$\boxed{13.6\ \text{Ejercicios}}$

- Hallar la derivada direccional de la función de P en dirección de v.

⑦ $g(x,y) = \sqrt{x^2+y^2}$, $P(3,4)$, $V = 3i - 4j$

$$\nabla g = \frac{x}{\sqrt{x^2+y^2}}\, i + \frac{y}{\sqrt{x^2+y^2}}\, j$$

$$\nabla g = (3,4) = \frac{3}{5}\, i + \frac{4}{5}\, j$$

$$u = \frac{V}{\|V\|} = \frac{3}{5} i - \frac{4}{5} j$$

$$D_u g\,(3,4) = \nabla g\,(3,4) \cdot u = \boxed{-\frac{7}{25}}$$

⑪ $h(x,y,z) = xyz$, $P(2,1,1)$, $V = \langle 2,1,2 \rangle$

$$\nabla h = yz\,i + xz\,j + xy\,k$$
$$\nabla h\,(2,1,1) = i + 2i + 2k$$

$$u = \frac{V}{\|V\|} = \frac{2}{3} i + \frac{1}{3} j + \frac{2}{3} k$$

$$D_u h(2,1,1) = \nabla h(2,1,1) \cdot u = \frac{8}{3}$$

(13) Hallar la derivada direccional de la función en dirección de $u = \cos\theta i + \operatorname{sen}\theta j$

- $F(x,y) = x^2 + y^2$, $\theta = \pi/4$

$\vec{u} = \cos\theta\, i + \operatorname{sen}\theta\, j = \langle \cos\theta, \operatorname{sen}\theta$

$= \langle \cos(\pi/4), \operatorname{sen}(\pi/4) \rangle = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$F_x = 2x$, $f_y = 2y$

$\nabla F = 2x i + 2y j$

$D_u F(x,y) = \nabla F \cdot u = \frac{2}{\sqrt{2}} x + \frac{2}{\sqrt{2}} y$

$= \cancel{2}x\left(\frac{\sqrt{2}}{\cancel{2}}\right) + \cancel{2}y\left(\frac{\sqrt{2}}{\cancel{2}}\right)$

$= \sqrt{2}(x+y)$

- Hallar la derivada direccional de la función en P en dirección de Q.

(17) $F(x,y) = x^2 + 3y^2$, $P(1,1)$, $Q(4,5)$

$\vec{PQ} = \langle 4-1, 5-1 \rangle = \langle 3, 4 \rangle$

$V = 3i + 4i$

$F_x = 2x \rightarrow 2(1) = 2$

$F_y = 6y \rightarrow 6(1) = 6$

$\nabla F(x,y) = \langle 2, 6 \rangle$

$\nabla F = 2(x i + 6 y j$, $\nabla F = 2i + 6j$

$u = \frac{V}{\|V\|} = \frac{3i}{5} + \frac{4}{5} j$

$D_u F(1,1) = \nabla F(1,1) \cdot u = \frac{6}{5} + \frac{24}{5} = \boxed{6}$

- *Hallar la gradiente de la función en el punto dado.*

(21) $F(x,y) = 3x + 5y^2 + 1$, $(2,1)$.

$F_x = 3$, $F_y = 10y$

$F_y(2,1) = 10(1) = 10$

$\nabla F(x,y) = 3i + 10yj$

$\nabla F(2,1) = \langle 3, 10 \rangle = 3i + 10j$

(25) $w = 3x^2 - 5y^2 + 2z^2$, $(1,1,-2)$

$w_x = 6x$

$w_x(1,1,-2) = 6(1) = \boxed{6}$

$w_y = -10y$

$w_y = (1,1,-2) = -10(1) = \boxed{-10}$

$w_z = 4z$

$w_z(1,1,-2) = 4(-2) = -8$

$\nabla F(1,1,-2) = \langle 6, -10, -8 \rangle$

$= \boxed{6i - 10j - 8k}$

• Hallar la gradiente de la función y le valor máximo de la derivada direccional en el punto dado

(39) $w = xy^2z^2$, $(2,1,1)$.

