14.2 Ejercicios

(3) Dar una integral para cada orden de integración y utilizar el orden más conveniente para evaluar la integral en la región R.

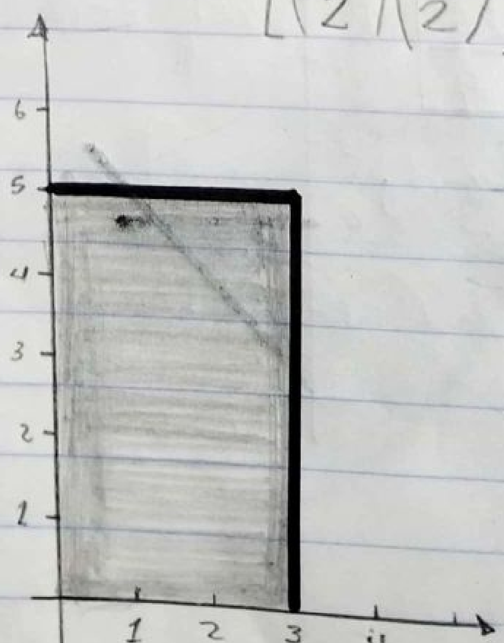
$$\int_R xy \, dA.$$

R: rectángulo con vértices (0,0), (0,5), (3,5), (3,0).

$$\int_0^5 \int_0^3 xy \, dx\,dy = \int_0^3 \int_0^5 xy \, dy\, dx$$

$$= \int_0^3 \left[\frac{1}{2} xy^2\right] dx = \frac{25}{2}\int_0^3 x \, dx$$

$$= \left[\left(\frac{25}{2}\right)\left(\frac{x^2}{2}\right)\right]_0^3 = \left[\frac{25}{4} x^2\right]_0^3$$



$$= \boxed{\frac{225}{4}}$$

(67) Trazar la región de integración. Despues evaluar lo integración iterada.

$$\int_0^2\int_x^2 x\sqrt{1+y^3}\,dy\,dx = \int_0^2\int_0^y x\sqrt{1+y^3}\,dx\,dy$$

$$= \int_0^y x\sqrt{1+y^3}\,dx = \left[\frac{x^2}{2}\sqrt{1+y^2}\right]_0^y = \frac{y^2}{2}\sqrt{1+y^3}$$

$$\int_0^2 \frac{y^2}{2}\sqrt{1+y^3}\,dy = \frac{1}{2}\int_0^2 y^2\sqrt{1+y^3}\,dy$$

$$\left[\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(1+y^3)^{3/2}\right]_0^2 = \frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(1+2)^{3/2}-0$$

$$\frac{1}{9}(27)-\frac{1}{9}(1)=\boxed{\frac{26}{9}}$$

$= [(\sin x) + (\sin x)(\cos x)] - [0 + (0)\cos x]$

$= \boxed{\sin x + \sin x \cos x}$

$\int_0^{\pi} (\sin x + \sin x \cos x)\, dx = \int_0^{\pi} \sin x\, dx + \int_0^{\pi} \sin x \cos x\, dx$

$= \left[-\cos x\right]_0^{\pi} + \left[\frac{1}{2}\sin^2\right]_0^{\pi} = \left[-\cos x + \frac{1}{2}\sin^2\right]_0^{\pi}$

$= -\cos \pi + \frac{1 \sin^2 \pi}{2} = 1 + 1 = \boxed{2}$

(25) $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y}}\, dx\, dy = \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y}}\, dx$

$= \frac{2x}{\sqrt{4-y}}\Big]_0^{\sqrt{4-y^2}} = \int_0^2 \frac{2x}{\sqrt{4-y}}\Big]_0^{\sqrt{4-y}} dy$

$\frac{2(\sqrt{4-y^2})}{\sqrt{4-y^2}} = \int_0^2 2\, dy = 2y\Big]_0^2 = 2(2) = \boxed{4}$

25/10/2023

Cálculo Vectorial

- Examen III

Integrales iteradas en el plano.

