Ejercicios de la unidad 15.3
Campos vectoriales conservativos e independencia de la trayectoria

En los ejercicios 5 a 10, determinar si el campo vectorial es o no conservativo.

5) $\vec{F}(x,y) = e^x(\operatorname{sen} y\, i + \cos y\, J)$

$$= \underbrace{e^x \operatorname{sen} y\, i}_{M} + \underbrace{e^x \cos y\, J}_{N}$$

Determinar si el campo vectorial es o no conservativo

$\frac{\partial M}{\partial y} = e^x \cos y$

$\therefore \vec{F}(x,y)$ es conservativo

$\frac{\partial N}{\partial x} = e^x \cos y(1)$

9) $\vec{F}(x,y,z) = \underbrace{y^2 z\, i}_{M} + \underbrace{2xyz\, J}_{N} + \underbrace{xy^2 K}_{P}$

$\frac{\partial P}{\partial y} = 2xy \;;\; \frac{\partial N}{\partial z} = 2xy$ ✓

$\therefore \vec{F}$ es conservativo

$\frac{\partial P}{\partial x} = y^2 \;;\; \frac{\partial M}{\partial z} = y^2$ ✓

$\frac{\partial N}{\partial x} = 2yz \;;\; \frac{\partial M}{\partial y} = 2yz$ ✓

En los ejercicios 11 a 24, hallar el valor de la integral de línea

$\int_C \vec{F} \cdot d\vec{r}$ (Sugerencia: si $\vec{F}$ es Conservativo, la integración Puede ser más Sencilla a través de una Trayectoria alternativa).

11) $\vec{F}(x,y) = 2xy\,i + x^2\,J$

a) $\vec{r}_1(T) = T\,i + T^2 J, \quad 0 \leq T \leq 1$

b) $\vec{r}_2(T) = T\,i + T^3 J, \quad 0 \leq T \leq 1$

a) $x = T, \quad y = T^2, \quad r'(T) = \langle 1, 2T \rangle\, dT$

$\vec{F}(x,y) = 2(T)(T^2) + (T)^2$

$\int_0^1 (2T^3 + T^2) \cdot \langle 1, 2T \rangle\, dT$

$\int_0^1 2T^3 + 2T^3\, dT = \int_0^1 4T^3\, dT = T^4 \Big|_0^1 = \underline{1}$

b) $x = T, \quad y = T^3, \quad r'(T) = \langle 1, 3T^2 \rangle\, dT$

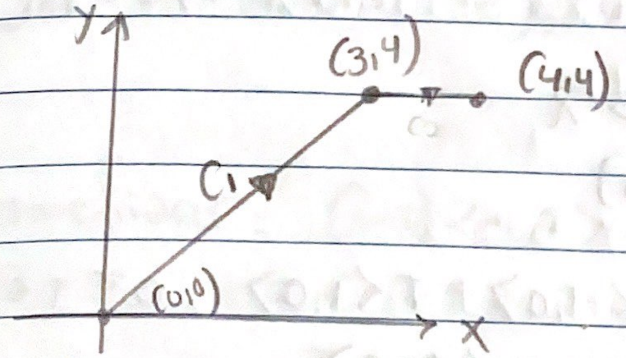
$\vec{F}(x,y) = 2(T)(T^3) + (T)^2$

$\int_0^1 (2T^4 + T^2) \cdot (1, 3T^2)\, dT$

$= \int_0^1 2T^4 + 3T^4\, dT = \int_0^1 5T^4\, dT = T^5 \Big|_0^1 = \underline{1}$

15) $\int_C y^2\,dx + 2xy\,dy$

a)



Parametrizar = $(1-T)\langle 0,0\rangle + T\langle 3,4\rangle \quad , 0 \le T \le 1$

$C_1 \Rightarrow \quad \langle 0,0\rangle + \langle 3T, 4T\rangle \quad = \langle 3T, 4T\rangle$

$x = 3T, \ y = 4T$

$dx = 3\,dT \quad dy = 4\,dT$

$\int_0^1 (4T)^2(3\,dT) + 2(3T)(4T)(4\,dT)$

$\int_0^1 48T^2 + 96T^2\,dT = \int_0^1 144T^2\,dT = \frac{144}{3}T^3 \Big|_0^1 = 48(1) = 48$

$C_2 \Rightarrow \ (1-T)\langle 3,4\rangle + T(4,4) \quad , 0 \le T \le 1$

$(3-3T, 4-4T) + (4T, 4T) = \langle 3+T, 4\rangle$

$x = 3+T \quad , dx = 1\,dT$

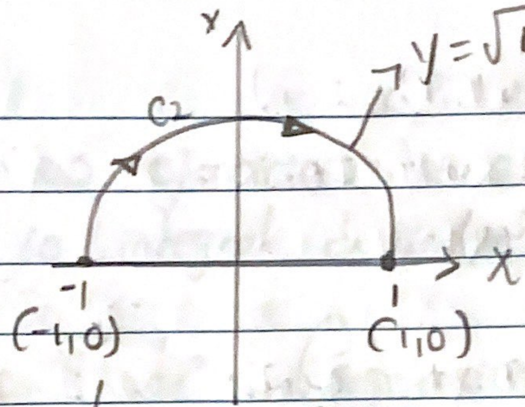
$y = 4 \qquad dy = 0$

$\int_0^1 (4)^2(1\,dT) + 2(3+T)(4)(0)$

$\int_0^1 16\,dT = 16T \Big|_0^1 = 16$

$= 48 + 16 = \boxed{64}$

b)



