Project: Modeling and Predicting the Performance of a 2024 100m Olympic Sprinter

Objective

- Apply concepts learned from Usain Bolt's 100m race modeling work done in class to analyze a sprinter from the 2024 Olympic Games in Paris.
- Use Geogebra for plotting our data, graph and slopefield.
- Use Excel for parameter estimation.
- Model the sprinter's performance based on provided race data and compare predictions to real-world results.
- Use the gathered data to make predictions about other races.

Introduction to the Hill-Keller Model

In class, we constructed the Hill-Keller model for modeling a 100m run for athletes. The differential equation that we found in class was

$$v'(t) = P - k \cdot v(t)$$

where P is a constant that depends on the runner's maximum propulsive effort, k is a resistance parameter, and $v(t)$ is the runner's velocity. We assumed $P = 11$ in our case with Usain Bolt, but we will estimate the best parameter to fit our data.

Project Steps

1. Data (Geogebra)

Use the collected data of split times and positions for the sprinter from the 2024 Olympic 100m and plot points.

2. Modeling the Motion (GeoGebra)

Velocity Function: Using the Hill-Keller model, solve the differential equation for velocity:

$$v'(t) = P - k \cdot v(t); \quad v(t_0) = 0$$

where P is a constant that depends on the runner's maximum propulsive effort, k is a positive constant, and $v(t)$ is the runner's velocity. We assumed $P = 11$ in our case with Usain Bolt, so use this until we find a better estimate later. (We will replace t_0 with an actual value later. For this step leave t_0 in your answer)

Graph in GeoGebra: Plot $v(t)$ using different values of k and observe how the curve fits the sprinter's performance. Adjust k to best match the provided data. (Make sure to screenshot a few of these and label your axis)

3. Position Function and Time Prediction

Once the velocity is known, integrate to find the position function. (Your position function should have the parameters P, and k and also t_0 for your initial time)

Graph in GeoGebra: Plot the position function $x(t)$ and compare it to the data points.

4. Parameter Estimation (Excel)

Using Excel, estimate the optimal value of k and P by minimizing the error between the modeled position $x(t)$ and the actual data points.

- Create a column for predicted positions based on your current estimate of k.
- Compute the error between predicted and actual positions.
- Use Excel's **Solver** tool to minimize the sum of squared errors by adjusting k.

5. Prediction for Other Distances

Predict Sprint Times: Using your estimated k and P, predict how fast the sprinter could run other distances like 200m and 60m. For 200m, solve:

$$x(t) = 200$$

and compare your results with the real-world times.

Validation: Evaluate how well your model fits the real-world data. Discuss potential sources of error or discrepancies between the model and reality.

6. Critical Analysis

Range of Validity: Discuss the limitations of the Hill-Keller model for short or long-distance races (e.g., 1500m or marathons). Predict these times using your model and explain why the predictions might be unrealistic.

Improving the Model: Suggest improvements or alternative models to better capture the sprinter's performance over different distances.

7. Conclusion

Summarize your findings and discuss how well the Hill-Keller model applies to your 2024 sprinter. Did the model accurately predict their performance? How does their race compare to Usain Bolt's?