$F_x(x,y) = 2xy$

Derivada parcial.

$\frac{\partial}{\partial x}(x^2y + C(y)) = 2xy$

$\int 2xy\, dx = 2y \int x\, dx$

$= 2y \cdot \frac{1}{2} x^2 + C(y)$.

$f(x,y) = x^2 y + C(y)$

¿Cuál es la Integral?

$\int_1^{2y} 2xy\, dx = x^2y \Big|_1^{2y}$

$= (2y)^2 - 1^2(y)$.

$= 4y^3 - y$

$= G(y) \rightarrow$ función que depende de y.

Halle la integrar de 0 hasta 4 de

$$\int_0^4 G(y)\,dy = \int_0^4 (4y^3 - y)\,dy$$

$$= \int_0^4 \Big[\underbrace{\int_1^{2y} 2xy\,dx}_{G(y)}\Big]\,dy$$

En general:

$$F' = f.$$

$$\int_c^d \left[\int_{h(y)}^{h_2(y)} f(x,y)\,dx\right] dy = \int_c^d \left[F(x,y)\Big|_{h_1(y)}^{h_2(y)}\right]$$

$$= \int_c^d \left[F(h(y), y) - F(h_1(y), y)\right] dy$$

$$\int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\right] dx = \int_a^b \Big[F(x,y)\Big]_{g_1(x)}^{g_2(x)}\,dx$$

$$= \int_a^b \left[F(x, g_2(x)) - F(x, g_1(x))\right] dx.$$

· Evaluar integrales iteradas

(25) $$\underbrace{\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y^2}}\, dx}_{I}\, dy = \int_0^2 2\, dy = 2y \Big|_0^2$$

$$= 2(2-0) = \boxed{4}$$

$$I = \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y^2}}\, dx = \frac{2}{\sqrt{4-y^2}}\, x \Big|_0^{\sqrt{4-y^2}}$$

En base a este resultado se integró en función de "y"

$$= \frac{2}{\sqrt{4-y^2}}\left[\sqrt{4-y^2} - 0\right] = \boxed{2}$$

(24) $$\int_0^2 \underbrace{\int_{3y^2-6y}^{2y-y^2} 3y\, dx}_{I}\, dy.$$

Pag siguiente

se representó en función de y en base al resultado de la integral de $\boxed{\int dx}$

$$I = \int_{3y^2-6y}^{2y-y^2} 3y\, dx = 3y\left[x\right]_{3y^2-6y}^{2y-y^2}$$

$$= 3y\left[2y - y^2 - (3y^2 - 6y)\right]$$

$$= 3y\left[2y - y^2 - 3y^2 + 6y\right]$$

$$= 3y\left[-4y^2 + 8y\right]$$

$$= \boxed{-12y^3 + 24y^2}$$

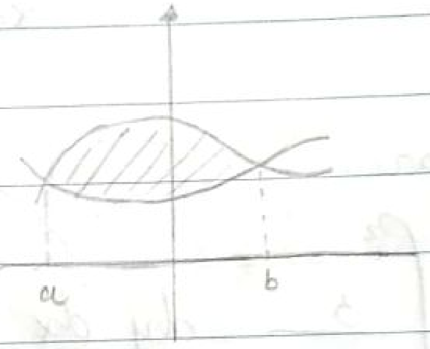
$$\int_0^2 \int_{3y^2-6y}^{2y-y^2} 3y \, dx \, dy = \int_0^2 (-12y^3 + 24y^2) \, dy$$

$$= -3y^4 + 8y^3 \Big|_0^2 = -3(2)^4 + 8(2)^3$$

$$= -48 + 64 = \boxed{16}$$

- Integral iterada Tipo I.