$x^2+y^2=1$

Parametrizar $= (1-T)\langle -1,0\rangle + T\langle 1,0\rangle$, $0 \le T \le 1$

$= \langle -1+T, 0\rangle + \langle T, 0\rangle$

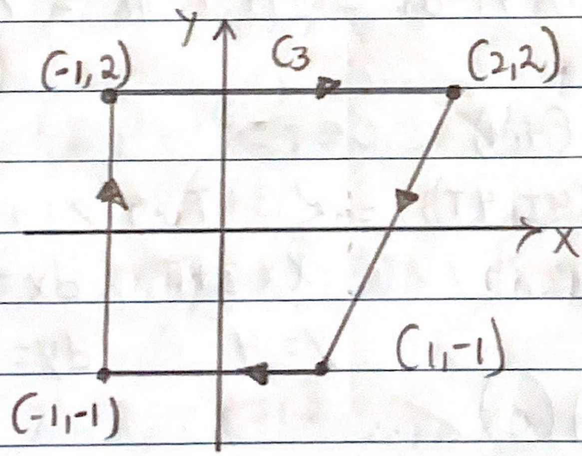
$= \langle -1+2T, 0\rangle$ $\quad x = -1+2T$

$dx = 2\,dT$

$y=0 \quad dy=0$

$$\int_0^1 0^2(2dT) + 2(-1+2T)(0)(0) = \int_0^1 0 = \boxed{0}$$

c)



Parametrizar $= (1-T)\langle -1,-1\rangle + T\langle -1,2\rangle$ $\quad 0 \le T \le 1$

C_1 $\quad = (-1+T, -1+T) + \langle -T, 2T\rangle$

$= \langle -1, -1+3T\rangle$

$x=-1 \qquad y=-1+3T$

$dx=0 \qquad dy = 3\,dT$

$$\int_0^1 (-1+3T)^2(0) + 2(-1)(-1+3T)(3\,dT)$$

$$\int_0^1 -6(-1+3T)\,dT = \int_0^1 6 - 18T\,dT = 6T - \frac{18T^2}{2}\Big|_0^1$$

$$= 6 - 9$$

$$= \boxed{-3}$$

Parametrizar = $(1-T)\langle -1,2\rangle + T\langle 2,2\rangle \quad , 0 \le T \le 1$

C2

$$= \langle -1+T, 2-2T\rangle + \langle 2T, 2T\rangle$$

$$= \langle -1+3T, 2\rangle$$

$x = -1+3T \qquad y = 2$

$dx = 3\,dT \qquad dy = 0$

$$\int_0^1 (2)^2(3\,dT) + 2(-1+3T)(2)(0) = \int_0^1 12\,dT = 12T\Big|_0^1$$

$$= \boxed{12}$$

Parametrizar = $(1-T)\langle 2,2\rangle + T\langle 1,-1\rangle \quad , 0 \le T \le 1$

C3

$$= \langle 2-2T, 2-2T\rangle + \langle T, -T\rangle$$

$= \langle 2-T, 2-3T\rangle \qquad x = 2-T \quad dx = -1\,dT$

$y = 2-3T \quad dy = -3\,dT$

$$\int_0^1 (2-3T)^2(-1\,dT) + 2(2-T)(2-3T)(-3\,dT)$$

$$\int_0^1 (-4 + 12T - 9T^2 - 18T^2 + 48T - 24)\,dT$$

$$\int_0^1 -27T^2 + 60T - 28\,dT = \frac{-27}{3}T^3 + \frac{60}{2}T^2 - 28T\Big|_0^1$$

$$= -9 + 30 - 28$$

$$= \boxed{-7}$$

Parametrizar = $(1-T)\langle 1,-1\rangle + T\langle -1,-1\rangle$ $\quad 0 \le T \le 1$

