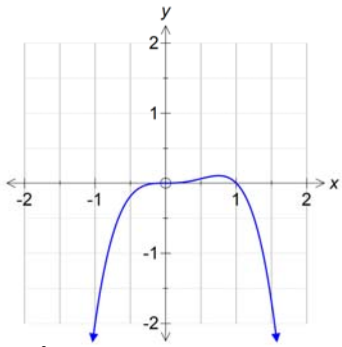

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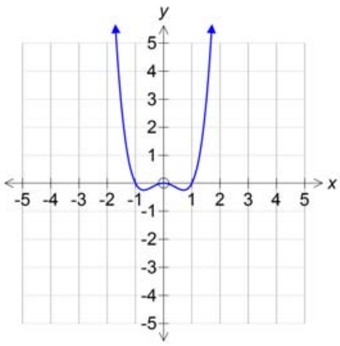
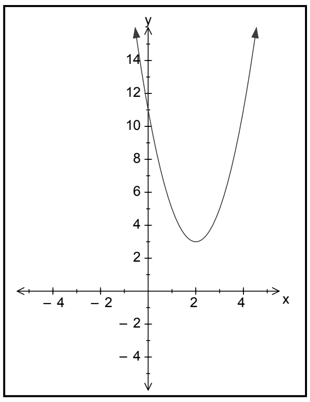


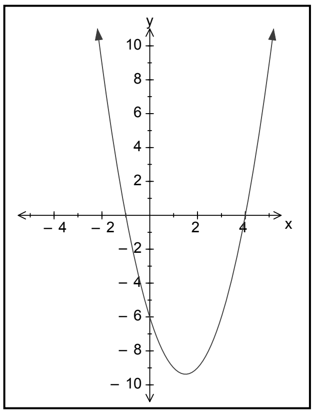
A: $y = x^3(1-x)$ B: $y = x(1-x^2)$ C: $y = x^2(x^2-1)$

D: $y = x^2(1-x)$ E: $y = x(x^2-1)$ F: $y = x^2(1-x^2)$

- **Example:** Find the rule for:

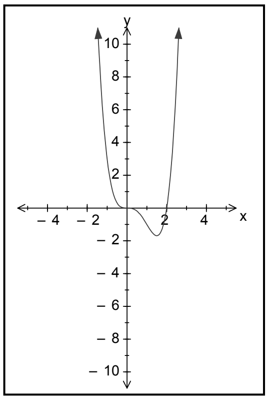


- **Example:** Find the rule for:

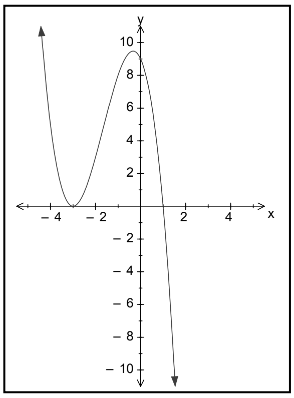


- *Graphical Calculator can be used for this example.*
    - *Insert Lists & Spreadsheet*
    - *X-values – List1*
    - *Y-values – List2*
    - *Regression – Menu – Statistics - Calculations*

- **Example:** Find the rule for: (1, -1) on curve.



- **Example:** Find the rule for:



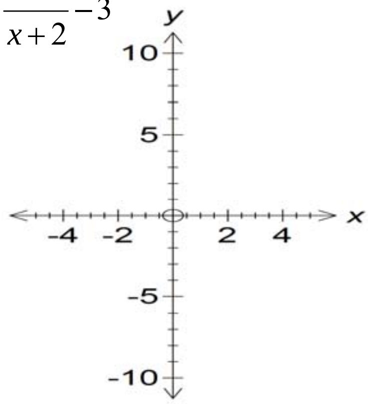
- **Ex4A** Q 8, 9; **Ex4B** Q 1, 2, 3, 4, 7, 8 , 9; **Ex4G** Q 1, 3, 4, 5, 6, 7, 8; **Ex3G** 3, 4, 5, 6, 7ab, 8, 9

Transformations of $f(x) = x^p;\ p = -1, -3, \ldots$

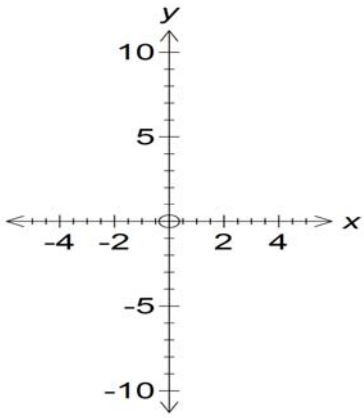
- $\dfrac{1}{x^p} \to \dfrac{a}{(n(x-b))^p} + c \quad or \quad af(n(x-b)) + c$
- **Examples:** Sketch the graph of:
- *(i)* $f(x) = \dfrac{2}{x}$ *(ii)* $f(x) = \dfrac{1}{2x^3}$ *(iii)* $f(x) = \dfrac{4}{x-2}$

(iv) $f(x) = \dfrac{4}{2-x}$ *(v)* first show that $f(x) = \dfrac{-3x-3}{x+2}$ is equal to $f(x) = \dfrac{3}{x+2} - 3$