$$\int_a^b \int_{g_1(x)}^{g_2(x)} F(x,y) \, dy \, dx$$



- Región de integración

$g_1(x) \leq y \leq g_2(x)$

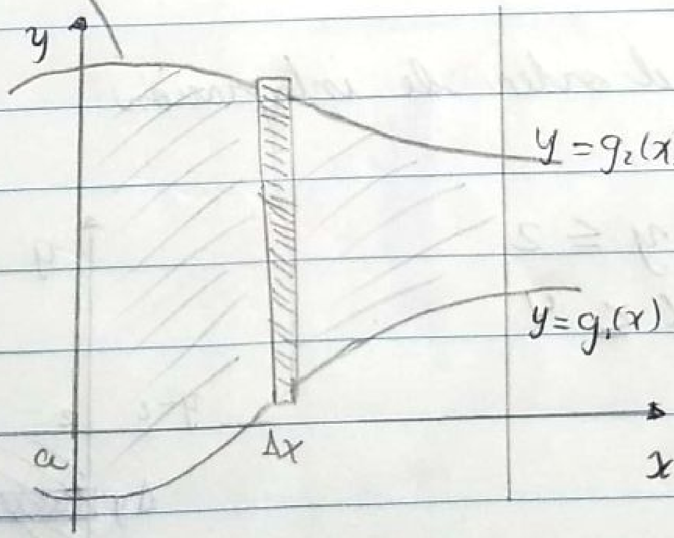
$a \leq x \leq b$

$y = g_1(x)$

$y = g_2(x)$

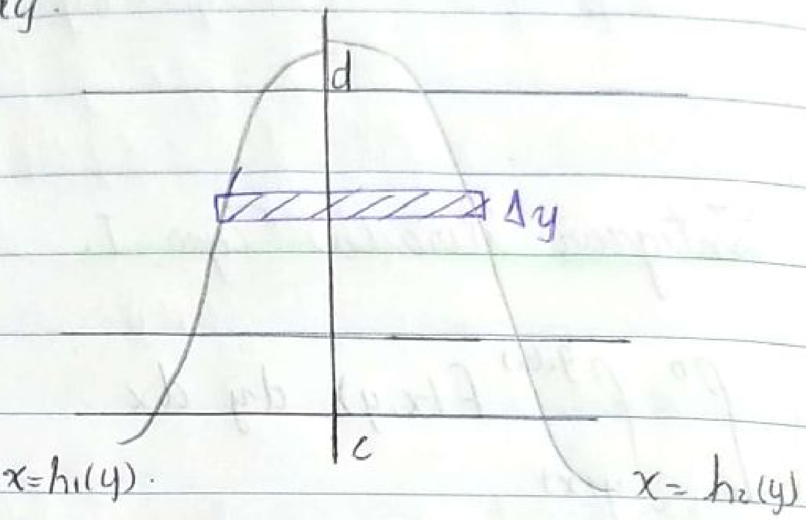
$x = a$

$x = b$.



Integral iterado de Tipo II

$$\int_c^d \int_{h_1(y)}^{h_2(y)} F(x,y)\,dx\,dy.$$



$h_1(y) \le x \le h_2(y)$

$c \le y \le d$

- Evaluar

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{2+y^3}\,dy\,dx \longrightarrow \int_0^2 \int_0^{y^2} \frac{3}{2+y^3}\,dx\,dy$$

