Ambik Baral

Symbol No...............

RE - 231'B'

SEE 2078(2022)

Subject: Additional Mathematics

Time: 3: 00 hrs **F.M.:** 100

Attempt all the questions. All the working must be shown.

Group 'A' $[5 \times (1 + 1) = 10]$

1. (a) If $f(x), q(x), d(x)$ and $r(x)$ represent polynomial, quotient, divisor and remainder respectively, write the relation among them.

Solution:
Required relation is
$$f(x) = d(x) \times q(x) + r(x)$$

(b) What is the geometric mean between two positive numbers m and n? Write it.

Solution:
Geometric mean $= \sqrt{mn}$

2. (a) Express in words : $\lim_{x \to a^-} f(x)$

Solution:
Left hand limit of $f(x)$ at $x = a$.

(b) If matrix $M = \begin{bmatrix} a & -b \\ c & a \end{bmatrix}$, What is the value of $|M|$? Write it.

Solution:
$$|M| = \begin{vmatrix} a & -b \\ c & a \end{vmatrix} = a^2 + bc$$

3. (a) Write the condition of coincident of a pair of lines represented by the equation $ax^2 + 2hxy + by^2 = 0$.

Solution:
The condition of coincident is
$$h^2 = ab$$

(b) Which geometric figure will be formed when a plane intersects a cone parallel to the generator ? Write it.

Solution:
Parabola will be formed when a plane intersects a cone parallel to the generator.

4. (a) Write $\cos 2A$ in terms of $\tan A$.

Solution:

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

(b) If $\cos A = 0.5 \ \ (0^\circ < A < 90^\circ)$, what is the value of A? Write it.

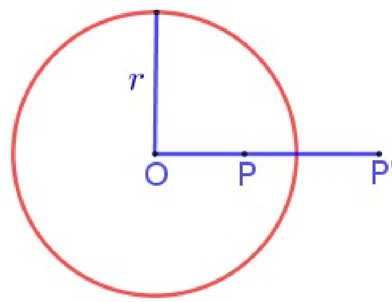
Solution: The value of A is 60°.

5. (a) If $\vec{a} = (x_1, y_1)$ and $\vec{b} = (x_2, y_2)$, write the value of $\vec{a}.\vec{b}$

Solution:

$$\vec{a}.\vec{b} = x_1.x_2 + y_1.y_2$$

(b) In the given figure, O is the centre of inversion circle and r is the radius. If P' is the inversion point of the point P, write the relation among OP, OP' and r



Solution:
The required relation is

$$OP \times OP' = r^2$$

. **Group 'B'** [$13 \times 2 = 26$]

6. (a) If $f(x) = 3x + a$ and $ff(6) = 10$, find the value of a.

Solution:

$$
\begin{aligned}
\text{Given, } & f(x) = 3x + a \\
& ff(6) = 10 \\
\text{Now,} & \\
& ff(6) = 10 \\
\text{or, } & f(f(6)) = 10 \\
\text{or, } & f(3 \times 6 + a) = 10 \\
\text{or, } & f(18 + a) = 10 \\
\text{or, } & 3(18 + a) + a = 10 \\
\text{or, } & 54 + 3a + a = 10 \\
\text{or, } & 4a = 10 - 54 \\
\text{or, } & 4a = -44 \\
\text{or, } & a = \frac{-44}{4} \\
\therefore \quad & a = -11
\end{aligned}
$$

(b) If the polynomial $x^3 - 6x^2 + 11x - p$ is divided by $(x - 2)$, the remainder is -4, find the value of p using remainder theorem.

Solution: Let, $f(x) = x^3 - 6x^2 + 11x - p$
Zero of divisor $(x - 2)$ is 2.

$$
\begin{aligned}
\text{Remsinder } & = -2 \\
\text{or, } & f(2) = -4 \\
\text{or, } & 2^3 - 6(2)^2 + 11(2) - p = -4 \\
\text{or, } & 8 - 24 + 22 - p = -4 \\
\text{or, } & 6 - p = -4 \\
\text{or, } & 6 + 4 = p \\
\therefore \quad & p = 10
\end{aligned}
$$

(c) Find the vertex of the parabola having equation $y = 2x^2 + 4x + 3$.

Solution: Given $y = 2x^2 + 4x + 3.....(i)$
Comparing equation (i) with $y = ax^2 + bx + c$, we get
$a = 2, b = 4$ and $c = 3$

$$\begin{aligned}
\text{Vertex of parabola } &= \left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)\\
&= \left(\frac{-4}{2\times 2}, \frac{4\times 2\times 3-4^2}{4\times 2}\right)\\
&= \left(\frac{-4}{4}, \frac{24-16}{8}\right)\\
&= \left(-1, \frac{8}{8}\right)\\
&= (-1, 1)
\end{aligned}$$

Alternative

Given,

$$\begin{aligned}
y &= 2x^2+4x+3\\
&= 2(x^2+2x)+3\\
&= 2(x^2+2.x.1+1^2)-2+3\\
&= 2(x+1)^2+1\\
\therefore y &= 2(x+1)^2+1 ...(ii)
\end{aligned}$$

Comparing equation $(ii)$with $y = a(x-h)^2+k$

We get,

Vertex$(h, k) = (-1, 1)$

7. (a) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & m \\ -1 & 2 \end{bmatrix}$ and $AB = I$ where I is an 2×2 identity matrix, then find the value of m.

