Sección 3.1

27) $y_p'' + Py_p' + Qy_p = f(x)$

$y_c'' + Py_c' + Qy_c = 0$

$(y_p+y_c)'' + P(y_p+y_c)' + Q(y_p+y_c) =$

$y_p'' + y_c'' + Py_p' + Py_c' + Qy_p + Qy_c =$

$\underbrace{y_p''+Py_p'+Qy_p}_{f(x)} + \underbrace{y_c''+Py_c'+Qy_c}_{0} = f(x)$

$y_p+y_c \longrightarrow$ es Solución

32) $A(x)y'' + B(x)y' + C(x)y = 0 \qquad A(x) \neq 0$

a. $W(y_1,y_2) = W$ Demuestre que $A(x)\frac{dw}{dx} = (y_1)Ay_2'' - (y_2)Ay_1''$

$W(y_1,y_2) \neq 0$ puesto que y_1 e y_2 son slnes de la ec. dif.cial homogenea asociada

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_1'y_2 = w$$

$$\frac{dw}{dx} = \frac{d(y_1y_2' - y_1'y_2)}{dx} = (\cancel{y_1'y_2'} + y_1y_2'') - (y_1''y_2 + \cancel{y_1'y_2'})$$

$$A(x)\frac{dw}{dx} = (y_1y_2'' - y_1''y_2)\,A(x)$$

$$A(x)\frac{dw}{dx} = A(x)y_1y_2'' - A(x)y_2y_1''$$

$Ay_1'' + By_1' + Cy_1 = 0 \Rightarrow Ay_1'' = -By_1' - Cy_1$

$Ay_2'' + By_2' + Cy_2 = 0 \Rightarrow Ay_2'' = -By_2' - Cy_2$

$$A(x)\frac{dw}{dx} = (y_1)(-By_2' - Cy_2) - (y_2)(-By_1' - Cy_1)$$

$$A(x)\frac{dw}{dx} = -B(y_1y_2' - y_2y_1') \cancel{- Cy_1y_2} + \cancel{Cy_1y_2}$$

$$A(x)\frac{dw}{dx} = -B(x)\,W(x)$$

(b.) $w_{(x)} = Ke^{\left(-\int \frac{B(x)}{A(x)}dx\right)}$

$$\frac{\cancel{A(x)}}{\cancel{A(x)}}\frac{dw}{dx} + \frac{B(x)}{A(x)}w = 0 \longrightarrow \text{Ecuación Lineal de Primer Orden}$$

$P(x) = \frac{B(x)}{A(x)}$

$Q(x) = 0$

$f(x) = e^{\int P(x)dx}$

$$e^{\int \frac{B(x)}{A(x)}dx}\frac{dw}{dx} + e^{\int \frac{B(x)}{A(x)}dx}\frac{B(x)}{A(x)}w = 0$$

$$\int \cancel{dx}\frac{d\left(we^{\int \frac{B}{A}dx}\right)}{\cancel{dx}} = \int 0\,dx$$

$$we^{\int \frac{B}{A}dx} + C_1 = C_2$$

$$we^{\int \frac{B}{A}dx} = K$$

$$w_{(x)} = Ke^{-\int \frac{B}{A}dx}$$

Para que $w=0$, $K=0$

ya que $e^{-\int \frac{B}{A}dx} \neq 0$ Siempre

(51) $ax^2y'' + bxy' + cy = 0$

$v = \ln x$

$\frac{dv}{dx} = \frac{1}{x}$

$$y' = \frac{dy}{dx} = \frac{dy}{dv}\cdot\frac{dv}{dx} = \frac{dy}{dv}\cdot\frac{1}{x}$$

$$y'' = \frac{d^2y}{dx} = \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\frac{1}{x} - \frac{dy}{dv}\cdot\frac{1}{x^2} = \frac{d^2y}{dv^2}\cdot\frac{1}{x^2} - \frac{dy}{dv}\cdot\frac{1}{x^2}$$