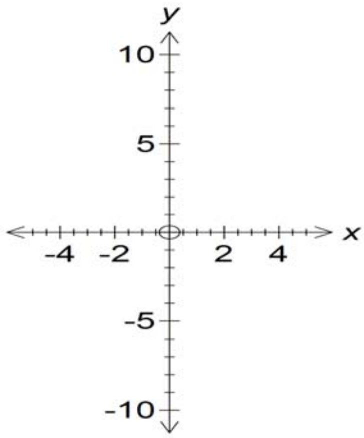
(i)



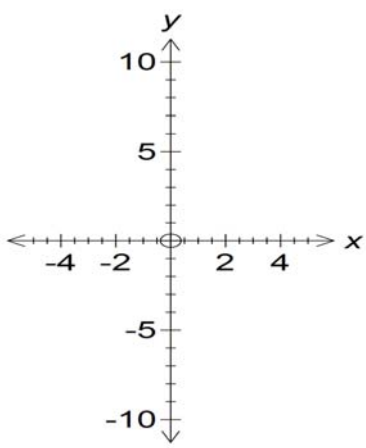
(ii)



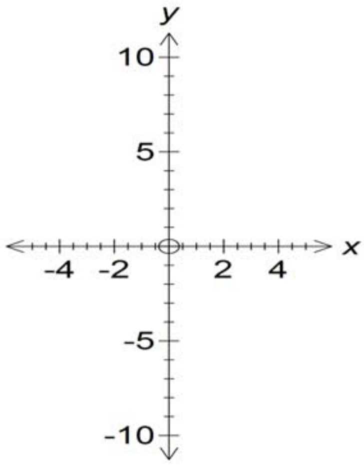
(iii)



(iv)



(v) $\boxed{f(x) = \dfrac{-3x-3}{x+2} =}$



- **Ex3A** Q 2, 3 aefghk, 4, 5 be, 8b; **Ex3B** Q 1, 5 ab, 7 **Ex3D** Q 4 c; **Ex3E** Q 4af; **Ex3F** 1 abef, 2 dgij, 3, 4, 5 ab

Hint Ex 3F Q4

$$y = \frac{4x+5}{2x+3} = \frac{2(2x+3)-1}{2x+3} = 2 - \frac{1}{2x+3}$$

Or synthetic division

$$2x+3=0$$

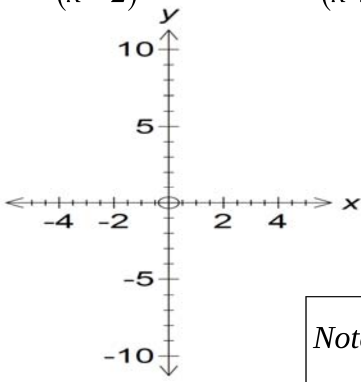
$$x = -\frac{3}{2}$$ first (divide all terms by 2)

$$y = \frac{4x+5}{2x+3} = \frac{2x+\frac{5}{2}}{x+\frac{3}{2}}$$

Transformations of $f(x)=x^{p}; p=-2,-4,...$

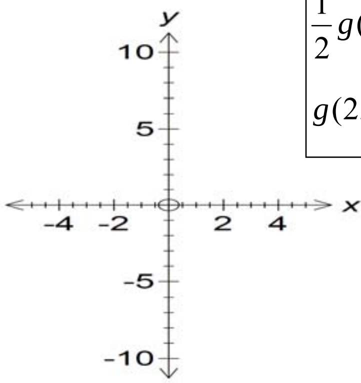
- $\dfrac{1}{x^{p}} \to \dfrac{a}{\left(n(x-b)\right)^{p}}+c$
- **Examples:** Sketch the graph of:
- *(i)* $f(x)=\dfrac{2}{x^{2}}$ *(ii)* $f(x)=\dfrac{1}{2x^{4}}$ *(iii)* $f(x)=\dfrac{4}{(x-2)^{2}}$ *(iv)* $f(x)=\dfrac{-3}{(x+2)^{2}}$ *(v)* $f(x)=\dfrac{3}{(x+2)^{2}}-3$

(i)

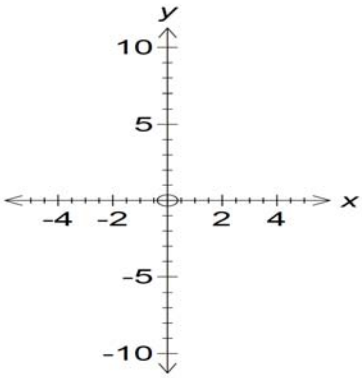


$$Note: g(x)=\frac{1}{x^{2}}$$
$$\frac{1}{2}g(x)=\frac{1}{2x^{2}}; \quad dilation\ \frac{1}{2} \quad from \quad x-axis$$
$$g(2x)=\frac{1}{(2x)^{2}}; \quad dilation\ \frac{1}{2} \quad from \quad y-axis$$

