[MAA 2.2] QUADRATICS

SOLUTIONS

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O. Practice questions

1. (a) $\Delta = 9+16=25$, $x=\dfrac{3\pm 5}{2}$ so $x=4$ or $x=-1$

(b) $x^2-3x=0 \Leftrightarrow x(x-3)=0 \Leftrightarrow x=0$ or $x=3$

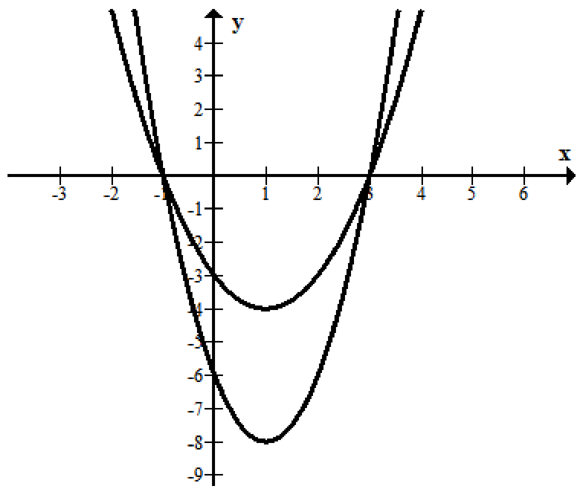
(c) $x^2-4=0 \Leftrightarrow x^2=4 \Leftrightarrow x=\pm 2$

2. (a) $\Delta = 4+12=16$, $x=\dfrac{2\pm 4}{2}$ so $x=3$ or $x=-1$

(b) $\Delta = 16+48=64$, $x=\dfrac{4\pm 8}{4}$ so $x=3$ or $x=-1$

(c) For *f*, vertex (1,-4). For g, vertex (1,-8)

(d)



3.

	$f(x)=2x^2-12x+10$	$f(x)=2x^2-12x+18$	$f(x)=2x^2-12x+23$
Discriminant	$\Delta=64$	$\Delta=0$	$\Delta=-40$
y-intercept	10	18	23
Roots	1, 5	3 (double),	No real roots,
Factorisation	$f(x)=2(x-1)(x-5)$	$f(x)=2(x-3)^2$	No factorization
axis of symmetry	$x=3$	$x=3$	$x=3$
Vertex	V(3,-8)	V(3,0)	V(3,5)
Vertex form $f(x)=a(x-h)^2+k$	$f(x)=2(x-3)^2-8$	$f(x)=2(x-3)^2$	$f(x)=2(x-3)^2+5$
Solve $f(x)\geq 0$	$x\leq 1$ or $x\geq 5$	$x\in R$	$x\in R$
Solve $f(x)>0$	$x<1$ or $x>5$	$x\in R-\{3\}$	$x\in R$
Solve $f(x)\leq 0$	$1\leq x\leq 5$	$x=3$	No solutions (It is always positive)
Solve $f(x)<0$	$1<x<5$	No solutions (It is always positive or 0)	No solutions (It is always positive)

4. (a) (i) $x=10 \quad x=20$

(ii) $y=4(x-10)(x-20)$

(b) (i) (15,-100)

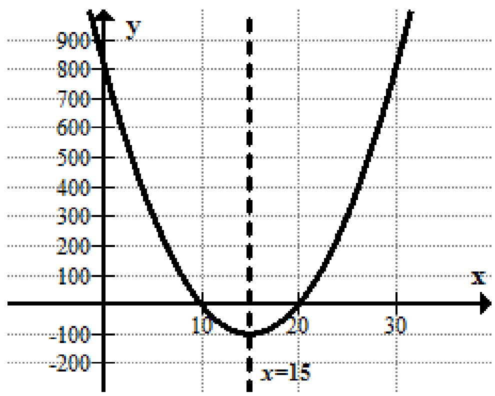
(ii) $y=4(x-15)^2-100$

(iii) $x=15$

(iv) $y_{\min}=-100$

(c) $y=800$

(d)