→ Rect verticales

→ Pag sigu

Cambiando el orden de integración

R: $\sqrt{x} \le y \le 2$

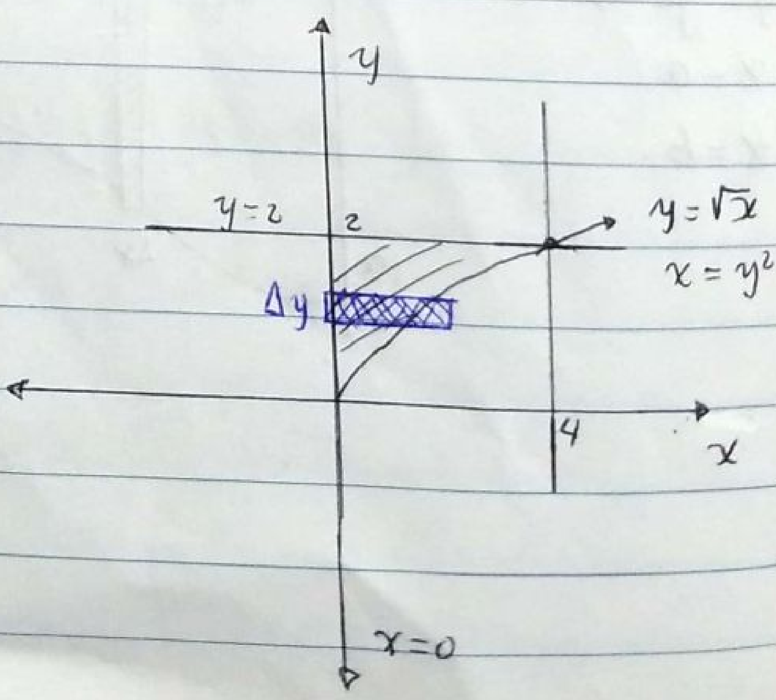
$0 \le x \le 4$

$y = \sqrt{x}$

$y = 2$

$x = 0$

$x = 4$



$$\int_0^2\int_0^{y^2} \frac{3}{2+y^3}\,dx\,dy$$

Por sustitución

$u = 2+y^3$

$du = 3y^2\,dy$

$$\int \frac{du}{u} = \ln|u| + c$$
$$= \ln|2+y^3| + c$$

$$\int_0^2 \frac{3}{2+y^3}\left[x\right]_0^{y^2} dy$$

$$\int_0^2 \frac{3}{2+y^3}\, y^2\, dy = \ln|2+y^3|\Big|_0^2$$

$$= \ln|2+2^3| - \ln|2+0^3|$$
$$= \ln 5$$

$$\int_0^1\int_y^1 \operatorname{sen}(x^2)\,dx\,dy$$

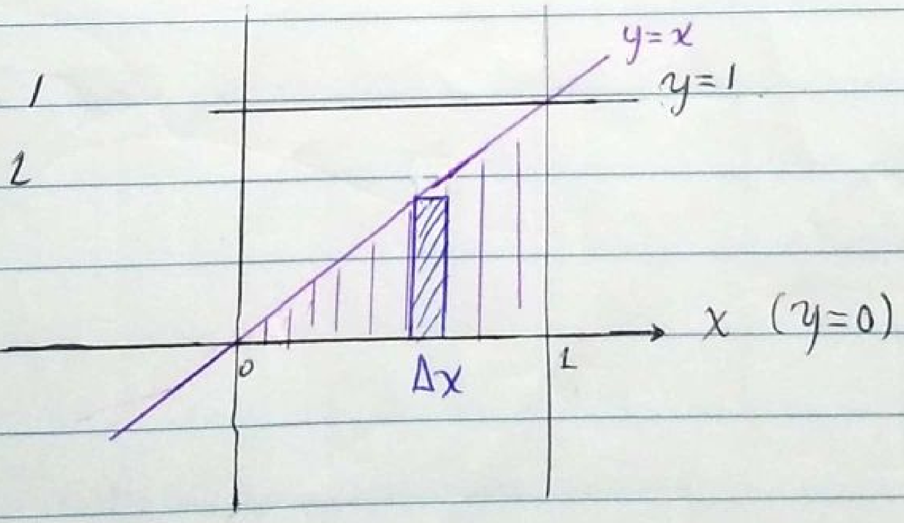
Rect horizontales

Cambio de orden → Pag siguiente

$$\int_0^1\int_0^x \underbrace{\operatorname{sen} x^2\, dy\,dx}_{I}$$

R: $y \le x \le 1$

$0 \le y \le 1$



$y = x$

$x = 1$

$y = 0$

$y = 1$

R: $0 \le y \le x$

$0 \le x \le 1$

$\int$