Sainik School Examination Board
PRE-SEE 2078

Time: 3:00 hrs — Optional Mathematics — **F.M. :** 100

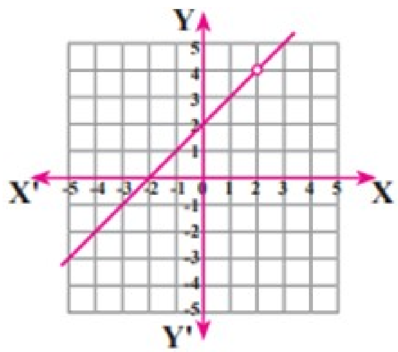
Candidates are required to answer in their own words as far as practicable. The figures in the margin represent the full marks.

Attempt **all** the questions.

Group 'A' $[5 \times (1+1) = 10]$

1. (a) What is the period of the function $f(x) = \cos x$?
   (b) State factor theorem.

2. (a) State with reason whether the graph of the function is continuous or discontinuous at $x = 2$.



   (b) Write down the determinant of matrix $A = [-4]$.

3. (a) What is the condition of coincidence of pair of lines represented by the equation $ax^2 + 2hxy + by^2 = 0$?
   (b) Name the conic section which is formed when the intersecting plane is parallel to the generator of cone?

4. (a) Express $\sin 2A$ in terms of $\tan A$.
   (b) Write down the sum $\cos\alpha + \cos\beta$ as the product of sine or cosine.

5. (a) The position vectors two points A and B are $\vec{a}$ and $\vec{b}$ respectively. What is the position vector of the mid-point M of the segment AB?
   (b) In an inversion transformation, if P' is the image of the P and r is the radius of inversion circle with centre O, write the relation of OP, OP' and r.

Group 'B' $[13 \times 2 = 26]$

6. (a) If $f(x) = x + 2$ and $g(x) = 50 - x^2$, find the value of $(gof)(5)$.
   (b) A function $f : R - \{2\} \to R$ is definde by $f(x) = \dfrac{3}{x-2}$, find $f^{-1}(x)$.
   (c) Find the vertex of parabola $y = x^2 + 2x - 3$

7. (a) Show that the matrices $P = \begin{pmatrix} 7 & 3 \\ 5 & 2 \end{pmatrix}$ and If $Q = \begin{pmatrix} -2 & 3 \\ 5 & -7 \end{pmatrix}$ are inverse to each other.
   (b) According to Cramer's rule, find the value of D_1 and D_2 for the system of equations $4x + y = 6$ and $3x + 2y = 7$.

8. (a) Find the slopes of lines $3x + 2y + 1 = 0$ and $\dfrac{x}{4} + \dfrac{y}{6} = 1$ then write the relation between them.
   (b) Find the single equation for the pair of straight lines represented by $3x + y = 0$ and $x–2y = 0$.

9. (a) If $\cos 30° = \dfrac{\sqrt{3}}{2}$, prove that:

$$\sin 15° = \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

   (b) Prove that: $\dfrac{\cos 10° - \cos 70°}{\sin 10° + \sin 70°} = \dfrac{1}{\sqrt{3}}$
   (c) If $A + B + C = 180°$, prove that:

$$\tan A + \tan B + \tan C = \tan A . \tan B . \tan C$$

10. (a) If $\overrightarrow{OA} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} k+2 \\ k+1 \end{pmatrix}$ and $\angle AOB = 90°$, find the value of k.
   (b) The position vectors of the points P and Q are $\vec{i} - 6\vec{j}$ and $3\vec{i}$ respectively. Find the position vector of point M such that $\overrightarrow{PM} = \overrightarrow{MQ}$.

(c) In a continuous data, the quartile deviation is 10 and the lower quartile is 30, find the upper quartile and coefficient of the quartile deviation.

.

Group 'C' [$11 \times 4 = 44$]

11. Solve: $x^3 - 6x^2 + 11x - 6 = 0$

12. Optimize the objective function F (x, y) = 3x + 4y subject to the constraints $x + y \leq 6, x - y \leq 4, x \geq 0, y \geq 0$.

13. For a real valued function $f(x) = 7x - 1$,
    (a) What are the values of $f(x)$ at $x = 4.9, 4.99$ and 4.999 ?
    (b) What are the values of $f(x)$ at $x = 5.1, 5.01$ and 5.001 ?
    (c) Find the values of $f(5)$, $\lim_{x \to 5^-} f(x)$ and $\lim_{x \to 5^+} f(x)$.
    (d) Is this function f continuous at $x = 5$? Give reason

14. By using matrix method, solve the following system of equations:
$$4x – y = 11 \text{ and } 7x + 2y = 8$$

15. Find the equation of the altitude of triangle PQR with vertices $P(2, 3), Q(-4, 1)$ and $R(2, 0)$ drawn from the vertex P.

16. Find the value of
$$\left(1 + \sin\frac{\pi^c}{8}\right)\left(1 - \sin\frac{3\pi^c}{8}\right)\left(1 + \sin\frac{5\pi^c}{8}\right)\left(1 - \sin\frac{7\pi^c}{8}\right)$$

17. Solve: $\sqrt{3}\sin\theta + \cos\theta = \sqrt{2}$ [$0° \leq \theta \leq 360°$]

18. There are two posts of height 45 m and 15 m respectively. If the angle of elevation of the top of the first post as observed from the foot of the second post is 60°, what is the angle of elevation of the top of the second post observed from the foot of the first post?

19. If a matrix $\begin{bmatrix} 1 & a \\ b & 3 \end{bmatrix}$ transforms the unit square into the parallelogram $\begin{bmatrix} 0 & c & 5 & 2 \\ 0 & 2 & d & 3 \end{bmatrix}$, find the values of a, b, c and d.

20. Find the mean deviation from median and its coefficient of the given data.

Class Interval	0 - 6	6 - 12	12 - 18	18 - 24	24 - 30
Frequency	8	10	12	9	5

21. Find the standard deviation and its coefficient of variation from the given data.:

Marks	0 -10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number	8	12	20	40	12	8

Group 'D' [$4 \times 5 = 20$]

22. There are some varieties of butterflies in a central zoo. The number butterflies of each variety forms a geometric progression. The fourth and seventh varieties consist of 54 and 1458 butterflies respectively.
    (i) Find the number of butterflies in the first variety.
    (ii) Calculate the number of butterflies of the 6th variety.
    (iii) If the total number of all varieties of butterflies is 6560, find the number of varieties of butterflies in a zoo.

23. On a wheel there are three points $(5, 7), (-1, 7)$ and $(5, -1)$ located such that the distance from a fixed point to these points is always equal. Find the coordinates of the fixed point and then derive the equation of representing the locus that contains all three points.

24. Prove by vector method that the diagonals of a rectangle are equal.

25. The coordinates of vertices of a quadrilateral $ABCD$ are $A(3, 4), B(1, 0), C(3, 1)$ and $D(5, -1)$. Rotate this quadrilateral about origin through 270 in anti-clockwise direction. Reflect this image of quadrilateral about $y = x$. Write the name of transformation which denotes the combined transformation of above two transformations.

$*\sigma * Ambik * \epsilon *$