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CALCULUS I

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2nd Partial Project

**APPLICATIONS OF MOTION:
POSITION, VELOCITY, AND ACCELERATION**

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In this activity we will be using the concepts of position, velocity, and acceleration, as well as all of the themes that have been in class, to solve what's presented to us.

For that, we need to remember these concepts: Position, velocity, and acceleration all describe the motion of an object. *Position* is given as a function of x with respect to time, $x(t)$. *Velocity* is the object's speed and direction, or change in position over time. And *acceleration* is the name given to any process where the velocity changes.

The objective of this activity is to show what we've been learning throughout high school regarding mathematics.

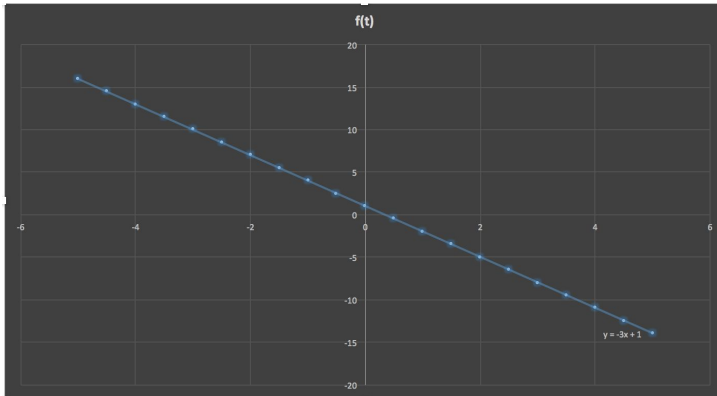
Our team chose to work with table F, which is shown below:

t	f(t)	g(t)	h(t)	F(t)	G(t)	H(t)
-5	16	7.828427	2	5.000335	0.142857	-213
-4.5	14.5	7.738613	1.5	5.000553	0.153846	-163.375
-4	13	7.645751	1	5.000912	0.166667	-122
-3.5	11.5	7.54951	0.5	5.001503	0.181818	-88.125
-3	10	7.44949	0	5.002479	0.2	-61
-2.5	8.5	7.345208	-0.5	5.004087	0.222222	-39.875
-2	7	7.236068	-1	5.006738	0.25	-24
-1.5	5.5	7.12132	-1.5	5.011109	0.285714	-12.625
-1	4	7	-2	5.018316	0.333333	-5
-0.5	2.5	6.870829	-1.5	5.030197	0.4	-0.375
0	1	6.732051	-1	5.049787	0.5	2
0.5	-0.5	6.581139	-0.5	5.082085	0.666667	2.875
1	-2	6.414214	0	5.135335	1	3
1.5	-3.5	6.224745	0.5	5.22313	2	3.125
2	-5	6	1	5.367879	N. P.	4
2.5	-6.5	5.707107	1.5	5.606531	-2	6.375
3	-8	5	2	6	-1	11
3.5	-9.5	N. P.	2.5	6.648721	-0.66667	18.625
4	-11	N. P.	3	7.718282	-0.5	30
4.5	-12.5	N. P.	3.5	9.481689	-0.4	45.875
5	-14	N. P.	4	12.38906	-0.33333	67

We decided to use Desmos and Excel to graph each one of the functions, and determine from the graph the equation of the function.

Here is the graph, equation, and derivatives of the first function:

$$f(t) = -3x + 1$$



- We have a linear function that intercepts with the y-axis on (0, 1).

Having the equation of position, which is $f(t) = -3x + 1$, we can find the equation of velocity, by taking its derivative; and the equation of acceleration by taking the derivative of the equation of velocity.

Equation of velocity:

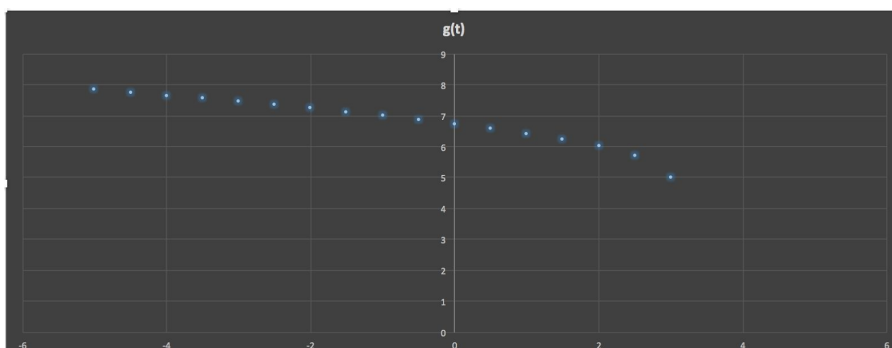
$$F(t) = -3x + 1$$
$$F'(t) = -3$$

Equation of acceleration:

$$F'(t) = -3$$
$$F''(t) = 0$$

Here is the graph, equation, and derivatives of the second function:

$$g(t) = (-x + 3)^{\frac{1}{2}} + 5$$



- This is an exponential graph reflected on the x-axis, with a movement of five units up and three units to the right.
- When the graph reaches the coordinates (3,5) it stops existing, at least for the next four values of t.

Equation of velocity:

$$g(t) = (-x + 3)^{1/2} + 5$$

$$g'(t) = \frac{1}{2} (-x + 3)^{-1/2}$$

$$g'(t) = -\frac{1}{2} (x - 3)^{-1/2}$$

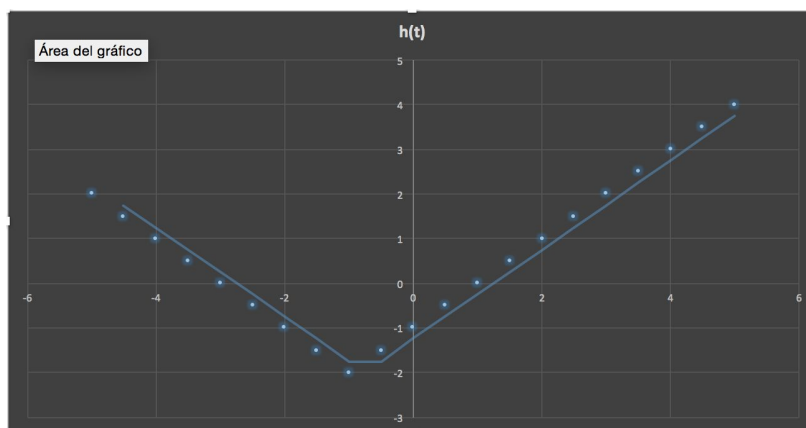
Equation of acceleration:

$$g'(t) = \frac{1}{2} (x - 3)^{-1/2}$$

$$g''(t) = \frac{1}{4} (x - 3)^{-1}$$

Here is the graph, equation, and derivatives of the third function:

$$h(t) = |x + 1| - 2$$



- We can clearly see this is an example of an absolute value graph, commonly called vector (has a lot to do with the character that appears in Despicable Me). With a transformation of two units downwards and one unit left.

Equation of velocity:

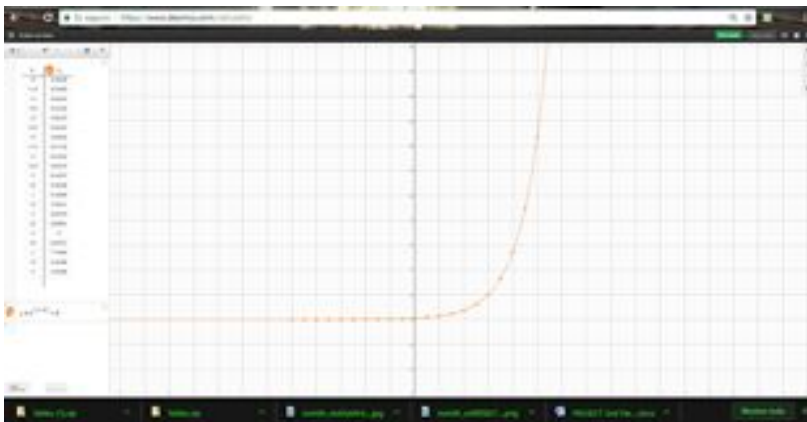
$$h(t) = |x+1| - 2 = \sqrt{(x+1)^2} - 2 \rightarrow ((x+1)^2)^{1/2}$$
$$h'(t) = \frac{1}{2} ((x+1)^2)^{-1/2} \cdot 2(x+1) \cdot 2$$
$$\frac{1}{2\sqrt{(x+1)^2}} \cdot 2x+2 = \frac{x}{\sqrt{(x+1)^2}} + \frac{1}{\sqrt{(x+1)^2}}$$
$$h'(t) = \frac{x+1}{|x+1|}$$