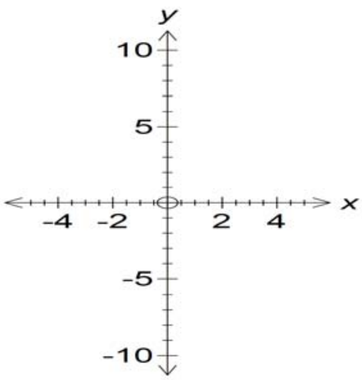
(ii)



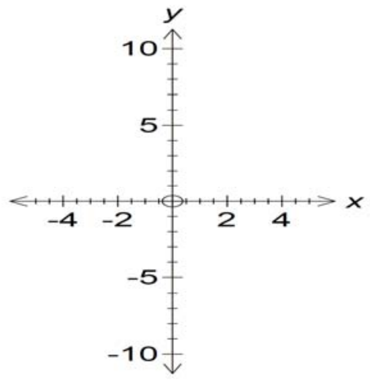
(iii)



(iv)



(v) dilation of factor __, a translation of ____ units down (________ direction of the *y*-axis) and a translation of ___ units to the left (_________ direction of the *x*-axis):



- **Ex3A** Q 3 bcij, 5cd, 7c, 8a; **Ex3B** Q 2, 5d, 6, 10 ae, 11 bde; **Ex3C** Q4 cd; **Ex3D** Q 1c, 4 efg, 6; **Ex3E** Q 1 b, 2 cd, 3a, 4bc; **Ex 3F** 1 cdg, 2h, 5c

Transformations of functions of the form $f(x) = x^{\frac{p}{q}}$

- $x^{\frac{p}{q}} \to a(n(x-b))^{\frac{p}{q}} + c \quad OR \quad x^{\frac{p}{q}} \to a\sqrt[q]{(n(x-b))^{p}} + c$

- **Ex3A**Q 6, 7e, 8c; **Ex3B** Q 3, 5c, 8, 9, 10 bcd, 11 cfg; **Ex 3C** Q 2a, 3, 4 befg; **Ex3D** Q 1b, 4b, 5, 7; **Ex3E** Q 1, 3 de, 4de ; **Ex3F** Q 2 abcef, 5 def

Determining rules for $f(x) = x^n$

Example: It is known that the points (1, 5) and (4, 2) lie on a curve with the equation $y = \frac{a}{x} + b$. Find the values of a and b.

Solution:

Example 2: It is known that the points (2, 1) and (10, 6) lie on a curve with equation $y = a\sqrt{x-1} + b$. Find the equation.

Solution:

- **Ex3H** Q 1, 2, 3, 4, 5, 6, 7, 8

Transformations using Matrices:

- (x', y') is called the image of (x, y).
- the transformations are written as follows:

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\substack{2\times 2 \quad matrix\\ dilations/reflections}}\begin{bmatrix}x\\y\end{bmatrix} + \underbrace{\begin{bmatrix} \\ \end{bmatrix}}_{\substack{2\times 1 \quad matrix\\ translations}}$$

- You can have more than one dilation/reflection matrix.
- Remember: multiply rows by columns, add/subtract elements in the same position.
- The transformation matrices are:

Reflection in the x-axis	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$	$(x, y) \to (x, -y)$ $f(x) \to -f(x)$
Reflection in the y-axis	$\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$	$(x, y) \to (-x, y)$ $f(x) \to f(-x)$
Reflection in the line $y=x$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$(x, y) \to (y, x)$ $f(x) \to f(y)$
Dilation of factor a from the x-axis	$\begin{bmatrix}1 & 0\\0 & a\end{bmatrix}$	$(x, y) \to (x, ay)$ $f(x) \to af(x)$
Dilation of factor k from the y-axis (note $n = 1/k$)	$\begin{bmatrix}k & 0\\0 & 1\end{bmatrix}$	$(x, y) \to (kx, y)$ $f(x) \to f\left(\frac{x}{k}\right)$
Translation Matrix (add)	$\begin{bmatrix}b\\c\end{bmatrix}$	$(x, y) \to (x+b, y+c)$ $f(x) \to f(x-b)+c$

Example 1: find the image of the point (2, 3) under:

a a reflection in the x-axis **b** a dilation of factor 4 from the y-axis

Example 2: Consider a linear transformation such that (1, 0) → (3, -1) and (0, 1) → (-2, 4). Find the image of (-3, 5)

Example 3: A transformation is defined by the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. Find the equation of the graph of $y = \sin(x) + x$, under this transformation.

Solution:

1. Write the dilations in terms of matrices	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix}$
2. Multiply matrices	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \times x + 0 \times y \\ 0 \times x + 3 \times y \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$
3. Determine the result in terms of *x'* and *y'* & rearrange to make *x* and *y* the subject of each equation.	$x' =$ $\qquad$ $y' =$ $x =$ $\qquad$ $y =$
4. Sub each into the original equation.	
5. Rearrange to make *y'* the subject	
6. Then drop the '	