$w_x = y^2z^2$

$w_x(2,1,1) = (1)^2(1)^2 = \boxed{1}$

$w_y = x\,2y\,z^2$

$w_y = (2,1,1) = (2)\,2(1)(1)^2 = \boxed{4}$

$w_z = xy^2\,2z$

$w_z(2,1,1) = (2)(1)^2\,2(1) = \boxed{4}$

$\nabla w(2,1,1) = \langle 1,4,4 \rangle = 1i + 4j + 4k$.

Valor Máximo $\|\nabla w(2,1,1)\| = \sqrt{33}$

• Utilizar la función $f(x,y) = 3 - \frac{x}{3} - \frac{y}{2}$

(44) Hallar $\nabla F(x,y)$.

$F_x = \frac{-1}{3}$, $F_y = \frac{-1}{2}$

$\nabla F(x,y) = \left\langle -\frac{1}{3}, -\frac{1}{2} \right\rangle$

$\boxed{= -\frac{1}{3}i - \frac{1}{2}j}$

(45) Hallar el valor máximo de la derivada direccional en $(3,2)$.

$$\|\nabla F(3,2)\| = \sqrt{\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{9}+\frac{1}{4}}$$

$$= \sqrt{\frac{13}{36}} = \boxed{\frac{1}{6}\sqrt{13}}$$

- Halla un vector normal a la curva de nivel $f(x,y) = c$ en P.

(51) $F(x,y) = 6 - 2x - 3y$

$c = 6, \quad P(0,0)$.

$F(x,y) = c$

$\cancel{6} - 2x - 3y = \cancel{6} \rightarrow -2x - 3y = 0$

$F_x = -2 \rightarrow F_x(0,0) = -2$

$F_y = -3 \rightarrow F_y(0,0) = -3$

$\nabla F(0,0) = \langle -2, -3 \rangle$

$= -2i - 3j$

(57) $f(x,y) = 3x^2 - 2y^2$, $c=1$, $P(1,1)$.

a) Encontrar la gradiente de la función de P.

$f_x = 6x$ | $f_y = -4y$

$f_x(1,1) = 6(1) = 6$ | $f_y(1,1) = -4(1) = -4$

$\nabla f(1,1) = \langle 6, -4 \rangle = 6i - 4j$

b) Encontra el vector un vector normal unitario para la curva de nivel $f(x,y) = c$ en P.

$\nabla f(x,y) = \langle 6x, -4y \rangle$

$\nabla f(1,1) = \langle 6, -4 \rangle = 6i - 4j$

$= 3i - 2j$

$\frac{\nabla f(1,1)}{\|\nabla f\|} = \frac{\langle 3,-2 \rangle}{\sqrt{(3)^2+(-2)^2}} = \frac{\langle 3,-2 \rangle}{\sqrt{13}} = \frac{3i-2j}{\sqrt{13}}$

$= \frac{1}{\sqrt{13}}(3i - 2i)$ → Vector normal en la curva $3x^2 - 2y^2 = 1$ a $(1,1)$.

c) Encontrar la recta tangente a la curva de nivel $f(x,y) = c$ en P.

$m = \frac{f_x}{f_y}\Big|_{(1,1)} \rightarrow \frac{-6x}{-9y} = \frac{-6(1)}{-9(1)} = \frac{-6}{-9} = \boxed{\frac{3}{2}}$

$y - y_0 = m(x - x_0)$

$y - 1 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x - \frac{3}{2} + 1 \rightarrow \boxed{y = \frac{3}{2}x - \frac{1}{2}}$

linea tangente

d) Trazar la curva de nivel, el vector unitario normal y la recta tangente en el plano xy.

$3x^2 - 2y^2 = 1 \rightarrow \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle = \langle 0.83, -0.55 \rangle$

$y = \frac{3}{2}x - \frac{1}{2} \rightarrow x = 1, y = 1$

$x = = -\frac{1}{2}$