Pythagorean Identities Worksheet Supplement

This supplement accompanies the GeoGebra worksheet *Pythagorean Identities in Trigonometry*, found at http://tube.geogebra.org/m/1462041. The worksheet provides a geometric understanding of the 3 familiar Pythagorean identities,

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$
$$\tan^2\theta + 1 = \sec^2\theta \tag{2}$$
$$1 + \cot\theta = \csc^2\theta, \tag{3}$$

and also explores geometric discovery of several more. The user should prove the identities after discovering them geometrically. For example, we see a right triangle with the legs 1 and $\tan\theta$, and hypotenuse $\sec\theta$. From this we conclude that $1+\tan^2\theta = \sec^2\theta$ must be true. We would prove as follows:

$$\begin{aligned} 1+\tan^2\theta &= \sec^2\theta \\ 1+\frac{\sin^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \\ \cos^2\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right) &= \cos^2\theta\left(\frac{1}{\cos^2\theta}\right) \\ \cos^2\theta+\sin^2\theta &= 1. \end{aligned}$$

We consider the final identity as a well known identity and use it as a conclusion for the proof. In practice, simplifying to any of equations (1), (2), or (3) is sufficient.

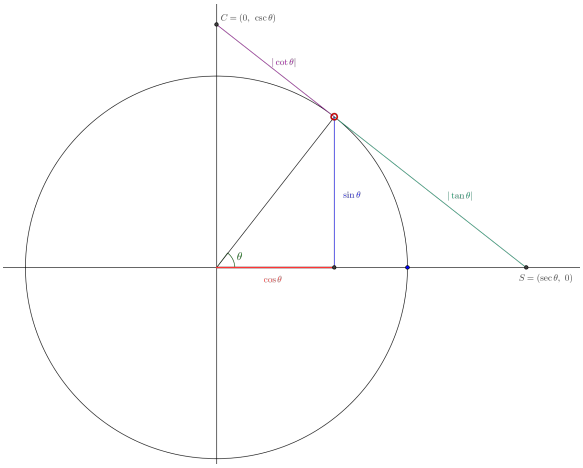
Before proceeding, try to prove the identity $\tan\theta + \cot\theta = \sec\theta\csc\theta$. It may make some of the identities in this worksheet easier to prove. It is recommended that you follow these steps as you progress through the worksheet.

1. Familiarize yourself with the basics of the worksheet. Toggle the first 2 checkboxes and the checkbox *tan, cot, sec, csc*. Also, move the terminal point of θ along the unit circle. When you are ready to proceed, reset the worksheet.

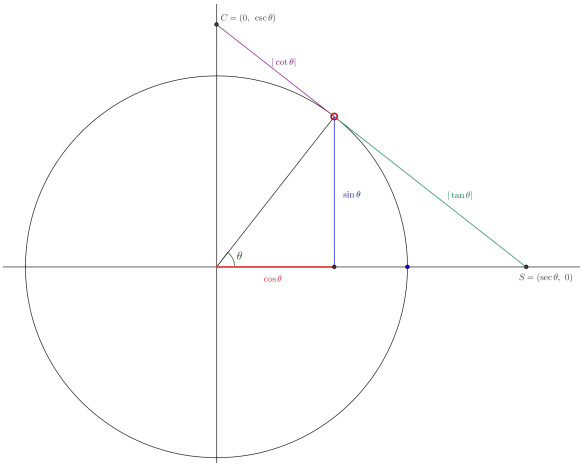
2. Locate the right triangles associated with the 3 basic Pythagorean identities. Try to do this without clicking the checkboxes, but use them if you need assistance.

3. Locate 3 more right triangles (without either of the advanced projects checked). Use these triangles to identify 3 more Pythagorean identities. Try to do it without hints, but there is a checkbox for hints provided if you need it. List and prove your newly discovered Pythagorean identities below. Highlight the associated triangle.

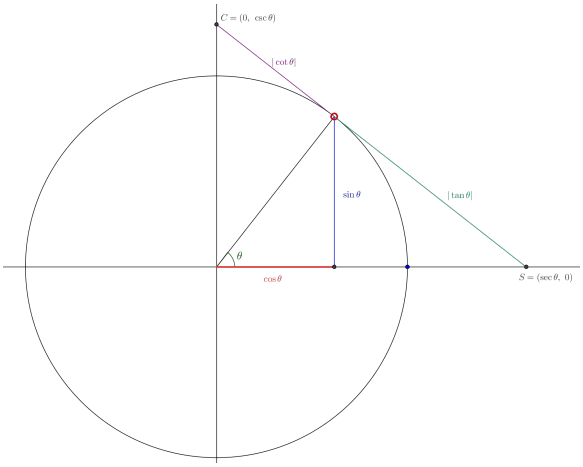
(a)


$C = (0,\ \csc\theta)$
$|\cot\theta|$
$\sin\theta$
$|\tan\theta|$
θ
$\cos\theta$
$S = (\sec\theta,\ 0)$

(b)


$C = (0,\ \csc\theta)$
$|\cot\theta|$
$\sin\theta$
$|\tan\theta|$
θ
$\cos\theta$
$S = (\sec\theta,\ 0)$

(c)


$C = (0,\ \csc\theta)$
$|\cot\theta|$
$\sin\theta$
$|\tan\theta|$
θ
$\cos\theta$
$S = (\sec\theta,\ 0)$

Advanced Project 1.

1. Find trigonometric functions for the lengths of a, b, and c as defined after selecting *Advanced Project 1*. Remember that they are lengths and therefore absolute value functions unless the function is a even power function, where the result is always positive anyway. A hint is provided in the worksheet if needed.

   (a) $a =$

   (b) $b =$

   (c) $c =$

2. List and prove 2 Pythagorean identities based on observing right triangles.

Advanced Project 2.

1. Find trigonometric functions for the lengths of a_2, b_2, and c_2 as defined after selecting *Advanced Project 2*.

   (a) $a_2 =$

   (b) $b_2 =$

   (c) $c_2 =$

2. List and prove 2 Pythagorean identities based on observing right triangles.

Bonus Question: Find the coordinates of the points P (shown with advanced project 1 selected), and B (shown with advanced problem 2 selected). The coordinates will be trigonometric functions of θ.