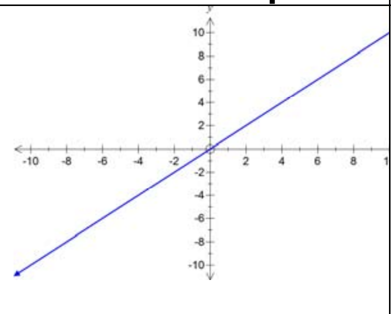
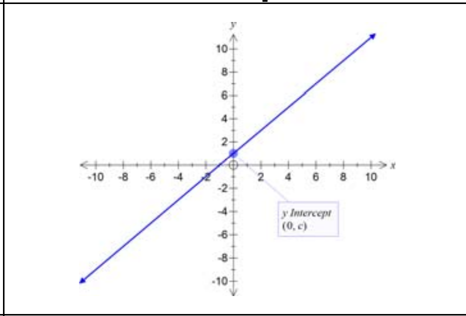
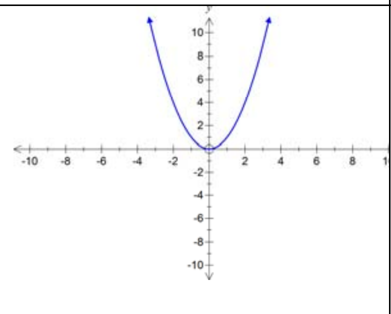
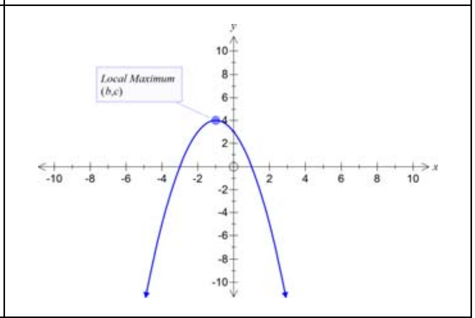
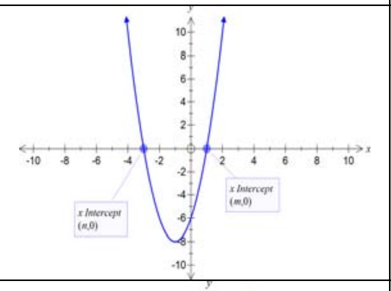
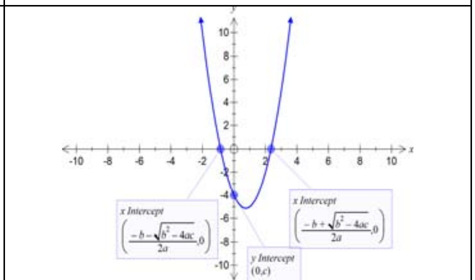
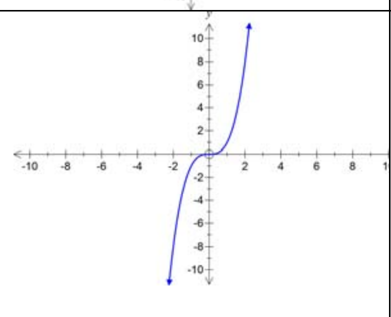
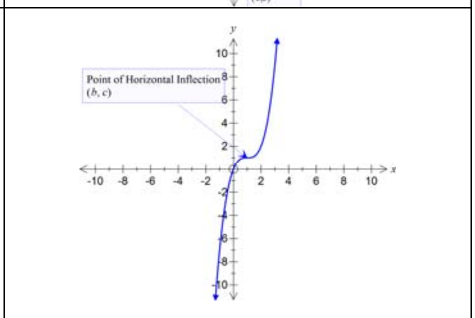
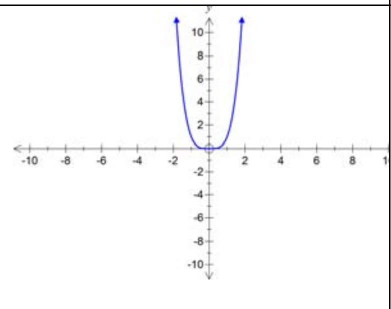
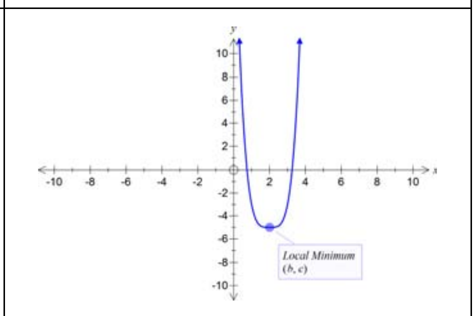
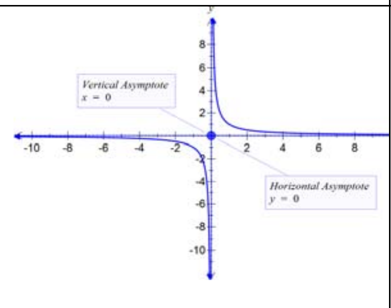
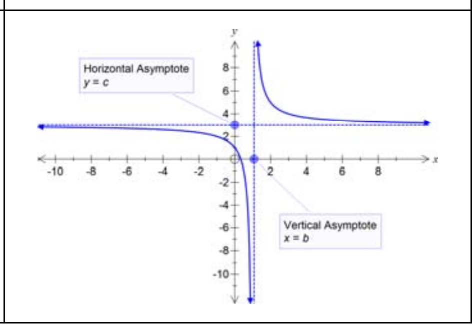
Example 4: A transformation is described by the matrix equation **A(X + B) = X′**, where