Solution: Given,$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & m \\ -1 & 2 \end{bmatrix}$ and $AB = I$

Now,

$$\begin{aligned}
AB &= I\\
\text{or, } \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & m \\ -1 & 2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\\
\text{or, } \begin{bmatrix} 4-3 & 2m+6 \\ 2-2 & m+4 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\\
\text{or, } \begin{bmatrix} 1 & 2m+6 \\ 0 & m+4 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned}$$

Equating corresponding elements, we get,

$$m + 4 = 1$$
$$\text{or, } m = 1 - 4$$
$$\therefore m = -3$$

(b) If the sum of two numbers is 16 and their differcnce is 4, express those equations in matrix form.

Solution: Let, required two numbers be x and y.
By question,
$x + y = 16$ and $x - y = 4$
Expressing these equations in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \end{bmatrix}$$

8. (a) Find the equations of a pair of lines represented by the equation $x^2 - 2x - 2y - y^2 = 0$.

Solution: Given,

$$\text{or, } x^2 - 2x - 2y - y^2 = 0$$
$$\text{or, } x^2 - y^2 - 2x - 2y = 0$$
$$\text{or, } (x - y)(x + y) - 2(x + y) = 0$$
$$\text{or, } (x - y)(x + y - 2) = 0$$

$$\text{Either, } x - y = 0 \quad (1)$$
$$\text{Or, } x + y - 2 = 0 \quad (2)$$

Hence, required equations are

$$x + y = 0 \text{ and } x - y - 2 = 0$$

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(b) Find the obtuse angle between a pair of straight lines represented by an equation $x^2 - 4xy + y^2 = 0$.

Solution: Given,$x^2 - 4xy + y^2 = 0...(i)$
Comparing this equation with $ax^2 + 2hxy + by^2 = 0$,
We get,

$$a = 1, \qquad b = 1, \qquad h = -2$$

Let θ be the angle between two lines.
Then,

$$
\begin{aligned}
\tan\theta &= \pm\frac{2\sqrt{h^2-ab}}{a+b} \\
&= \pm\frac{2\sqrt{(-2)^2-1\times 1}}{1+1} \\
&= \pm\frac{2\sqrt{3}}{2} \\
&= \pm\sqrt{3}
\end{aligned}
$$

Taking positive,

$$
\begin{aligned}
\tan\theta &= \sqrt{3} \\
\tan\theta &= \tan 60^\circ \\
\therefore \theta &= 60^\circ \\
\text{Required obtuse angle } &= 180^\circ - 60^\circ = 120^\circ
\end{aligned}
$$

9. (a) Prove that: $\sqrt{1+\sin\theta} = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}$

Solution:

$$
\begin{aligned}
\text{L.H.S. } &= \sqrt{1+\sin\theta} \\
&= \sqrt{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \\
&= \sqrt{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2} \\
&= \cos\frac{\theta}{2} + \sin\frac{\theta}{2} \\
&= \text{ RHS}
\end{aligned}
$$

(b) Prove that: $\sin 105^\circ . \cos 15^\circ = \frac{1}{4}\left(2+\sqrt{3}\right)$

Solution:

$$\begin{aligned} \text{LHS} &= \sin 105^\circ . \cos 15^\circ \\ &= \frac{1}{2}[2\sin 105^\circ . \cos 15^\circ] \\ &= \frac{1}{2}[\sin(105^\circ + 15^\circ) + \sin(105^\circ - 15^\circ) \\ &= \frac{1}{2}[\sin 120^\circ + \sin 90^\circ] \\ &= \frac{1}{2}[\sin 60^\circ + 1] \\ &= \frac{1}{2}\left(\frac{\sqrt{3}}{2} + 1\right) \\ &= \frac{1}{2}\left(\frac{\sqrt{3}+2}{2}\right) \\ &= \frac{1}{4}\left(2+\sqrt{3}\right) \\ &= \text{ RHS} \end{aligned}$$

(c) If $\cos 3\theta - \sin 2\theta = 0$, find the value of θ under $0^\circ \leq \theta \leq 90^\circ$.

Solution:

Given,

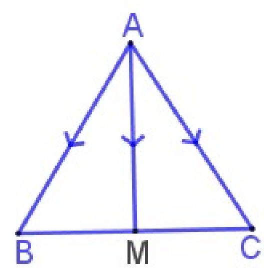
$$\begin{aligned} & \cos 3\theta - \sin 2\theta = 0 \\ \text{or, } & \cos 3\theta = \sin 2\theta \\ \text{or, } & \cos 3\theta = \cos(90^\circ - 2\theta) \\ \text{or, } & 3\theta = 90^\circ - 2\theta \\ \text{or, } & 3\theta + 2\theta = 90^\circ \\ \text{or, } & 5\theta = 90^\circ \\ \text{or, } & \theta = \frac{90^\circ}{5} \\ \therefore \quad & \theta = 18^\circ \end{aligned}$$

10. (a) If $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$, prove that $\vec{a} \perp \vec{b}$.