Equation of acceleration:

$$h'(t) = \frac{x+1}{|x+1|} \left(\frac{u}{v} \right)$$
$$u = |x+1| \quad v = |x+1|$$
$$u' = 1 \quad v' = 1$$
$$(1) \frac{|x+1| - 1|x+1|}{|x+1|^2} = 0$$
$$h''(t) = 0$$

Here is the graph, equation, and derivatives of the fourth function:

$$F(t) = e^{(x-3)} + 5$$



- In the scatter plot of $F(t)$, we can clearly see that it follows an exponential shape. This can be easily deduced due to the curve going upwards and the values of $F(t)$ increasing exponentially.
- There's also the fact that the values of $F(t)$ do not cross the y value of 5, indicating that there's an asymptote there.
- The one other characteristic about the scatter plot is that it goes through the "y axis" in a very specific point, that being 5.

Equation of velocity:

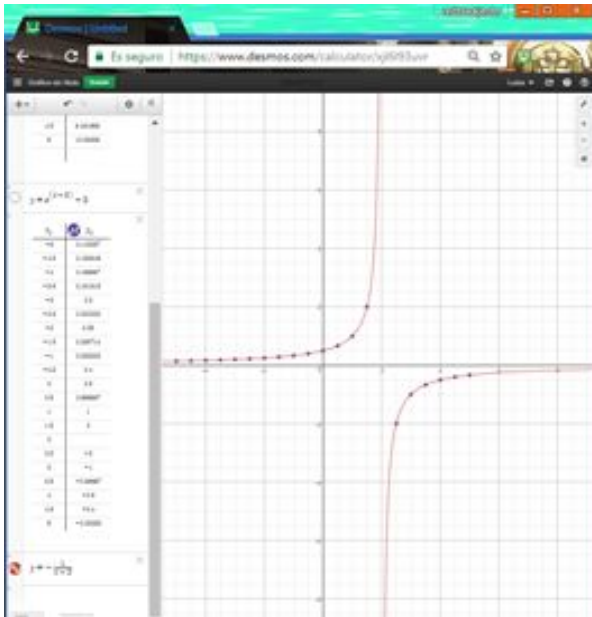
$$F(t) = e^{x-3} + 5$$
$$F'(t) = (e^{x-3} \cdot 1) + \emptyset$$
$$\boxed{F'(t) = e^{x-3}}$$

Equation of acceleration:

$$F'(t) = e^{x-3}$$
$$F''(t) = (e^{x-3} \cdot 1)$$
$$\boxed{F''(t) = e^{x-3}}$$

Here is the graph, equation, and derivatives of the fifth function:

$$G(t) = -1/x-2$$



- For $G(t)$ The scatter plot provides us with a clear pattern that has two curves divided by two different values of "y" and "x".
- None of the points in the scatter plot cross $x=2$ nor $y=0$, which indicates the presence of two asymptotes. The points tend to approach the y asymptote but never touches it because it is an exponential function.

Equation of velocity:

$$G(t) = \frac{-1}{x-2}$$

$$u = -1 \quad v = x-2$$

$$u' = \emptyset \quad v' = 1$$

$$G'(t) = \frac{(x-2)(\emptyset) - (1)(-1)}{(x-2)^2}$$

$$G'(t) = \frac{1}{(x-2)^2}$$

Equation of acceleration:

$$G'(t) = \frac{1}{(x-2)^2}$$

$$u = 1 \quad v = (x-2)^2$$

$$u' = \emptyset \quad v' = 2(x-2)$$

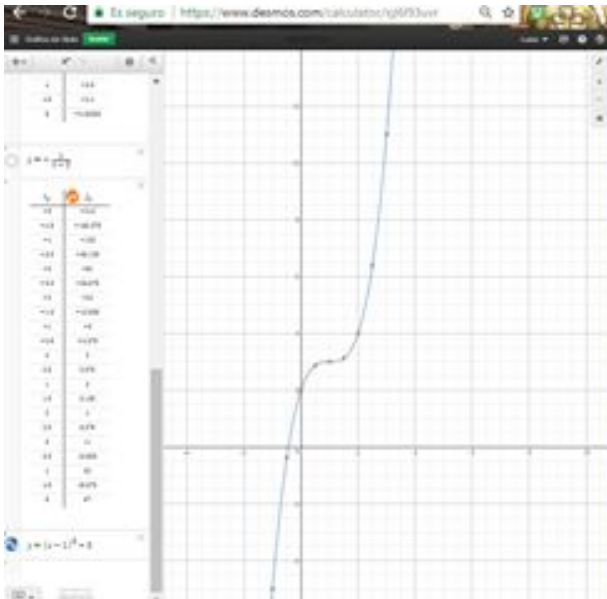
$$G''(t) = \frac{(x-2)^2 \cdot \emptyset - 2(x-2) \cdot 1}{(x-2)^4}$$