5. (a) (i) $x=10 \quad x=20$

(ii) $y=-4(x-10)(x-20)$

(b) (i) (15, 100)

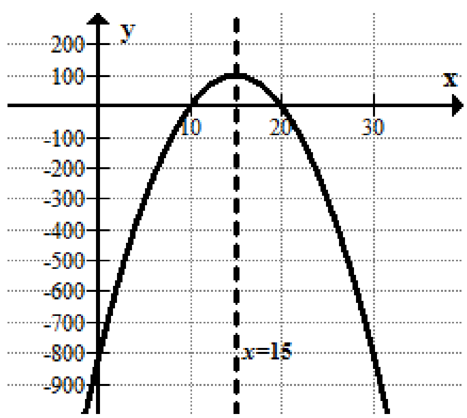
(ii) $y=-4(x-15)^2+100$

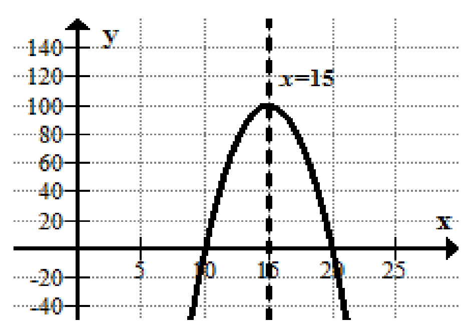
(iii) $x=15$

(iv) $y_{\max}=100$

(c) $y=-800$

(d)





6. (a) $x=4$

(b) $y=12$ since (8,12) is symmetric to (0,12) about $x=4$

(c) $y=5$ since (1,5) is symmetric to (7,5) about $x=4$

7. (a) (i) $x^2 - 3x + 4 = x + 1 \Leftrightarrow x^2 - 4x + 3 = 0 \ \Leftrightarrow \ x = 1$ or $x = 3$

Points (1,2) and (3,4) (**OR** directly obtained by GDC graph)

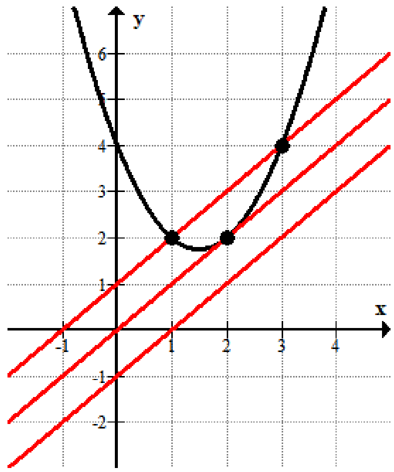
(ii) $x^2 - 3x + 4 = x \Leftrightarrow x^2 - 4x + 4 = 0 \ \Leftrightarrow \ x = 2$

Point (2,2) (**OR** directly obtained by GDC graph)

(iii) $x^2 - 3x + 4 = x - 1 \Leftrightarrow x^2 - 4x + 5 = 0$ no real solution

No Point of intersection

(b)



A. Exam style questions (SHORT)

8. (a) $x^2 - 3x - 10 = 0 \Rightarrow x = 5$ or $x = -2$

(b) $x^2 - 3x - 10 = (x - 5)(x + 2)$

9. (a) $p = -\frac{1}{2}$, $q = 2$ or vice versa

(b) By symmetry C is midway between p, $q \Rightarrow x$-coordinate is $\frac{-\frac{1}{2}+2}{2} = \frac{3}{4}$

10. (a) $f(x) = 0$

$x = \frac{1 \pm \sqrt{9}}{2}$

intercepts are $(-1, 0)$ and $(2, 0)$ (accept $x = -1$, $x = 2$)

(b) $x_v = \frac{x_1 + x_2}{2}$ **OR** $x_v = -\frac{b}{2a}$

$x_v = 0.5$

11. $(7 - x)(1 + x) = 0 \Leftrightarrow x = 7$ or $x = -1$

B: $x = \frac{7 + (-1)}{2} = 3$

$y = (7 - 3)(1 + 3) = 16$

12. $y = (x + 2)(x - 3) = x^2 - x - 6$

Therefore, $p = -1$, $q = -6$

OR

$0 = 4 - 2p + q$

$0 = 9 + 3p + q$

$p = -1$, $q = -6$

13. (a) $f(x) = 0 \Leftrightarrow 2x(4 - x) = 0 \Leftrightarrow x = 4, x = 0$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0))

(b) (i) $x = 2$ (must be equation)

(ii) substituting $x = 2$ into $f(x) \Rightarrow y = 8$

14.