Solution:

$$
\begin{aligned}
&\text{Given,}\\
&(\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2\\
&\text{or, } (\vec{a})^2 + 2\vec{a}.\vec{b} + (\vec{b})^2 = (\vec{a})^2 - 2\vec{a}.\vec{b} + (\vec{b})^2\\
&\text{or, } 2\vec{a}.\vec{b} = -2\vec{a}.\vec{b}\\
&\text{or, } 2\vec{a}.\vec{b} + 2\vec{a}.\vec{b} = 0\\
&\text{or, } 4\vec{a}.\vec{b} = 0\\
&\text{or, } \vec{a}.\vec{b} = 0\\
&\implies \vec{a} \perp \vec{b}
\end{aligned}
$$

(b) In the adjoining figure, M is the mid point of BC, prove that: $\frac{1}{2}\left(\overrightarrow{AB}+\overrightarrow{AC}\right) = \overrightarrow{AM}$



Solution:

$$
\begin{aligned}
\text{LHS } &= \frac{1}{2}\left(\overrightarrow{AB}+\overrightarrow{AC}\right)\\
&= \frac{1}{2}\left(\overrightarrow{AM}+\overrightarrow{MB}+\overrightarrow{AM}+\overrightarrow{MC}\right)\\
&= \frac{1}{2}\left(2\overrightarrow{AM}+\overrightarrow{MB}+\overrightarrow{BM}\right)\\
&= \frac{1}{2}\left(2\overrightarrow{AM}+\overrightarrow{MB}-\overrightarrow{MB}\right)\\
&= \frac{1}{2}\left(2\overrightarrow{AM}\right)\\
&= \overrightarrow{AM}\\
&= \text{ RHS}
\end{aligned}
$$

(c) In the first quartile of a grouped data is 15 and the quartile deviation is 30, then find the coefficient of the quartile deviation.

Solution: Given, First Quartile $(Q_1) = 15$
Quartile Deviation$(Q.D.) = 30$

Now,

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2}$$
$$\text{or, } 30 = \frac{Q_3 - 15}{2}$$
$$\text{or, } 60 = Q_3 - 15$$
$$\text{or, } 60 + 15 = Q_3$$
$$\therefore Q_3 = 75$$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$
$$= \frac{75 - 15}{75 + 15}$$
$$= \frac{60}{90}$$
$$= 0.67$$

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Group 'C' [$11 \times 4 = 44$]

11. Solve by using factor theorem $2x^3 - 3x^2 - 11x + 6 = 0$.

Solution:

Given, $2x^3 - 3x^2 - 11x + 6 = 0$
Let, $f(x) = 2x^3 - 3x^2 - 11x + 6$
Possible roots are $\pm 1, \pm 2, \pm 3...$
Here,

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 11 \times (-2) + 6$$
$$= -16 - 12 + 22 + 6$$
$$= -28 + 28$$
$$= 0$$

$\therefore$ $(x + 2)$ is a factor of $f(x)$.

Now, using synthetic division,

-2	2	-3	-11	6
	$\downarrow$	-4	14	-6
	2	-7	3	$0 \rightarrow R$

$$\begin{aligned} \text{Quotient } &= 2x^2 - 7x + 3 \\ &= 2x^2 - 6x - x + 3 \\ &= 2x(x-3) - 1(x-3) \\ &= (x-3)(2x-1) \end{aligned}$$

$$\therefore f(x) = (x+2)(x-3)(2x-1)$$

Either	Or	Or
$x+2=0$	$x-3=0$	$2x-1=0$
or, $x=-2$	or, $x=3$	or, $x=\frac{1}{2}$

$$\therefore x = -2, 3, \frac{1}{2}$$

Alternative Method

Given, $2x^3 - 3x^2 - 11x + 6 = 0$
Let, $f(x) = 2x^3 - 3x^2 - 11x + 6$
Possible roots are $\pm 1, \pm 2, \pm 3...$
Here,

$$\begin{aligned} f(-2) &= 2(-2)^3 - 3(-2)^2 - 11 \times (-2) + 6 \\ &= -16 - 12 + 22 + 6 \\ &= -28 + 28 \\ &= 0 \end{aligned}$$

$\therefore$ $(x+2)$ is a factor of $f(x)$.
Now,

$$\begin{aligned} & 2x^3 - 3x^2 - 11x + 6 = 0 \\ \text{or, } & 2x^2(x+2) - 7x^2 - 11x + 6 = 0 \\ \text{or, } & 2x^2(x+2) - 7x(x+2) + 3x + 6 = 0 \\ \text{or, } & 2x^2(x+2) - 7x(x+2) + 3(x+2) = 0 \\ \text{or, } & (x+2)(2x^2 - 7x + 3) = 0 \\ \text{or, } & (x+2)[2x^2 - (6+1)x + 3] = 0 \\ \text{or, } & (x+2)[2x^2 - 6x - x + 3] = 0 \\ \text{or, } & (x+2)[2x(x-3) - 1(x-3)] = 0 \\ \text{or, } & (x+2)(x-3)(2x-1) = 0 \end{aligned}$$

Either	Or	Or
$x+2=0$	$x-3=0$	$2x-1=0$
or, $x=-2$	or, $x=3$	or, $x=\frac{1}{2}$

$$\therefore x = -2, 3, \frac{1}{2}$$

12. The students of class ten of a certain school collected a sum of Rs. 2750 from the some arts of the amounts they had brought for tiffin. It was planned to distribute cash prizes from the collected amount for the first ten students who secured distinct marks in the exam. If each cash prize is Rs. 50 less than the preceeding prizes, how much cash prize will the topper student receive ? Find it.