C_4 $\quad = \langle 1-T, -1+T\rangle + \langle -T, -T\rangle$

$= \langle 1-2T, -1\rangle$

$x = 1-2T \qquad y = -1$

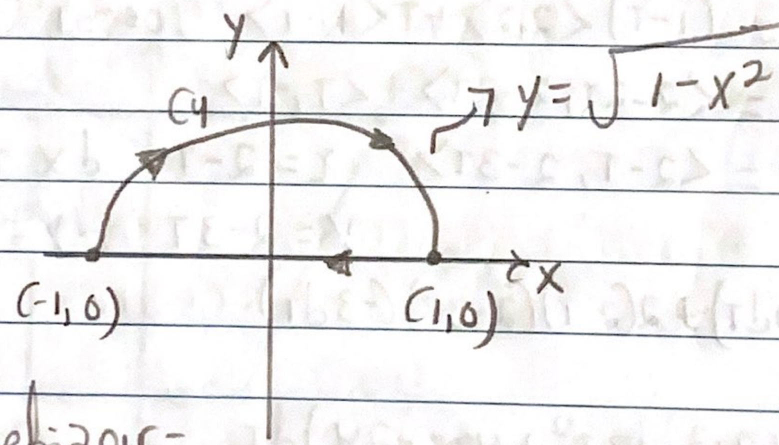
$dx = -2\,dT \qquad dy = 0$

$$\int_0^1 (-1)^2(-2\,dT) + 2(1-2T)(-1)(0)$$

$$\int_0^1 -2\,dT = -2T\Big|_0^1 = \boxed{-2}$$

$$= -3 + 12 - 7 - 2 = \boxed{0}$$

d)



Parametrizar =

$(1-T)\langle -1,0\rangle + T\langle 1,0\rangle$, $\quad 0 \le T \le 1$

$\langle -1+T, 0\rangle + \langle T, 0\rangle = \langle -1+2T, 0\rangle$

$x = -1+2T \qquad y = 0$

$dx = 2\,dT \qquad dy = 0$

$$\int_0^1 (0)^2(2\,dT) + 2(-1+2T)(0)(0) = \int_0^1 0 = 0$$

$= (1-T)\langle 1,0\rangle + T\langle -1,0\rangle$, $0 \le T \le 1$

$\langle 1-T, 0\rangle + \langle -T, 0\rangle = \langle 1-2T, 0\rangle$ $x = 1-2T$ $y = 0$

$dx = -2dT$ $dy = 0$

$\int_0^1 (0)^2(-2dT) + 2(1-2T)(0)(0) = 0$

$= 0 + 0 = \boxed{0}$

En los ejercicios 25 a 34, evaluar la integral de línea utilizando el Teorema fundamental de las integrales de línea. utilizar un sistema algebraico por computadora y verificar los resultados

25) $\int_C (3y\,i + 3x\,j) \cdot d\vec{r}$

C : Curva suave desde (0,0) hasta (3,8)

$\vec{F}(x,y) = \underbrace{3y}_{M}\,i + \underbrace{3x}_{N}\,j$

Determinar si el campo es conservativo

$\frac{dM}{dy} = 3$ $\frac{dN}{dx} = 3$ $\therefore \vec{F}$ es conservativo

Determinar la función potencial

$F = \int f_x\,dx = \int 3y\,dx = 3yx + C(y)$

$F = \int f_y\,dy = \int 3x\,dy = 3xy + C(x)$

$f(x,y) = 3xy + K$

$\to \int_C \vec{F} \cdot d\vec{r} = 3xy \Big|_{(0,0)}^{(3,8)}$

$= 3(3)(8) - 3(0)(0)$

$= \boxed{72}$

27) $\int_C \cos x \operatorname{sen} y \cdot dx + \operatorname{sen} x \cos y \, dy$

C: Curva suave desde $(0, -\pi)$ hasta $\left(\frac{3\pi}{2}, \frac{\pi}{2}\right)$

$\vec{F}(x,y) = \underbrace{\cos x \operatorname{sen} y}_{M}\, i + \underbrace{\operatorname{sen} x \cos y}_{N}\, j$

Determinar si el Campo es Conservativo

$\frac{\partial M}{\partial y} = \cos x \cos y$ $\longrightarrow$ $\therefore \vec{F}$ es Conservativo

$\frac{\partial N}{\partial x} = \cos x \cos y$

Determinar la función Potencial

$F = \int f_x \, dx = \int \cos x \operatorname{sen} y \, dx = \operatorname{sen} y \int \cos x \, dx$
$= \operatorname{sen} y \cdot \operatorname{sen} x + C(y)$

$F = \int f_y \, dx = \int \operatorname{sen} x \cos y \, dy = \operatorname{sen} x \int \cos y \, dy =$
$\operatorname{sen} x \cdot \operatorname{sen} y + C(x)$

$F(x,y) = \operatorname{sen} x \cdot \operatorname{sen} y + K$

$$\int_C \vec{F} \cdot d\vec{r} = \operatorname{sen} x \cdot \operatorname{sen} y \Big|_{(0,-\pi)}^{(3\pi/2,\, \pi/2)}$$

$= \left(\operatorname{sen}(3\pi/2) \cdot \operatorname{sen}(\pi/2)\right) - \left(\operatorname{sen}(0) \cdot \operatorname{sen}(-\pi)\right)$

$= (-1 \cdot 1) - (0 \cdot 0)$

$= (-1) - 0 = \boxed{-1}$