Expression	+ – 0
a	–
c	–
$b^2 - 4ac$	0
$-\frac{b}{2a}$	+
b	+

15.

Expression	+ – 0
a	–
c	0
$b^2 - 4ac$	+
$-\frac{b}{2a}$	+
b	+

16.

Expression	+ – 0
a	+
c	–
$b^2 - 4ac$	+
$-\frac{b}{2a}$	+
b	–

17. (b) Vertex is (3, 5)

(a) Directly $f(x) = (x - 3)^2 + 5$

OR $f(x) = x^2 - 6x + 14 = x^2 - 6x + 9 - 9 + 14 = (x - 3)^2 + 5$

18. (a) $2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8 = 2(x - 2)^2 - 3$

OR vertex at $(2, -3) \Rightarrow y = 2(x - 2)^2 - 3$

=> $a = 2, \ p = 2, \ q = -3$

(b) Minimum value of $f(x) = -3$

19. (a) Vertex is (– 0.5, 1.5)

(b) $f(x) = 2(x + 0.5)^2 + 1.5$

20. (a) Vertex is (– 0.5, – 0.75)

(b) $f(x) = -(x + 0.5)^2 - 0.75$

21. (a) $q = -2, r = 4$ or $q = 4, r = -2$

(b) $x = 1$ (must be an equation)

(c) substituting (0, –4) into the equation: $-4 = -8p \Leftrightarrow p = \frac{4}{8}\left(= \frac{1}{2}\right)$

22. (a) Since the vertex is at (3, 1)

$h = 3, k = 1$

(b) (5, 9) is on the graph $\Rightarrow\ 9 = a(5-3)^2 + 1$

$\Leftrightarrow\ 9 = 4a + 1 \Leftrightarrow 4a = 8 \Leftrightarrow a = 2$

(c) $y = 2(x-3)^2 + 1 = 2(x^2 - 6x + 9) + 1 = 2x^2 - 12x + 19$

23. (a) $h = 3\ \ k = 2$

(b) $y \le 2$

(c) $f(x) = -(x-3)^2 + 2 = -x^2 + 6x - 9 + 2\ = -x^2 + 6x - 7$

24. (a) (i) $h = -1$, (ii) $k = 2$

(b) $a(1+1)^2 + 2 = 0 \Leftrightarrow a = -0.5$

25. (a) (i) $p = 1, q = 5$ (or $p = 5, q = 1$)

(ii) $x = 3$ (must be an equation)

(b) $y = (x-1)(x-5) = x^2 - 6x + 5 = (x-3)^2 - 4$

OR For $x = 3, y = -4 \Rightarrow y = (x-3)^2 - 4$

26. (a) (i) $m = 3$ (ii) $p = 2$

(b) $0 = d(1-3)^2 + 2$ **OR** $0 = d(5-3)^2 + 2$ **OR** $2 = d(3-1)(3-5)$

$d = -\frac{1}{2}$

27. (a) $p = -2\ \ q = 4$ (or $p = 4, q = -2$)

(b) $y = a(x+2)(x-4)$

$8 = a(6+2)(6-4)\ \Leftrightarrow\ 8 = 16a \Leftrightarrow\ a = \frac{1}{2}$

(c) $y = \frac{1}{2}(x+2)(x-4) = \frac{1}{2}(x^2 - 2x - 8) = \frac{1}{2}x^2 - x - 4$