Solution: Here,

$$S_{10} = 2750$$
$$d = -50$$

We know,

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ \text{or, } 2750 &= \frac{10}{2}[2a + (10-1)d] \\ \text{or, } 2750 &= 5[2a + (9)(-50)] \\ \text{or, } \frac{2750}{5} &= 2a - 450 \\ \text{or, } 550 &= 2a - 450 \\ \text{or, } 550 + 450 &= 2a \\ \text{or, } 1000 &= 2a \\ \text{or, } \frac{1000}{2} &= a \\ \text{or, } 500 &= a \\ \therefore \quad a &= 500 \end{aligned}$$

$\therefore$ The topper student will receive Rs 500.

13. Prove that the function

$$f(x) = \begin{cases} 2x - 1 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \quad \text{at } x = 2 \\ x + 1 & \text{for } x > 2 \end{cases}$$

is continuous at the point $x = 2$.

Solution: Given,

$$f(x) = \begin{cases} 2x - 1 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \quad \text{at } x = 2 \\ x + 1 & \text{for } x > 2 \end{cases}$$

Left hand limit at $x = 2$

x	1.9	1.99	1.999	$x \to 2$
$f(x) = 2x - 1$				$f(x) \to 3$

$$\therefore \lim_{x \to 2^-} f(x) = 3$$

Right hand limit at $x = 2$

x	2.1	2.01	2.001	$x \to 2$
$f(x) = x + 1$				$f(x) \to 3$

$$\therefore \lim_{x \to 2^+} f(x) = 3$$

Functional value at $x = 2$

$$f(x) = 3$$
$$\therefore f(2) = 3$$

Since,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) = 3,$$

Given function $f(x)$ is continuous at $x = 2$

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14. Solve by Cramer's rule

$$\frac{6}{y} + \frac{10}{x} = 3; \quad \frac{3}{y} - \frac{21}{x} = -5$$

Solution: Let, $\frac{1}{y} = a$ and $\frac{1}{x} = b$, then

$$6a + 10b = 3...(i)$$

$$3a - 21b = -5...(ii)$$

Coeff of a	Coeff of b	Constant Term
6	10	3
3	−21	−5

Now,

$$D = \begin{vmatrix} 6 & 10 \\ 3 & -21 \end{vmatrix}$$
$$= -126 - 30$$
$$= -156$$
$$D_a = \begin{vmatrix} 3 & 10 \\ -5 & -21 \end{vmatrix}$$
$$= -63 + 50$$
$$= -13$$
$$D_b = \begin{vmatrix} 6 & 3 \\ 3 & -5 \end{vmatrix}$$
$$= -30 - 9$$
$$= -39$$

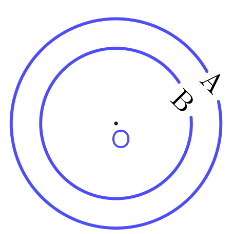
Now, By Cramer's Rule,

$$a = \frac{D_a}{D}$$
$$\text{or, } \frac{1}{y} = \frac{-13}{-156}$$
$$\text{or, } \frac{1}{y} = \frac{1}{12}$$
$$\therefore \quad y = 12$$

Again,

$$b = \frac{D_b}{D}$$
$$\text{or, } \frac{1}{x} = \frac{-39}{-156}$$
$$\text{or, } \frac{1}{x} = \frac{1}{4}$$
$$\therefore \quad x = 4$$

15. In the given figure, A and B are two concentric circles. If the equation of circle A is $x^2 + y^2 + 4x - 6y - 3 = 0$ and radius of circle B is 2 units, find the equation of circle B.



Solution: The equation of circle A is

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

Comparing this equation with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

We get,

$$g = 2, f = -3, c = -3$$

Center $(-g, -f) = (-2, 3)$

$\therefore$ Center of circle B,$(h, k) = (-2, 3)$ Radius of circle B, $r = 2$ units
Equation of circle B is

$$\begin{aligned} &(x-h)^2 + (y-k)^2 = r^2 \\ \text{or, } &(x+2)^2 + (y-3)^2 = 2^2 \\ \text{or, } &x^2 + 4x + 4 + y^2 - 6y + 9 = 4 \\ \therefore\ &x^2 + y^2 + 4x - 6y + 9 = 0 \end{aligned}$$

This is required equation.

16. Prove that: $\tan 45^\circ \sec 40^\circ + \tan 60^\circ \text{cosec } 40^\circ = 4$

Solution:

$$\begin{aligned} \text{LHS} &= \tan 45^\circ \sec 40^\circ + \tan 60^\circ \text{cosec } 40^\circ \\ &= 1 \times \frac{1}{\cos 40^\circ} + \sqrt{3} \times \frac{1}{\sin 40^\circ} \\ &= \frac{1}{\cos 40^\circ} + \frac{\sqrt{3}}{\sin 40^\circ} \\ &= \frac{\sin 40^\circ + \sqrt{3}\cos 40^\circ}{\sin 40^\circ . \cos 40^\circ} \\ &= \frac{\frac{1}{2}\sin 40^\circ + \frac{\sqrt{3}}{2}\cos 40^\circ}{\frac{1}{2}\sin 40^\circ . \cos 40^\circ} \\ &= \frac{\cos 60^\circ . \sin 40^\circ + \sin 60^\circ . \cos 40^\circ}{\frac{1}{4} \times 2\sin 40^\circ . \cos 40^\circ} \\ &= \frac{\sin(60^\circ + 40^\circ)}{\frac{1}{4}\sin 80^\circ} \\ &= \frac{\sin 100^\circ}{\frac{1}{4}\sin(180^\circ - 100^\circ)} \\ &= \frac{4\sin 100^\circ}{\sin 100^\circ} \\ &= 4 \\ &= \text{RHS} \end{aligned}$$

