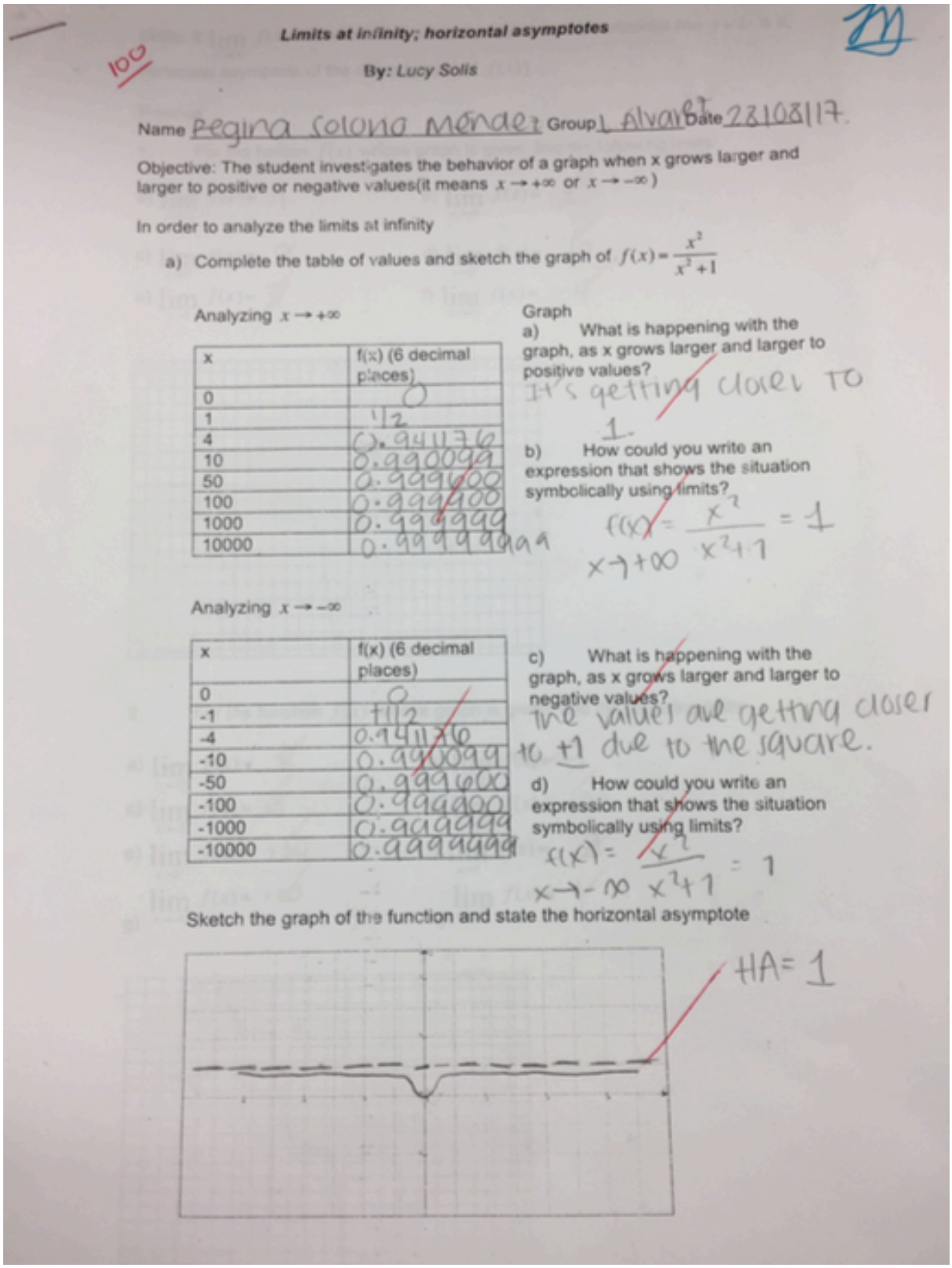
Limits at infinity; horizontal asymptotes



Limits at infinity; horizontal asymptotes

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Objective: The student investigates the behavior of a graph when x grows larger and larger to positive or negative values(it means $x \to +\infty$ or $x \to -\infty$)

In order to analyze the limits at infinity

a) Complete the table of values and sketch the graph of $f(x) = \frac{x^2}{x^2+1}$

Analyzing $x \to +\infty$

x	f(x) (6 decimal places)
0	0
1	1/2
4	0.941176
10	0.990099
50	0.999600
100	0.999900
1000	0.999999
10000	0.99999999

Graph

a) What is happening with the graph, as x grows larger and larger to positive values?

It's getting closer to 1.

b) How could you write an expression that shows the situation symbolically using limits?

$\lim_{x \to +\infty} f(x) = \frac{x^2}{x^2+1} = 1$

Analyzing $x \to -\infty$

x	f(x) (6 decimal places)
0	0
-1	1/2
-4	0.941176
-10	0.990099
-50	0.999600
-100	0.999900
-1000	0.999999
-10000	0.99999999

c) What is happening with the graph, as x grows larger and larger to negative values?

The values are getting closer to +1 due to the square.

d) How could you write an expression that shows the situation symbolically using limits?