$$G''(t) = \frac{2(x-2)}{(x-2)^4}$$

$$G''(t) = \frac{2}{(x-2)^3}$$

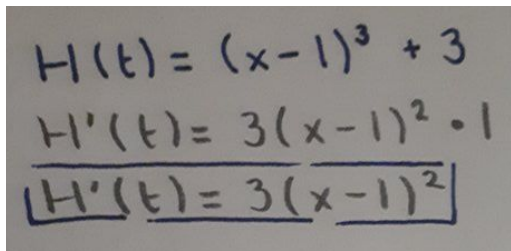
Here is the graph, equation, and derivatives of the sixth and final function:

$$H(t) = (x-1)^3 + 3$$



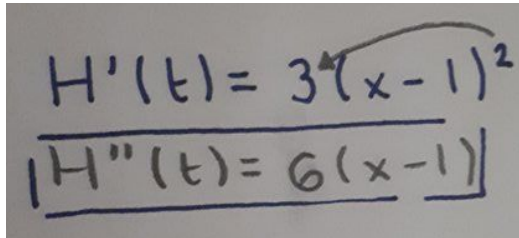
- In $H(t)$ the pattern seems to come from minus infinity and starts to stop going up at a specific value of y (That value being $y=3$), then starts to go upwards into infinity.
- This particular shape of the scatter plot involves two curves united by a single strand, the shape itself indicates that it is a power function.

Equation of velocity:



Handwritten equations showing the derivation of velocity and acceleration from a position function. The first equation is $H(t) = (x-1)^3 + 3$. The second equation is $H'(t) = 3(x-1)^2 \cdot 1$. The third equation, which is boxed, is $H'(t) = 3(x-1)^2$.

Equation of acceleration:



Handwritten equations showing the derivation of acceleration from the velocity function. The first equation is $H'(t) = 3(x-1)^2$. The second equation, which is boxed, is $H''(t) = 6(x-1)$.

After finishing the activity and analyzing the information we got, we can conclude that:

A graph can always be deduced with the proper analyzing of the shape of the points themselves. Sometimes there may be no clear pattern in a scatter plot, but that's when we must look for the nearest points and try to connect them, isolating the rest, but obtaining a tendency that one can follow. This is especially important when you work on a business and have to deliver a solid graph depicting the tendency a product follows or the budget they will have to produce a certain product.

Luisa

It's important to remember past topics and have the capacity to observe and analyze a graph, in order to properly determine the equation that this one follows. We can also see a relation between physics and calculus. Our graphs/equations are of position (which in physics we write as something like: $x = x_i + v_i * t + 1/2at^2$, which will resemble any equation of position we obtain in calculus), and by taking the derivative, we obtain the equation of velocity (for which the formula reads: $v = v_i + at$, which will again, be like any equation of velocity we obtain), and finally, by taking the derivative of the equation of velocity, we may obtain the equation of the acceleration; thus confirming that calculus and physics have a lot to do with each other.

Cinthia

With this project we learned how to analyze graphs and its behaviors according to what the function says. In the graphs, there may be patterns where it is difficult to see the pattern but we could just see the points around in order to follow the tendency. We also applied what we already know about derivatives and how we are supposed to apply the rules. We must always retake algebra so we can graph.

Lorena

Knowing the different behaviors of the parent functions and its transformations, we can deduce the graph's equation. Plus, a graphing utility can become very helpful if you want to go ahead and guess the function's equation. I learned why derivatives are important in our calculus' class and daily life for that matter. How they can be applied and used. It also worked out as a pretty good review about functions.

Andrea

References:

- *What is velocity? - Definition from WhatIs.com.* (n.d.). Retrieved October 06, 2017, from <http://whatis.techtarget.com/definition/velocity>
- *Motion, Position, Velocity, And Acceleration.* (2015). *Chegg Study*. Retrieved 6 October 2017, from <http://www.chegg.com/homework-help/definitions/motion-position-velocity-and-acceleration-29>
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