SEE(Grade Promotion)2078(2022)

Optional First Mathematics

Based on the given instructions, give creative answers in your own style.

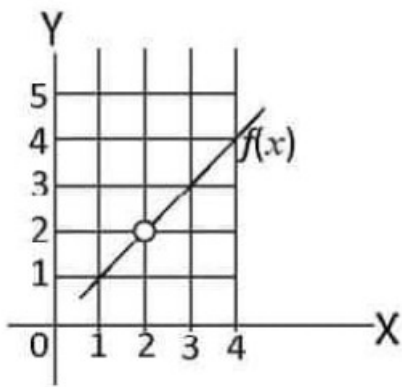**Time:** 3: 00 hrs   **F.M.:** 100

Attempt all the questions. All the working must be shown.

Group 'A'   [5 × (1 + 1) = 10]

1. (a) What type of algebraic function is $f(x) = mx + c$? Write it.

   (b) Write the arithmetic mean between a and b.

2. (a) In the given figure, at which point the function $f(x)$ is discontinuous ? Write it.

   (b) If matrix A is a singular matrix, what is the determinant of A ? Write it.

3. (a) Write the condition of parallism of the straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$.

   (b) Which geometric figure will be formed when a plane intersects a cone parallel to the base of the cone? Write it.

4. (a) Express $\cos 2A$ in terms of $\sin A$.

   (b) Express $\sin C + \sin D$ in the form of product of sine or cosine.

5. (a) If the angle between vectors $\vec{a}$ and $\vec{b}$ is θ, what is their scalar product? Write it.

   (b) In inversion transformation, if an object lies inside the circle, then in which region of the circle the image lies? Write it.

   Group 'B'   [3 × (2 + 2 + 2) + 2 × (2 + 2) = 26]

6. (a) If $f(x) = 3x + 4$, find the value of $ff(2)$.

   (b) If $g(x) = 2x - 5$, find $g^{-1}(x)$.

   (c) Find two geometric means between 3 and 81.

7. (a) If matrix $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and matrix $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find $|AB|$

   (b) If $M = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$, find M^{-1}

8. (a) Find the acute angle between two straight lines having slopes 2 and -3.

   (b) Prove that the lines having equations $2x + 3y = 5$ and $6x - 4y = 10$ are perpendicular to each other.

9. (a) Prove that : $\dfrac{1 - \cos 2A}{sin2A} = \tan A$

   (b) If $\tan \dfrac{A}{2} = \dfrac{3}{4}$, find the value of $\sin A$.

   (c) Solve: $2 \sin \theta = \operatorname{cosec} \theta$ $(0° \leq \theta 90°)$

10. (a) If $|\vec{a}| = 3, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = \dfrac{15}{2}$, find the angle between $\vec{a}$ and $\vec{b}$.

    (b) If the position vectors of the points A and B are $4\vec{i} + 2\vec{j}$ and $2\vec{i} - 5\vec{j}$ respectively, find the position vector of the mid-point of AB.

(c) In the grouped data if the first quartile is 20 and the third quartile is 60, find the quartile deviation and its coefficient.

Group 'C' $[11 \times 4 = 44]$

11. Solve: $x^3 - 9x^2 + 26x - 24 = 0$

12. Find the sum of the first 15 terms of an arithmetic series whose nineth term is 40 and nineteenth term is 60.

13. If
$$f(x) = \begin{cases} x + 4 & \text{for } x < 2 \\ 4x - 2 & \text{for } x \geq 2 \end{cases} \text{ at } x = 2$$

is a real valued function, then prove that $f(x)$ is continuous at $x = 2$?

14. Solve by Cramer's rule

$$2x + 3y = 13 \text{ and } 4x - y = 5$$

15. Find the centre and radius of the circle having equation $x^2 + y^2 - 4x + 6y - 12 = 0$

16. Withouth using calculator or table, find the value of $\cos 20°. \cos 40°. \cos 80°$.

17. If $A + B + C = 180°$, then prove that:

$$\sin 2A + \sin 2B - \sin 2C = 4\cos A. \cos B. \sin C$$

18. From a fixed point the angle of elevation of the top of a tower is observed to be 30°. Walking 60 m towards the tower from that point the angle of elevation of the top of the tower is found to be 45°. If the points of observation and the foot of the tower lie on the same plane and straight line, find the height of the tower.

19. Find a 2x2 matrix which transforms a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ into the parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

20. Calculate the mean deviation of the data given below from the median.

Class interval	0-20	20-40	40-60	60-80	80-100
Frequency	2	3	4	5	6

21. Find the standard deviation of the data given below.

Marks obtained	0 - 10	10 - 20	20 -30	30 - 40	40 - 50
No. of students	3	5	12	6	4

Group 'D' $[4 \times 5 = 20]$

22. Solve graphically: $x^2 - 5x + 6 = 0$

23. If the angle between a pair of straight lines represented by the equation $x^2 - Kxy - 6y^2 = 0$ is 45°, find the positive value of K. Also find the separate equations of these lines.

24. In the given figure, PQR is a semi-circle and O is its centre. Prove by vector method that $\angle$PQR is a right angle.
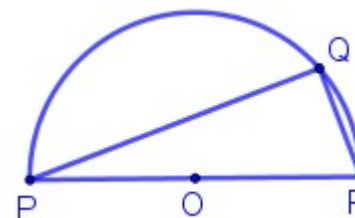
25. Reflect $\triangle ABC$ with vertices $A(3,5), B(1,2)$ and $C(6,3)$ on the Y-axis. Again rotate the image so formed through $+90°$ about the centre $(0,0)$. Write the co-ordinates of the vertices of the images thus obtained from the combined transformation. Also represent the object and the images on the same graph paper.

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