$\lim_{x \to -\infty} f(x) = \frac{x^2}{x^2+1} = 1$

Sketch the graph of the function and state the horizontal asymptote

HA = 1

[Note: If $\lim_{x\to\infty} f(x) = L$ where L is a real number then the horizontal line $y = L$ is a horizontal asymprote of the curve (graph) of $f(x)$]

Practice

1. For the funtion $f(x)$ whose graph is given, find the following limits

a) $\lim_{x\to-\infty} f(x) =$ 1

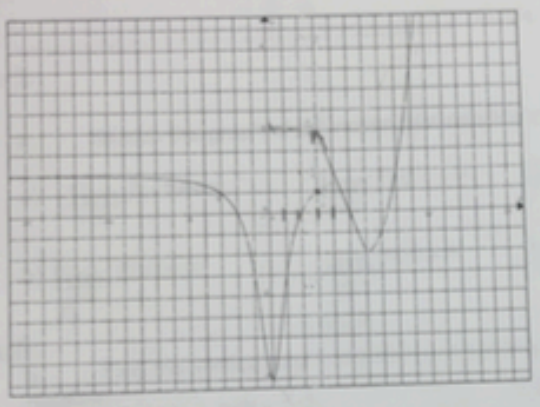
b) $\lim_{x\to+\infty} f(x) =$ ∞

c) $\lim_{x\to3^+} f(x) =$ 2

d) $\lim_{x\to3^-} f(x) =$ 1/2

e) $\lim_{x\to3} f(x) =$ DNE

f) $\lim_{x\to0} f(x) =$ -4



2. For the function $f(x)$ whose graph is given, find the following limits

a) $\lim_{x\to-2^+} f(x) =$ 3

b) $\lim_{x\to2^-} f(x) =$ 9

c) $\lim_{x\to-2} f(x) =$ DNE

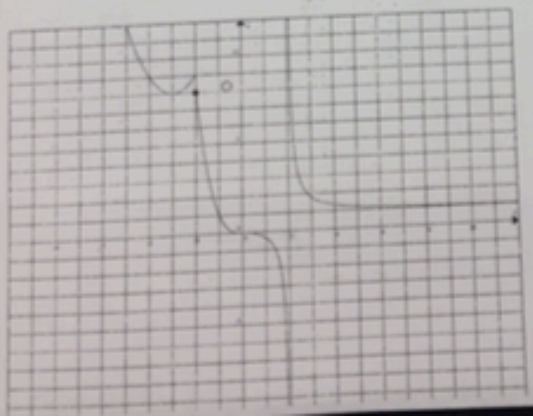
d) $\lim_{x\to2^+} f(x) =$ $-\infty$

e) $\lim_{x\to-2^+} f(x) =$ $+\infty$

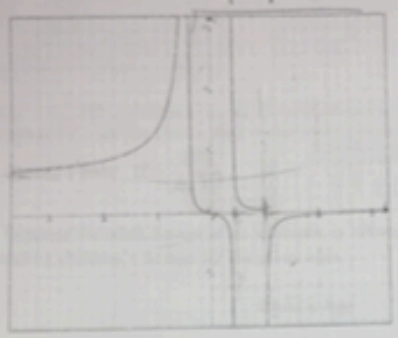
f) $\lim_{x\to2} f(x) =$ DNE

g) $\lim_{x\to-\infty} f(x) =$ $+\infty$

h) $\lim_{x\to+\infty} f(x) =$ 1



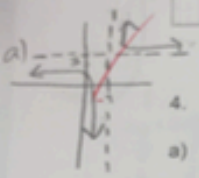
3. Find an estimation of the infinite limits, limits at infinity, and asymptotes for the function $f(x)$ (give the answer using integer numbers) whose graph is given below.



Asymptotes → VA: $x=-3, 2, 5$
HA: $y=0, 3$

Infinite limits
$\lim_{x\to -3^+} f(x) = +\infty$
$\lim_{x\to 3^-} f(x) = +\infty$
$\lim_{x\to -3} f(x) = +\infty$ →

a)



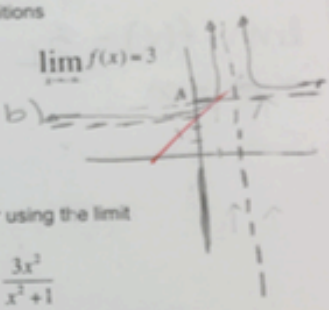
4. Sketch the graph of a function that satisfies all the given conditions

a) $\lim_{x\to -2^-} f(x) = +\infty$ $\lim_{x\to -2^+} f(x) = -\infty$ $\lim_{x\to \infty} f(x) = 3$ $\lim_{x\to -\infty} f(x) = 3$

b) $\lim_{x\to -2} f(x) = \infty$ $\lim_{x\to \infty} f(x) = 4$ $\lim_{x\to -\infty} f(x) = 3$

VA → $\lim_{x\to -2}$ HA → $\lim_{x\to \infty}$

b)



5. Find the vertical and horizontal asymptotes, write the answer using the limit notation

a) $f(x)=\frac{2x}{x+4}$ VA: $x=-4$ HA: $y=2$

b) $f(x)=\frac{2x^2}{x^2-4}$ VA: $x=\pm 2$ HA: $y=2$

c) $f(x)=\frac{3x^2}{x^2+1}$ VA: — HA: $y=3$

a) $\lim_{x\to -4} f(x) = \infty$

$\lim_{x\to \infty} f(x) = 2$

b) $\lim_{x\to \pm 2} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = 2$

c) $\lim_{x\to} f(x) = \infty$

$\lim_{x\to \infty} f(x) = 3$

$\lim_{x \to 2^+} f(x) = +\infty$ ✓

$\lim_{x \to 2^-} f(x) = -\infty$ ✓

} VA $\to x$

$\lim_{x \to 5^+} f(x) = -\infty$ ✓

Limits at infinity.

$\lim_{x \to +\infty} f(x) = 0$ ✓

$\lim_{x \to -\infty} f(x) = 3$ ✓

} HA $\to y$