(d) $y = \frac{1}{2}(x-1)^2 - \frac{9}{2}$

28. (a) $f(x) = -10(x+4)(x-6)$

(b) **METHOD 1**

Vertex: $x = 1, y = -10(1+4)(1-6)$, Hence $f(x) = -10(x-1)^2 + 250$

METHOD 2

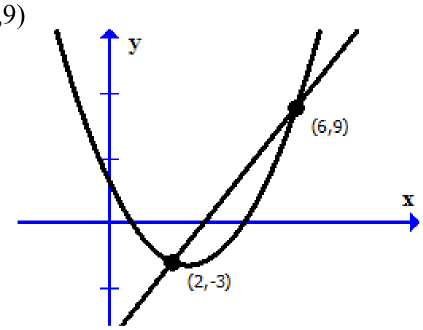
complete the square $f(x) = -10(x^2 - 2x - 24) = -10((x-1)^2 - 1 - 24) = -10(x-1)^2 + 250$

(c) $f(x) = -10(x+4)(x-6) = -10(x^2 - 6x + 4x - 24) = 240 + 20x - 10x^2$

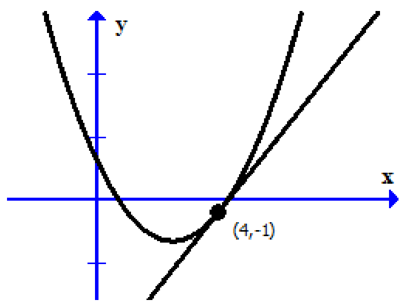
OR

$f(x) = -10(x-1)^2 + 250 = -10(x^2 - 2x + 1) + 250 = 240 + 20x - 10x^2$

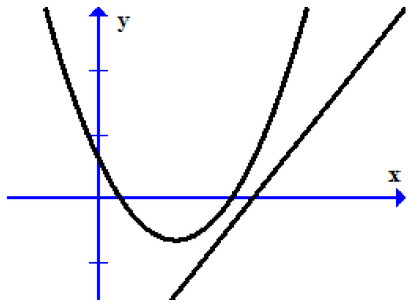
29. (2,-3) and (6,9)



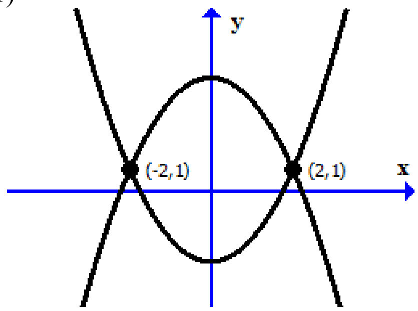
30. (4,-1)



31. no points of intersection



32. (-2,1) and (2,1)



33. $4x^2 + 4kx + 9 = 0$

Only one solution $\Rightarrow b^2 - 4ac = 0 \Rightarrow 16k^2 - 4(4)(9) = 0$

$k^2 = 9 \Leftrightarrow k = \pm 3$

But given $k > 0$, $k = 3$

34. One solution $\Rightarrow$ discriminant = 0

$$3^2 - 4k = 0 \Leftrightarrow 9 = 4k \Leftrightarrow k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$$

35. (a) $(k-3)^2 - 4 \times k \times 1 = 0$, $k^2 - 10k + 9 = 0$

$k = 1$, $k = 9$

(b) $k = 1$, $k = 9$

36. (a) $\Delta = 0 \Leftrightarrow (-4k)^2 - 4(2k)(1) = 0 \Leftrightarrow 16k^2 - 8k = 0 \Leftrightarrow 8k(2k - 1) = 0$

$k = \frac{1}{2}$

(b) vertex is on the x-axis $\Rightarrow$ $p \geq 0$

37. Discriminant $\Delta = (-2k)^2 - 4$, $\Delta > 0$

$(2k)^2 - 4 > 0 \Rightarrow 4k^2 - 4 > 0$

Solve $4k^2 - 4 = 0 \Leftrightarrow 4k^2 = 4 \Leftrightarrow k^2 = 1 \Leftrightarrow k = \pm 1$