17. If $A + B + C = \frac{\pi^c}{2}$, then prove that:

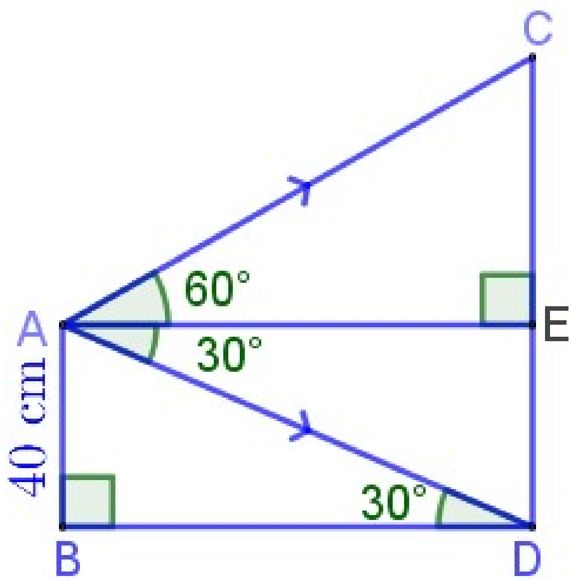
$$\cos^2 A + \cos^2 B + \cos^2 C = 2 + 2\sin A . \sin B . \sin C$$

Solution:

$$
\begin{aligned}
&\text{Given,}\\
A+B+C &= \frac{\pi^c}{2}\\
\text{or, } A+B &= \frac{\pi^c}{2} - C\\
\text{or, } \cos(A+B) &= \cos\left(\frac{\pi^c}{2} - C\right)\\
&= \sin C\\
\text{LHS} &= \cos^2 A + \cos^2 B + \cos^2 C\\
&= \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2} + \cos^2 C\\
&= \frac{1+\cos 2A + 1 + \cos 2B}{2} + \cos^2 C\\
&= \frac{2+\cos 2A + \cos 2B}{2} + \cos^2 C\\
&= \frac{2 + 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)}{2} + \cos^2 C\\
&= 1 + \cos(A+B)\cos(A-B) + \cos^2 C\\
&= 1 + \sin C\cos(A-B) + 1 - \sin^2 C\\
&= 2 + \sin C\cos(A-B) - \sin^2 C\\
&= 2 + \sin C[\cos(A-B) - \sin C]\\
&= 2 + \sin C[\cos(A-B) - \cos(A+B)]\\
&= 2 + \sin C.2\sin A\sin B\\
&= 2 + 2\sin A\sin B\sin C\\
&= \text{RHS}
\end{aligned}
$$

18. From the roof of a house 40 m high, the angles of elevation and depression of the top and foot of a tower are found to be 60° and 30° respectively. Find the height of the tower and the distance between the house and the tower.

Solution:



Let, AB be the height of house
CD be the height of tower
BD be the distance between house and tower
Then,
$AB = 40$ m
$\angle CAE = 60°$
$\angle EAD = 30° = \angle ADB$
Now,

$$\begin{aligned}
\text{In right angled } \Delta ABD & \\
\tan 30° &= \frac{AB}{BD} \\
\frac{1}{\sqrt{3}} &= \frac{40}{BD} \\
BD &= 40\sqrt{3} \\
\therefore AE &= BD = 40\sqrt{3} \\
\text{In right angled } \Delta AEC, & \\
\tan 60° &= \frac{CE}{AE} \\
\sqrt{3} &= \frac{CE}{40\sqrt{3}} \\
\therefore CE &= 120 \\
\text{Hence Height of tower } &= CE + ED \\
&= 120 + 40 \\
&= 160 \text{ m} \\
\text{Distance between tower and house and tower } &= 40\sqrt{3} \text{ m}
\end{aligned}$$

19. Find a 2×2 transformation matrix which transforms the quadrilateral $\begin{pmatrix}0 & -1 & -1 & 0\\ 0 & 0 & 1 & 1\end{pmatrix}$ into the quadrilateral $\begin{pmatrix}0 & 3 & 4 & 1\\ 0 & 3 & 4 & 1\end{pmatrix}$.

Solution:

Object matrix $(O) = \begin{pmatrix}0 & -1 & -1 & 0\\ 0 & 0 & 1 & 1\end{pmatrix}$

Image matrix $(I) = \begin{pmatrix}0 & 3 & 4 & 1\\ 0 & 3 & 4 & 1\end{pmatrix}$

Let, transformation matrix $(T.M.) = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$

We know,

$$I = T.M. \times O$$
$$\text{or, } I = \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0 & -1 & -1 & 0\\ 0 & 0 & 1 & 1\end{pmatrix}$$
$$\text{or, } I = \begin{pmatrix}0+0 & -a+0 & -a+b & 0+b\\ 0+0 & -c+0 & -c+d & 0+d\end{pmatrix}$$
$$\text{or, } \begin{pmatrix}0 & 3 & 4 & 1\\ 0 & 3 & 4 & 1\end{pmatrix} = \begin{pmatrix}0 & -a & -a+b & b\\ 0 & -c & -c+d & d\end{pmatrix}$$