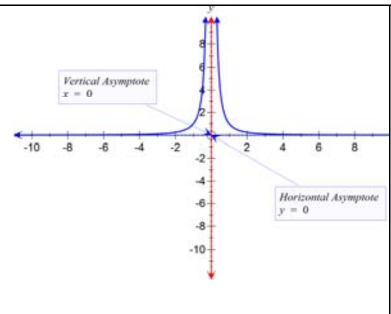
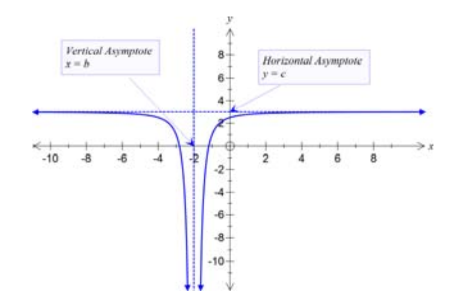
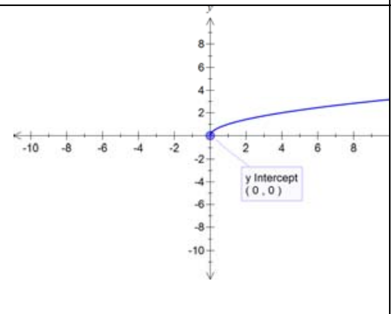
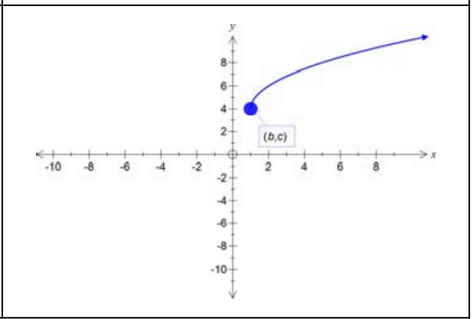
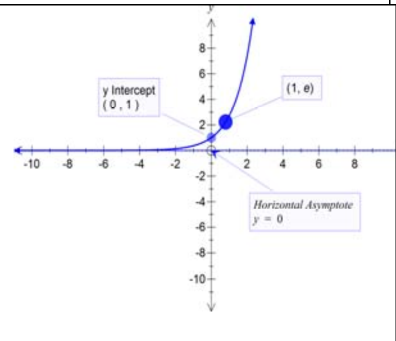
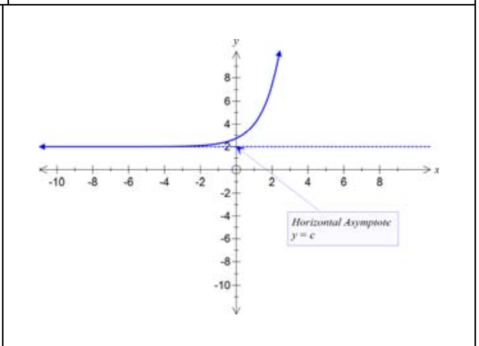
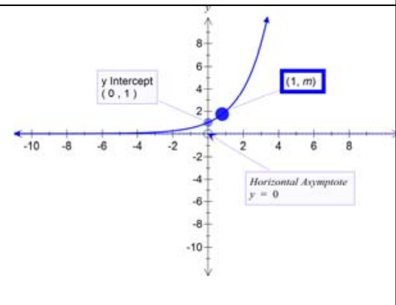
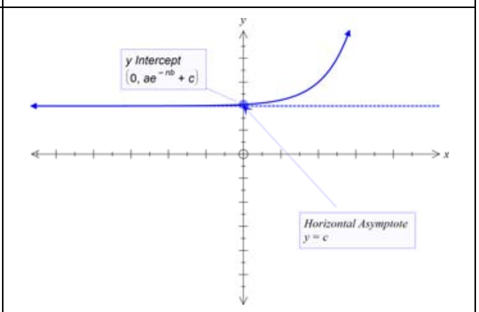
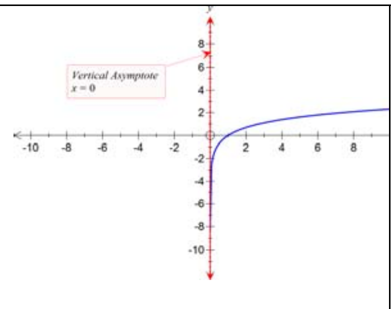
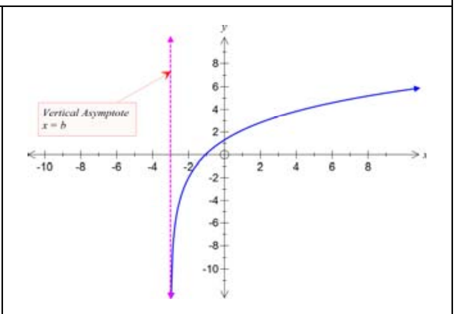
$$A = \begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