$$a\cancel{x^2}\left(\frac{d^2y}{dv^2}\cdot\frac{1}{\cancel{x^2}} - \frac{dy}{dv}\cdot\frac{1}{\cancel{x^2}}\right) + b\cancel{x}\left(\frac{dy}{dv}\cdot\frac{1}{\cancel{x}}\right) + cy = 0$$

$$a\frac{d^2y}{dv^2} - a\frac{dy}{dv} + b\frac{dy}{dv} + cy = 0$$

$$a\frac{d^2y}{dv^2} + (b-a)\frac{dy}{dv} + cy = 0$$

$$ar^2 + (b-a)r + c = 0 \longrightarrow$$

Se asumen r_1 y r_2 como reales y distintas

$$y(v) = C_1e^{r_1v} + C_2e^{r_2v}$$

$$y(x) = C_1e^{r_1\ln x} + C_1e^{r_2\ln x} = C_1x^{r_1} + C_2x^{r_2}$$

53) $x^2y'' + 2xy' - 12y = 0$

$ax^2y'' + bxy' + cy = 0$ $\quad a=1 \quad b=2 \quad c=-12$

$V = \ln x$

$a\frac{d^2y}{dV} + (b-a)\frac{dy}{dV} + cy = 0$

$\frac{d^2y}{dv} + (2-1)\frac{dy}{dv} - 12y = 0$

$r^2 + r - 12 = 0$ $\quad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$r_1 = 3 \quad r_2 = -4$

$y(v) = C_1e^{3v} + C_2e^{-4v}$

$y(x) = C_1e^{3\ln x} + C_2e^{-4\ln x}$

$y(x) = C_1x^3 + Cx^{-4}$

55) $x^2y'' + xy' = 0$

$ax^2y'' + bxy' + cy = 0$ $\quad a=1 \quad b=1 \quad c=0$

$a\frac{d^2y}{dV} + (b-a)\frac{dy}{dV} + cy = 0$ $\quad V = \ln x$

$\frac{d^2y}{dV} + \cancel{(1-1)\frac{dy}{dV}} + \cancel{(0)y} = 0$

$r^2 = 0 \quad r_1 = 0 \quad r_2 = 0$

$y(v) = c_1e^{rv} + c_2e^{rv}v$

$y(v) = (c_1 + c_2v)\cancel{e^{rv}}\, 1$

$y(x) = c_1 + c_2\ln x$

Sección 3.2

(19) $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$ $\quad$ $y(1)=6 \quad y'(1)=14 \quad y''(1)=22$

$y(x) = c_1y_1 + c_2y_2 + c_3y_3$ $\quad$ $y_1 = x \quad y_2 = x^2 \quad y_3 = x^3$

$y(x) = c_1x + c_2x^2 + c_3x^3$

$y(1) = c_1(1) + c_2(1)^2 + c_3(1)^3 \longrightarrow 6 = c_1 + c_2 + c_3$

$y'(x) = c_1 + 2c_2x + 3c_3x^2$

$y'(1) = c_1 + 2c_2(1) + 3c_3(1)^2 \longrightarrow 14 = c_1 + 2c_2 + 3c_3$

$y''(x) = 2c_2 + 6c_3x$

$y''(1) = 2c_2 + 6c_3(1) \longrightarrow 22 = (0)c_1 + 2c_2 + 6c_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 0 & 2 & 6 & 22 \end{array}\right] f_1(-1)+f_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 2 & 6 & 22 \end{array}\right] \begin{array}{l} f_2(-1)+f_1 \\ f_2(-2)+f_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 2 & 6 \end{array}\right] \begin{array}{l} f_3(-1)+f_2 \\ f_3(1/2) \\ f_3+f_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \begin{array}{l} c_1 = 1 \\ c_2 = 2 \\ c_3 = 3 \end{array}$$

$y(x) = x + 2x^2 + 3x^3$

31 $y'' + Py' + Qy = 0$

b $y'' - 2y' - 5y = 0$ $\quad y(0) = 1$, $y'(0) = 0$, $y''(0) = C$

$r^2 - 2r - 5 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$r_1 = 3{,}45$