Comparing the corresponding elements,
We get,

$$a = -3, b = 1, c = -3 \text{ and } d = 1$$
$$\therefore \text{ Transformation matrix } = \begin{pmatrix}-3 & 1\\ -3 & 1\end{pmatrix}$$

20. Find the mean deviation from the median and its coefficient from the data given below.

Marks obtained	10 - 20	20 - 30	30 - 40	40 - 50	50 -60
No. of Students	5	4	5	4	2

Solution:

C.I.	x	f	$c.f.$	$\lvert x - Md\rvert$	$f\lvert x - Md\rvert$
10 - 20	15	5	5	17	85
20 - 30	25	4	9	7	28
30 - 40	35	5	14	3	15
40 - 50	45	4	18	13	52
50 -60	55	2	20	23	46
		N = 20			$\Sigma f\lvert x - Md\rvert = 226$

$$\begin{aligned}
\text{Median class} &= \left(\frac{N}{2}\right)^{th} \text{ item}\\
&= \left(\frac{20}{2}\right)^{th} \text{ item}\\
&= (10)^{th} \text{ item}\\
\therefore \text{ Median Class} &= 30 - 40
\end{aligned}$$

$$\text{Here, } L = 30, f = 5, c.f. = 9, h = 10$$

Now,

$$\begin{aligned}
\text{Median} &= L + \frac{\frac{N}{2} - c.f.}{f} \times h\\
&= 30 + \frac{10-9}{5} \times 10\\
&= 30 + \frac{1}{5} \times 10\\
&= 30 + 2\\
&= 32
\end{aligned}$$

$$\begin{aligned}
\text{Now, Mean deviation from median } &= \frac{\Sigma f|x - Md|}{N}\\
&= \frac{226}{20}\\
&= 11.3\\
\text{Coefficient of M.D. from median } &= \frac{M.D.}{Median}\\
&= \frac{11.3}{32}\\
&= 0.35
\end{aligned}$$

21. Find the standard deviation and coefficient of variation from the given data.

Age in years	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20	20 - 24
No. of Students	7	7	10	15	7	6

Solution:

C.I.	x	f	fx	fx^2
0 - 4	2	7	14	28
4 - 8	6	7	42	252
8 - 12	10	10	100	1000
12 - 16	14	15	210	2940
16 - 20	18	7	126	2268
20 - 24	22	6	132	2904
		$N = 52$	$\Sigma fx = 624$	$\Sigma fx^2 = 9392$

$$\text{Standard Deviation}(\sigma) = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2}$$
$$= \sqrt{\frac{29392}{52} - \left(\frac{624}{52}\right)^2}$$
$$= \sqrt{180.62 - 144}$$
$$\therefore \text{Standard Deviation}(\sigma) = 6.05$$
$$\text{Mean}(\bar{x}) = \frac{\Sigma fx}{N}$$
$$= \frac{624}{52}$$
$$= 12$$
$$\text{Coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100\%$$
$$= \frac{6.05}{12} \times 100\%$$
$$= 50.43\%$$

Group 'D' [4 × 5 = 20]

22. Find the minimum value of the objective function $P = 3x + 4y$ under the given constraints.

$$x + y \geq 6, y \leq x \text{ and } x \leq 6$$

Solution: Given constraints are

$$x + y \geq 6 \ldots (i)$$
$$y \leq x \ldots (ii)$$
$$x \leq 6 \ldots (iii)$$

Boundary line of (i) is $x + y = 6$

x	6	0
y	0	6

$\therefore$ boundary line passes through $(6, 0)$ and $(0, 6)$

Now, taking (0,0) as testing point, we get

$0 + 0 > 6$

or, $0 > 6$ False

$\therefore$ solution set does not contain $(0, 0)$

Boundary line of (ii) is $y = x$

x	0	1
y	0	1

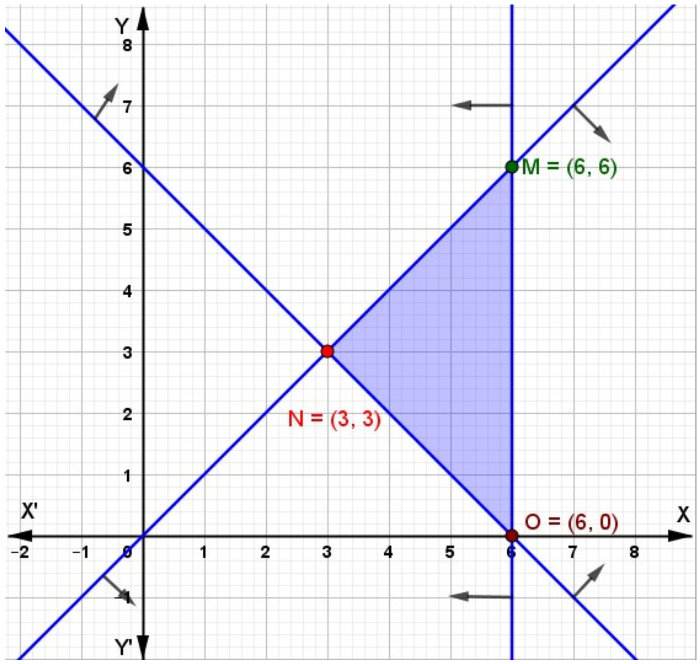
$\therefore$ boundary line passes through $(0,0)$ and $(1,1)$