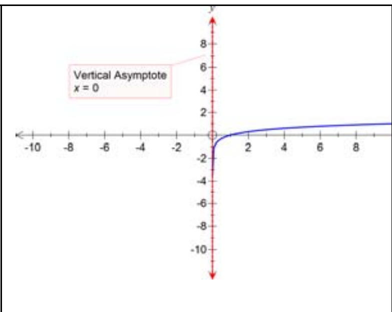
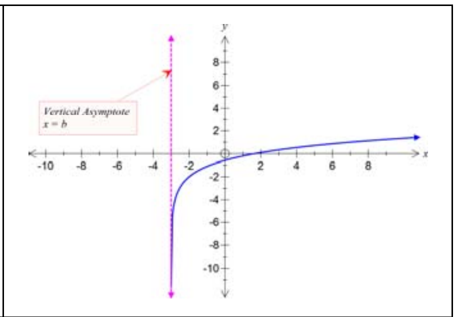
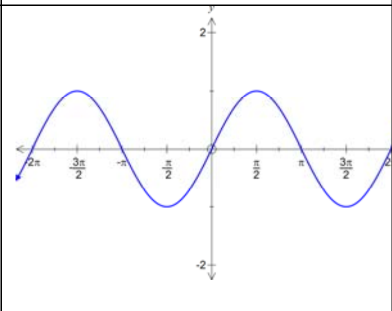
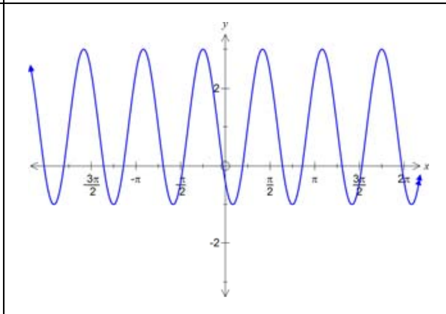
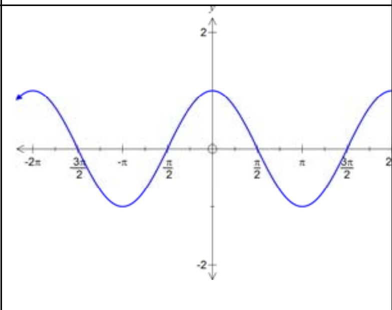
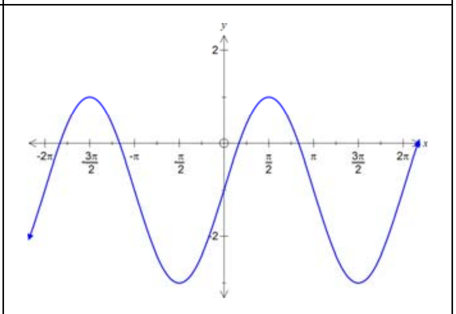
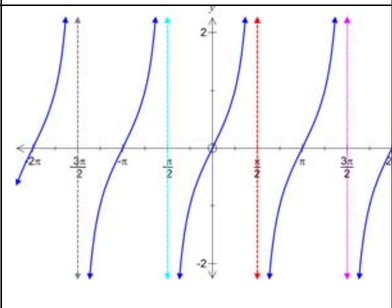
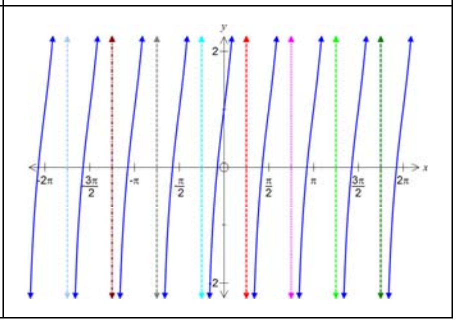
Find the image of the straight line with equation $y = 2x + 5$ under this transformation.

- **Ex3I** Q 1, 3, 4, 6, 7, 8, 9, 10, 11, 13, 15, 16

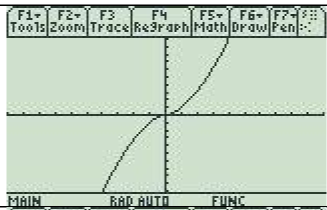
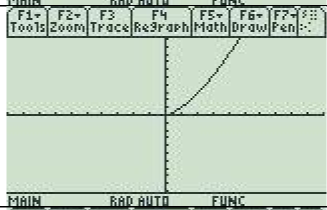
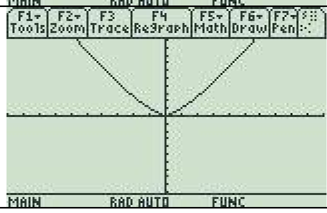
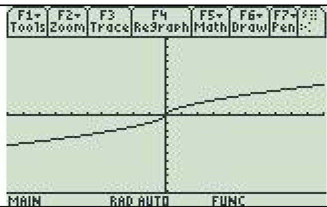
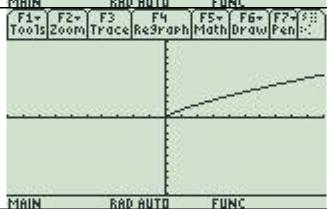
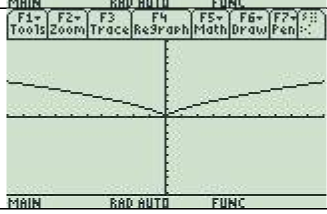
Transformations of standard graphs