$r_2 = -1{,}45$

$y(x) = C_1 e^{3{,}45x} + C_2 e^{-1{,}45x}$

$y(0) = C_1 \cancel{e^{3{,}45(0)}} + C_2 \cancel{e^{-1{,}45(0)}}$

$1 = C_1 + C_2$

$y'(x) = 3{,}45 C_1 e^{3{,}45x} - 1{,}45 C_2 e^{-1{,}45x}$

$y'(0) = 3{,}45 C_1 \cancel{e^{3{,}45(0)}} - 1{,}45 C_2 \cancel{e^{-1{,}45(0)}}$

$0 = 3{,}45 C_1 - 1{,}45 C_2$

$y''(x) = 11{,}9 C_1 e^{3{,}45x} + 2{,}1 C_2 e^{-1{,}45x}$

$y''(0) = 11{,}9 C_1 \cancel{e^{3{,}45(0)}} + 2{,}1 C_2 \cancel{e^{-1{,}45(0)}}$

$C = 11{,}9 C_1 + 2{,}1 C_2 \longrightarrow 11{,}9 C_1 + 2{,}1 C_2 - C = 0$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 3{,}45 & -1{,}45 & 0 \\ 11{,}9 & 2{,}1 & C \end{array}\right] \begin{array}{l} f_1(-3{,}45)+f_2 \\ f_1(-11{,}9)+f_3 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -4{,}9 & -3{,}45 \\ 0 & -9{,}8 & C-11{,}9 \end{array}\right] \begin{array}{l} f_2(-2)+f_3 \\ f_2(-\frac{10}{49}) \\ f_2(-1)+f_1 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 0{,}3 \\ 0 & 1 & 0{,}7 \\ 0 & 0 & C-5 \end{array}\right]$$

$C_1 = 0{,}3$

$C_2 = 0{,}7$

$C = 5$

35

$$w = \begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{bmatrix}$$

$$\frac{dw}{dx} = \underbrace{\begin{bmatrix} y_1' & y_2' & y_3' \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{bmatrix}}_{0} + \underbrace{\begin{bmatrix} y_1 & y_2 & y_3 \\ y_1'' & y_2'' & y_3'' \\ y_1'' & y_2'' & y_3'' \end{bmatrix}}_{0} + \begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1''' & y_2''' & y_3''' \end{bmatrix}$$

Ya que hay dos filas iguales el det. es $= 0$

$$\frac{dw}{dx} = \begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1''' & y_2''' & y_3''' \end{bmatrix}$$

(36) $y_2(x) = V(x)y_1(x)$

$y_2'(x) = V'(x)y_1(x) + V(x)y_1'(x)$

$y_2''(x) = V''(x)y_1(x) + V'(x)y_1'(x) + V'(x)y_1'(x) + V(x)y_1''(x) = V''(x)y_1(x) + 2V'(x)y_1'(x) + V(x)y_1''(x)$

$\left(V''(x)y_1(x) + 2V'(x)y_1'(x) + V(x)y_1''(x)\right) + P\left(V'(x)y_1(x) + V(x)y_1'(x)\right) + Q\left(V(x)y_1(x)\right) = 0$

$V(x)\cancel{\left(y_1''(x) + Py_1'(x) + Qy_1(x)\right)}^{\,0} + V'(x)\left(2y_1'(x) + Py_1(x)\right) + V''(x)\left(y_1(x)\right) = 0$

$V''(x)\left(y_1(x)\right) + V'(x)\left(2y_1'(x) + Py_1(x)\right) = 0$

(42) $\dfrac{(1-x^2)y''}{(1-x^2)} + \dfrac{2xy'}{(1-x^2)} - \dfrac{2y}{(1-x^2)} = 0 \quad (-1<x<1)$

$y_1(x) = x$

$y_2(x) = xV(x)$

$y_2'(x) = V(x) + xV'$

$y_2''(x) = V' + V' + xV''$
$= 2V' + xV''$

$y'' + \dfrac{2x}{1-x^2}y' - \dfrac{2}{1-x^2}y = 0$

$(2v' + xv'') + \dfrac{2x}{1-x^2}(v + xv') - \dfrac{2}{1-x^2}(xv) = 0$

$2v' + xv'' + \cancel{\dfrac{2xv}{1-x^2}} + \dfrac{2x^2v'}{1-x^2} - \cancel{\dfrac{2xv}{1-x^2}} = 0$