THEN $k < -1$ or $k > 1$

38. $\Delta = 9 - 4k > 0 \Leftrightarrow 9 > 4k \Leftrightarrow k < 2.25$

crosses the x-axis if $k = 1$ **or** $k = 2$

39. For $kx^2 - 3x + (k + 2) = 0$ to have two distinct real roots then $k \neq 0$ and $9 - 4k(k + 2) > 0$

$4k^2 + 8k - 9 < 0$, hence $-2.803 < k < 0.803$

Set of values of k is $-2.80 < k < 0.803$, $k \neq 0$

40.

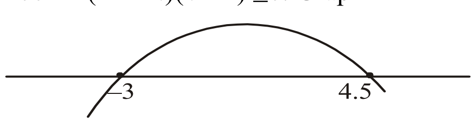
$\Delta = 4 - 4k(3k + 2)$ $\quad (= -12k^2 - 8k + 4, = -4(k+1)(3k-1))$

$\Delta = 0 \Rightarrow k = -1$, $k = \frac{1}{3}$

For 2 distinct roots, $\Delta > 0$

$-1 < k < \frac{1}{3}$

41. $100 - 4(1 + 2k)(k - 2) \geq 0$. Graph



$-3 \leq k \leq 4.5$ (accept $-3 < k < 4.5$)

42.

Using $b^2 - 4ac = (k-3)^2 - 4k(k-8)$

$-3k^2 + 26k + 9 = 0$

$\Rightarrow k = -\frac{1}{3}$, $k = 9$

$-3k^2 + 26k + 9 < 0$ $\quad (3k^2 - 26k - 9 > 0)$

$k < -\frac{1}{3}$ or $k > 9$

43. Let $f(x) = ax^2 + bx + c$ where $a = 1$, $b = (2 - k)$ and $c = k^2$.
Then for $a > 0$, $f(x) > 0$ for all real values of x if and only if
$b^2 - 4ac < 0 \Leftrightarrow (2 - k)^2 - 4k^2 < 0$
$\Leftrightarrow 4 - 4k + k^2 - 4k^2 < 0 \Leftrightarrow 3k^2 - 4k - 4 > 0$
$\Leftrightarrow (3k - 2)(k + 2) > 0 \Leftrightarrow k > \frac{2}{3}, k < -2$

44. $m(x + 1) \le x^2 \Rightarrow x^2 - mx - m \ge 0$

Hence $\Delta = b^2 - 4ac \le 0 \Rightarrow m^2 + 4m \le 0$

Now using a sketch of quadratic (or otherwise): $-4 \le m \le 0$

45. For intersection: $mx + 5 = 4 - x^2$ or $x^2 + mx + 1 = 0$.
For tangency: discriminant = 0
Thus, $m^2 - 4 = 0$, *so* $m = \pm 2$

46. $2x^2 + 2x - 1 = x^2 - m \Leftrightarrow x^2 + 2x + m - 1 = 0$.
$\Delta = 0 \Leftrightarrow 4 - 4(m - 1) = 0 \Leftrightarrow 8 - 4m = 0 \Leftrightarrow m = 2$

B. Exam style questions (LONG)

47. (a) Vertex is (4, 8)

(b) Substituting $-10 = a(7 - 4)^2 + 8 \Leftrightarrow a = -2$

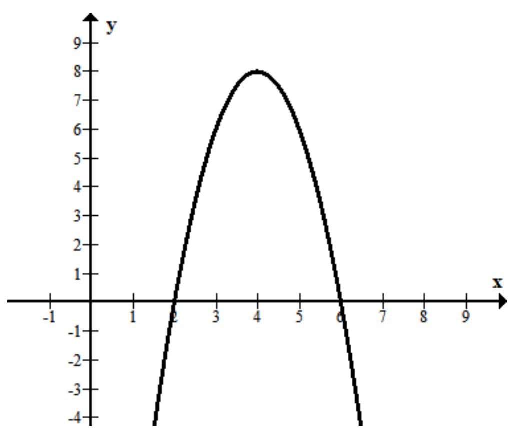
(c) For y-intercept, $x = 0$, $y = -24$

(d) $-2(x-4)^2 + 8 = 0 \Leftrightarrow 2(x-4)^2 = 8 \Leftrightarrow (x-4)^2 = 4 \Leftrightarrow x - 4 = \pm 2$