$f(x)$	Parent Graph	$af(n(x-b))+c$	Example
x		$ax+c$	
x^2		$a(n(x-b))^2+c$	
$a(x-m)(x-n)$		ax^2+bx+c	
x^3		$a(n(x-b))^3+c$	
x^4		$a(n(x-b))^4+c$	
$\frac{1}{x}$ or x^{-1}		$\frac{a}{n(x-b)}+c$	

$\frac{1}{x^2}$ or x^{-2}		$\frac{a}{(n(x-b))^2}+c$	
$x^{\frac{1}{2}}$ or $\sqrt{x}$		$a\sqrt{n(x-b)}+c$	
$x^{\frac{p}{q}}$ or $\sqrt[q]{x^p}$	See below	$a\sqrt[q]{n(x-b)^p}+c$	See below
e^x		$ae^{n(x-b)}+c$	
m^x		$am^{(n(x-b))}+c$	
$\log_e x$		$a\log_e(n(x-b))+c$	

$\log_{10} x$		$a\log_{10}(n(x-b))+c$	
$\sin x$		$a\sin(n(x-b))+c$	
$\cos x$		$a\cos(n(x-b))+c$	
$\tan x$		$a\tan(n(x-b))+c$	

$$y = x^{\frac{p}{q}}$$

	p	**q**	**Domain**	**Example graph**	**Equations**
p > q $\frac{p}{q} > 1$	p odd	q odd	R		$y = x^3$ $y = x^{\frac{5}{3}}$
	p odd	q even	$x \geq 0$		$y = x^{\frac{3}{2}}$
	p even	q odd	R		$y = x^2$ $y = x^{\frac{4}{3}}$
p < q $\frac{p}{q} < 1$	p odd	q odd	R		$y = x^{\frac{1}{3}}$ $y = x^{\frac{3}{5}}$
	p odd	q even	$x \geq 0$		$y = x^{\frac{1}{2}}$ $y = x^{\frac{3}{4}}$
	p even	q odd	R		$y = x^{\frac{2}{3}}$

VCAA EXAM QUESTIONS for TRANSFORMATIONS

2008

Question 8

The graph of the function $f: D \to R,\ f(x) = \dfrac{x-3}{2-x}$, where D is the maximal domain has asymptotes

A. $x = 3, \quad y = 2$

B. $x = -2, \quad y = 1$

C. $x = 1, \quad y = -1$

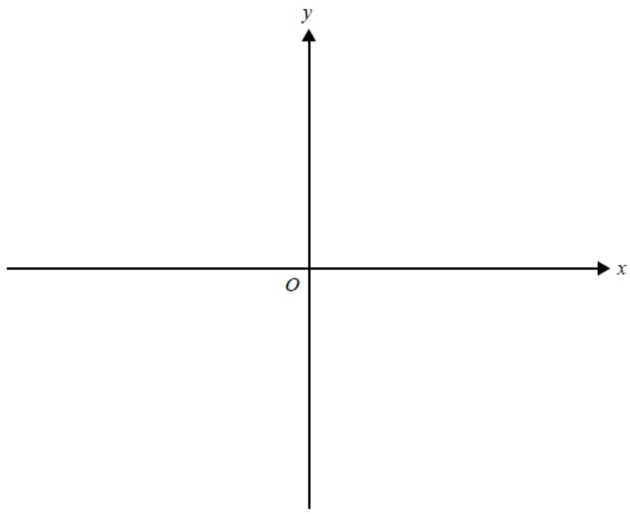
D. $x = 2, \quad y = -1$

E. $x = 2, \quad y = 1$

Question 2

On the axes below, sketch the graph of $f: R\backslash\{-1\} \to R, \quad f(x) = 2 - \dfrac{4}{x+1}$.

Label all axis intercepts. Label each asymptote with its equation.



2009

Question 2

At the point (1, 1) on the graph of the function with rule $y = (x-1)^3 + 1$

A. there is a local maximum.

B. there is a local minimum.

C. there is a stationary point of inflection.

D. the gradient is not defined.

E. there is a point of discontinuity.

Question 21

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x-axis at (2, 0) and (–3, 0). The maximum value of the derivative function is 10.

The value of x for which the graph of $y = f(x)$ has a local maximum is

A. -2

B. 2

C. -3

D. 3

E. $-\dfrac{1}{2}$

2011

Question 3

a. Consider the function $f: R \to R, f(x) = 4x^3 + 5x - 9$.

i. Find $f'(x)$

ii. Explain why $f'(x) \geq 5$ for all x.

1 + 1 = 2 marks

b. The cubic function p is defined by $p: R \to R, p(x) = ax^3 + bx^2 + cx + k$, where a, b, c and k are real numbers.

i. If p has m stationary points, what possible values can m have?

ii. If p has an inverse function, what possible values can m have?

1 + 1 = 2 marks

c. The cubic function q is defined by $q: R \to R, q(x) = 3 - 2x^3$.

i. Write down an expression for $q^{-1}(x)$.

ii. Determine the coordinates of the point(s) of intersection of the graphs of $y = q(x)$ and $y = q^{-1}(x)$.