$2u + xu' + \dfrac{2x^2}{1-x^2}u = 0$

$xu' + u\left(2 + \dfrac{2x^2}{1-x^2}\right) = 0$

$x\dfrac{du}{dx} = -u\left(2 + \dfrac{2x^2}{1-x^2}\right)$

$\displaystyle\int \frac{du}{u} = \int -\frac{2}{x} - \frac{2x}{1-x^2}\,dx$

$e^{\ln|u|} = e^{-2\ln x + \ln|1-x^2|}$

$u = x^{-2}(1-x^2)$

$u = (x^{-2} - 1)$

$\displaystyle\int \cancel{dx}\frac{dv}{\cancel{dx}} = \int x^{-2} - 1\,dx$

$v = \dfrac{x^{-1}}{-1} - x$

$y_2(x) = x(x^{-1} + x)$

$y_2(x) = 1 + x^2$

Sección 3.3

19 $y''' + y'' - y' - y = 0$

$r^3 + r^2 - r - 1 = 0$

$(r+1)(r+1)(r-1) = 0$

$r_1 = -1$
$r_2 = -1$
$r_3 = 1$

$y_c = c_1 e^{-x} + C_2 x e^{-x} + C_3 e^{x}$

39 $y(x) = (A + Bx + Cx^2) e^{2x}$

$y_1 = e^{r_1 x} \qquad e^{r_1 x} = e^{2x}$

$r_1 = 2 \qquad r_1 - 2 = 0$

$(r-2)^3 = 0 \qquad r^3 - 6r^2 + 12r - 8 = 0$

$y''' - 6y'' + 12y' - 8y = 0$

42 $y(x) = (A + Bx + Cx^2)\cos 2x + (D + Ex + Fx^2)\operatorname{Sen} 2x$

$y(x) = (A\cos 2x + Bx\operatorname{Cos} 2x + Cx^2\cos 2x)e^{0x} + (D\operatorname{Sen} 2x + Ex\operatorname{Sen} 2x + Fx^2\operatorname{Sen} 2x)e^{0x}$

$e^{(a \pm bi)x} = e^{0x}$

$\operatorname{Cos} 2x = \operatorname{Cos} bx$
$2 = b$
$a = r$

$a \pm bi = 0$

$(a)^2 = (\pm bi)^2$

$a^2 = -b^2$

$(r^2 + (2)^2)^3 = 0$

$r^6 + 12r^4 + 48r^2 + 64 = 0$

$y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0$

55 $x^3y''' - x^2y'' + xy' = 0$

$$v = \ln|x| \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$y' = \frac{dy}{dx} = \frac{dy}{dv}\cdot\frac{dv}{dx} = \frac{dy}{dv}\cdot\frac{1}{x}$$

$$y'' = \frac{d^2y}{dx} = \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\cdot\frac{1}{x} - \frac{dy}{dv}\cdot\frac{1}{x^2} = \frac{d^2y}{dv^2}\cdot\frac{1}{x^2} - \frac{dy}{dv}\cdot\frac{1}{x^2}$$

$$y''' = \frac{d^3y}{dx^3} = \frac{d^3y}{dv^3}\cdot\frac{dv}{dx}\cdot\frac{1}{x^2} - \frac{d^2y}{dv^2}\frac{2}{x^3} - \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\cdot\frac{1}{x^2} + \frac{dy}{dv}\cdot\frac{2}{x^3} = \frac{d^3y}{dv^3}\cdot\frac{1}{x^3} - \frac{d^2y}{dv^2}\frac{3}{x^3} + \frac{dy}{dv}\cdot\frac{2}{x^3}$$