$\Leftrightarrow x = 6$ or $x = 2$

OR by expanding and then solve, $x = 6$ or $x = 2$

(e)



48. (a) substituting (–4, 3)

$3 = a(-4)^2 + b(-4) + c \Rightarrow 16a - 4b + c = 3$

(b) $3 = 36a + 6b + c$

$-1 = 4a - 2b + c$

(c) $a = 0.25$, $b = -0.5$, $c = -3$ (accept fractions)

$f(x) = 0.25x^2 - 0.5x - 3$

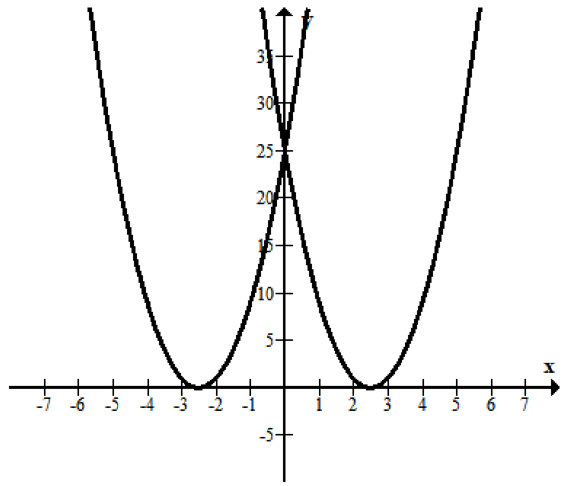
(d) $f(x) = 0.25(x - 1)^2 - 3.25$ (accept $h = 1$, $k = -3.25$, $a = 0.25$, or fractions)

49. (a) $\Delta = 0 \Leftrightarrow q^2 - 4(4)(25) = 0 \Leftrightarrow q^2 = 400 \Leftrightarrow q = 20, q = -20$

(b) $x = 2.5$

(c) (0, 25)

(d)



50. (a) line and graph intersect when $3x^2 - x + 4 = mx + 1 \Leftrightarrow 3x^2 - (1+m)x + 3 = 0$.

$\Delta = (1+m)^2 - 36$

(i) The line is tangent when $\Delta = 0 \Leftrightarrow (1+m)^2 = 36 \Leftrightarrow 1+m = \pm 6 \Leftrightarrow m = 5,\ m = -7$

(ii) Two points of intersection when $\Delta > 0 \Leftrightarrow m < -7,\ m > 5$

(iii) No points of intersection when $\Delta < 0 \Leftrightarrow -7 < m < 5$.

(b) When $m = 5$, $3x^2 - 6x + 3 = 0 \Leftrightarrow x^2 - 2x + 1 = 0 \Leftrightarrow x = 1$. Then $y = 6$. Point (1,6).

When $m = -7$, $3x^2 + 6x + 3 = 0 \Leftrightarrow x^2 + 2x + 1 = 0 \Leftrightarrow x = -1$. Then $y = 8$. Point (–1,8).

51. Let A(a,a^2) and B(b,0) be the points on the graph and on x-axis respectively. Then

$\frac{a+b}{2} = 5$ and $\frac{a^2+0}{2} = 2$, hence $a = \pm 2$ and $a = 8\, or\, 12$ respectively.

Therefore, A(2,4), B(8,0), or A(-2,4), B(12,0).

(b) Let C(c,c^2) be on the graph .

$(c-5)^2 + (c^2-2)^2 = 23^2 \Leftrightarrow c = 5 \ or \ c = -4.78$.

Therefore C(5,25) or C(-4.78,22.8)