2 + 2 = 4 marks

d. The cubic function g is defined by $g: R \to R, g(x) = x^3 + 2x^2 + cx + k$, where c and k are real numbers.

i. If g has exactly one stationary point, find the value of c.

ii. If this stationary point occurs at a point of intersection of $y = g(x)$ and $y = g^{-1}(x)$, find the value of k.

3 + 3 = 6 marks

Total 14 marks

2012

Question 8

The function $f: R \to R, f(x) = ax^3 + bx^2 + cx$, where a is a negative real number and b and c are real numbers.

For the real numbers $p < m < 0 < n < q$, we have $f(p) = f(q) = 0$ and $f'(m) = f'(n) = 0$.

The gradient of the graph of $y = f(x)$ is negative for

A. $(-\infty, m) \cup (n, \infty)$

B. (m, n)

C. $(p, 0) \cup (q, \infty)$

D. $(p, m) \cup (0, q)$

E. (p, q)

Question 16

The graph of a cubic function f has a local maximum at $(a, -3)$ and a local minimum at $(b, -8)$.

The values of c, such that the equation $f(x) + c = 0$ has exactly one solution, are

A. $3 < c < 8$

B. $c > -3$ or $c < -8$

C. $-8 < c < -3$

D. $c < 3$ or $c > 8$

E. $c < -8$

2014

Question 1

The point P (4, –3) lies on the graph of a function f. The graph of f is translated four units vertically up and then reflected in the y-axis.

The coordinates of the final image of P are

A. (–4, 1)

B. (–4, 3)

C. (0, –3)

D. (4, –6)

E. (–4, –1)

Question 12

The transformation $T: R^2 \to R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

maps the line with equation $x - 2y = 3$ onto the line with equation

A. $x + y = 0$

B. $x + 4y = 0$

C. $-x - y = 4$

D. $x + 4y = -6$

E. $x - 2y = 1$

Question 5 (13 marks)

Let $f: R \to R, f(x) = (x-3)(x-1)(x^2+3)$ and $g: R \to R, g(x) = x^4 - 8x$.

a. Express $x^4 - 8x$ in the form $x(x-a)\left((x+b)^2 + c\right)$. 2 marks

b. Describe the translation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. 1 mark

c. Find the values of d such that the graph of $y = f(x+d)$ has

i. one positive x-axis intercept 1 mark

ii. two positive x-axis intercepts. 1 mark

d. Find the value of n for which the equation $g(x) = n$ has one solution. 1 mark

e. At the point $(u, g(u))$, the gradient of $y = g(x)$ is m and at the point $(v, g(v))$, the gradient is $-m$, where m is a positive real number.

i. Find the value of $u^3 + v^3$. 2 marks

ii. Find u and v if $u + v = 1$. 1 mark

f. **i.** Find the equation of the tangent to the graph of $y = g(x)$ at the point $(p, g(p))$. 1 mark

ii. Find the equations of the tangents to the graph of $y = g(x)$ that pass through the point with coordinates $\left(\frac{3}{2}, -12\right)$. 3 marks

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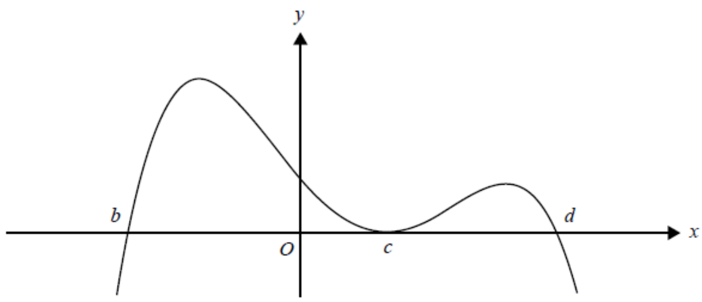
Question 3



The rule for a function with the graph above could be

A. $y = -2(x + b)(x - c)^2(x - d)$

B. $y = 2(x + b)(x - c)^2(x - d)$

C. $y = -2(x - b)(x - c)^2(x - d)$

D. $y = 2(x - b)(x - c)(x - d)$

E. $y = -2(x - b)(x + c)^2(x + d)$

Question 11

The transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$ is a

A. dilation by a factor of 2 from the y-axis.

B. dilation by a factor of 2 from the x-axis.

C. dilation by a factor of $\frac{1}{2}$ from the x-axis.

D. dilation by a factor of 8 from the y-axis.

E. dilation by a factor of $\frac{1}{2}$ from the y-axis.

Question 17

A graph with rule $f(x) = x^3 - 3x^2 + c$, where c is a real number, has three distinct x-intercepts.

The set of all possible values of c is

A. R

B. R^+

C. $\{0, 4\}$

D. $(0, 4)$

E. $(-\infty, 4)$

Question 20

If $f(x - 1) = x^2 - 2x + 3$, then $f(x)$ is equal to

A. $x^2 - 2$

B. $x^2 + 2$

C. $x^2 - 2x + 2$

D. $x^2 - 2x + 4$

E. $x^2 - 4x + 6$