$$\cancel{x^3}\left(\frac{d^3y}{dv^3}\cdot\frac{1}{\cancel{x^3}} - \frac{d^2y}{dv^2}\frac{3}{\cancel{x^3}} + \frac{dy}{dv}\cdot\frac{2}{\cancel{x^3}}\right) - \cancel{x^2}\left(\frac{d^2y}{dv^2}\cdot\frac{1}{\cancel{x^2}} - \frac{dy}{dv}\cdot\frac{1}{\cancel{x^2}}\right) + \cancel{x}\left(\frac{dy}{dv}\cdot\frac{1}{\cancel{x}}\right) = 0$$

$$\frac{d^3y}{dv^3} - 3\frac{d^2y}{dv^2} + 2\frac{dy}{dv} - \frac{d^2y}{dv^2} + \frac{dy}{dv} + \frac{dy}{dv} = 0$$

$$\frac{d^3y}{dv^3} - 4\frac{d^2y}{dv^2} + 4\frac{dy}{dv} = 0$$

$$r^3 - 4r^2 + 4r = 0 \qquad r_1 = 0$$
$$r(r^2 - 4r + 4) = 0 \qquad r_2 = 2$$
$$r(r-2)(r-2) = 0 \qquad r_3 = 2$$

$$y(v) = C_1\cancel{e^{(0)v}} + C_2e^{2v} + C_3ve^{2v}$$

$$y(x) = C_1 + C_2e^{2\ln x} + C_3\ln x\, e^{2\ln x} = C_1 + (C_2 + C_3\ln x)x^2$$

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58 $x^3y''' + 6x^2y'' + 7xy' + y = 0$

$$v = \ln|x| \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$y' = \frac{dy}{dx} = \frac{dy}{dv}\cdot\frac{dv}{dx} = \frac{dy}{dv}\cdot\frac{1}{x}$$

$$y'' = \frac{d^2y}{dx} = \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\cdot\frac{1}{x} - \frac{dy}{dv}\cdot\frac{1}{x^2} = \frac{d^2y}{dv^2}\cdot\frac{1}{x^2} - \frac{dy}{dv}\cdot\frac{1}{x^2}$$

$$y''' = \frac{d^3y}{dx^3} = \frac{d^3y}{dv^3}\cdot\frac{dv}{dx}\cdot\frac{1}{x^2} - \frac{d^2y}{dv^2}\frac{2}{x^3} - \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\cdot\frac{1}{x^2} + \frac{dy}{dv}\cdot\frac{2}{x^3} = \frac{d^3y}{dv^3}\cdot\frac{1}{x^3} - \frac{d^2y}{dv^2}\frac{3}{x^3} + \frac{dy}{dv}\cdot\frac{2}{x^3}$$

$$\cancel{x^3}\left(\frac{d^3y}{dv^3}\cdot\frac{1}{\cancel{x^3}} - \frac{d^2y}{dv^2}\frac{3}{\cancel{x^3}} + \frac{dy}{dv}\cdot\frac{2}{\cancel{x^3}}\right) + 6\cancel{x^2}\left(\frac{d^2y}{dv^2}\cdot\frac{1}{\cancel{x^2}} - \frac{dy}{dv}\cdot\frac{1}{\cancel{x^2}}\right) + 7\cancel{x}\left(\frac{dy}{dv}\cdot\frac{1}{\cancel{x}}\right) + y = 0$$

$$\frac{d^3y}{dv^3} - 3\frac{d^2y}{dv^2} + 2\frac{dy}{dv} + 6\frac{d^2y}{dv^2} - 6\frac{dy}{dv} + 7\frac{dy}{dv} + y = 0$$

$$\frac{d^3y}{dv^3} + 3\frac{d^2y}{dv^2} + 3\frac{dy}{dv} + y = 0$$

$$r^3 + 3r^2 + 3r + 1 = 0$$
$$(r+1)^3 = 0 \qquad r_1 = r_2 = r_3 = -1$$

$$y(v) = (C_1 + C_2v + C_3v^2)e^{-v}$$

$$y(x) = (C_1 + C_2\ln x + C_3(\ln x)^2)e^{-\ln x} = (C_1 + C_2\ln x + C_3[\ln x]^2)x^{-1}$$