Now, taking (1,0) as testing point, we get

$0 < 1$ (True)

$\therefore$ solution set of (ii) contains $(1,0)$

Also,$x \leq 6$ represents the region left from the line $x = 6$



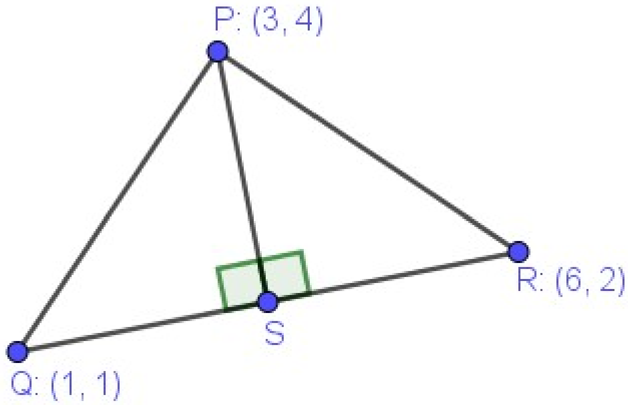
In the graph the shaded region is the common solution set. The vertices of common solution set are (6,0), (6,6) and (3,3).

Vertex	P $= 3x + 4y$	Remarks
(6,0)	$P = 3 \times 6 + 4 \times 0 = 18$	Minimum
(6,6)	$P = 3 \times 6 + 4 \times 6 = 42$	
(3,3)	$P = 3 \times 3 + 4 \times 3 = 21$	

Hence, the mimimum value of P is 18 at point $(6,0)$.

23. The co-ordinates of the vertices P, Q and R of ΔPQR are $(3,4), (1,1)$ and $(6,2)$ respectively. If PS is the altitude of ΔPQR, find the co-ordinates of the point S.

Solution:



Equation of QR is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$\text{or, } y - 1 = \frac{2-1}{6-1}(x - 1)$$
$$\text{or, } y - 1 = \frac{1}{5}(x - 1)$$
$$\text{or, } 5y - 5 = x - 1$$
$$\text{or, } 0 = x - 5y + 5 - 1$$
$$\text{or, } 0 = x - 5y + 4$$
$$\text{or, } x - 5y + 4 = 0$$
$$\therefore x = 5y - 4......(i)$$

Equation of PS which is perpendicular to QR is

$$5x = -y + k...(ii)$$

Since the line (ii) passes through the point P(3,4),
putting x =3 and y =4 in equation (ii), we get,

$$5 \times 3 = -4 + k$$
$$\text{or, } 15 + 4 = k$$
$$\therefore k = 19$$

Putting k = 19 in equation (ii),

$$5x = -y + 19.....(iii)$$

Putting the value of x from equation (i) to equation (iii), we get,

$$5(5y-4) = -y+19$$
$$\text{or, } 25y-20 = -y+19$$
$$\text{or, } 25y+y = 20+19$$
$$\text{or, } 26y = 39$$
$$\text{or, } y = \frac{39}{26}$$
$$\therefore y = \frac{3}{2}$$

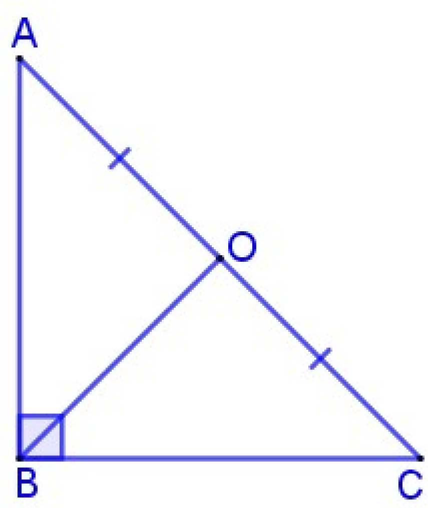
Putting $y = \frac{3}{2}$ in equation (i), we get,

$$x = 5 \times \frac{3}{2} - 5$$
$$\text{or, } x = \frac{15-8}{2}$$
$$\therefore x = \frac{7}{2}$$

Thus, the co-ordinates of the point $S = \left(\frac{7}{2}, \frac{3}{2}\right)$

24. In ΔABC, ∠ABC = 90° and O is the mid-point of side AC, then prove by vector method that: OA = OB = OC.

Solution:



Given: In ΔABC, $\angle ABC = 90°$ and O is the mid-point of side AC .
To Prove: $AO = OC = OB$

Proof:

S.N.	Statements	S.N.	Reasons
1.	$AO = OC$	1.	Given
2.	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$	2.	Triangle law of vector addition
3.	$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$ $= \overrightarrow{-OB} + \overrightarrow{AO}$ $= \overrightarrow{AO} - \overrightarrow{OB}$	3.	Triangle law of vector addition
4.	$\overrightarrow{AB}.\overrightarrow{BC} = (\overrightarrow{AO} + \overrightarrow{OB}).(\overrightarrow{AO} - \overrightarrow{OB})$ or, $0 = (\overrightarrow{AO})^2 - (\overrightarrow{OB})^2$ or, $0 = (AO)^2 - (OB)^2$ or, $(OB)^2 = (AO)^2$ or, OB = AO $\therefore$ AO = OB	4.	From (1) and (2) $\angle ABC = 90°$
5.	AO = OC = OB	5.	From (1) and (4)

Proved.

25. E denotes the enlargement about the centre $(3, 1)$ with a scale factor of 2 and R denotes the reflection on the line $y = x$. Find the image of ΔABC having the vertices $A(2, 3), B(4, 5)$ and $C(1, -2)$ under the combined transformation EoR. Draw both ΔABC and image $\Delta A'B'C'$ on the same graph paper.

Solution:

Here: $E : [(3, 1), 2]$ and $R : y = x$
Let object be (m, n).
Under the reflection on the line $y = x$,

$$(m, n) \rightarrow (n, m)$$

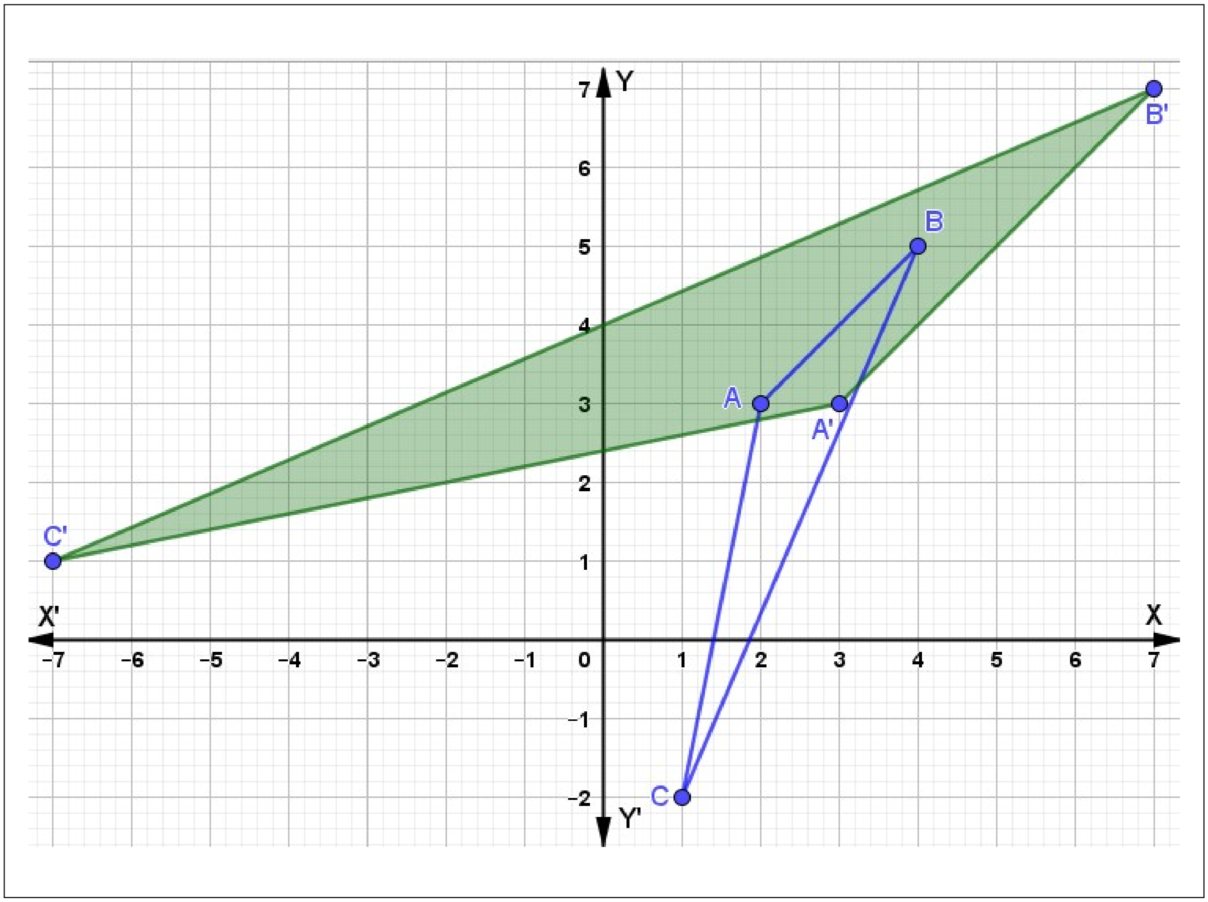
Under the Enlargement [(3,1),2]

$$(x, y) \rightarrow (kx - ka + a, ky - kb + b)$$
$$(n, m) \rightarrow (2x - 2 \times 3 + 3, 2y - 2 \times 1 + 1)$$
$$\therefore\ (n, m) \rightarrow (2n - 3, 2m - 1)$$

Hence Under EoR

$$(m, n) \rightarrow (2n - 3, 2m - 1)$$
$$A(2, 3) \rightarrow A'(2 \times 3 - 3, 2 \times 2 - 1) = A'(3, 3)$$
$$B(4, 5) \rightarrow B'(2 \times 5 - 3, 2 \times 4 - 1) = B'(7, 7)$$
$$C(1, -2) \rightarrow C'(2 \times (-2) - 3, 2 \times 1 - 1) = C'(-7, 1)$$

Hence, the vertices of image $\Delta A'B'C'$ are $A'(3, 3), B'(7, 7)$ and $C'(-7, 1)$.



All The Best