An Roinn Oideachais
agus Scileanna

Junior Cycle Mathematics

Contents

Introduction to junior cycle

Junior cycle education places students at the centre of the educational experience, enabling them to actively participate in their communities and in society and to be resourceful and confident learners in all aspects and stages of their lives. Junior cycle is inclusive of all students and contributes to equality of opportunity, participation and outcome for all.

The junior cycle allows students to make a greater connection with learning by focusing on the quality of learning that takes place and by offering experiences that are engaging and enjoyable for them, and relevant to their lives. These experiences are of a high quality, contribute directly to the physical, mental and social wellbeing of learners, and where possible, provide opportunities for them to develop their abilities and talents in the areas of creativity, innovation and enterprise. The learner's junior cycle programme builds on their learning to date and actively supports their progress in learning and in addition, supports them in developing the learning skills that will assist them in meeting the challenges of life beyond school.

Rationale

This mathematics specification provides students with access to important mathematical ideas to develop the mathematical knowledge and skills that they will draw on in their personal and work lives. This specification also provides students, as lifelong learners, with the basis on which further study and research in mathematics and many other fields are built.

Mathematical ideas have evolved across societies and cultures over thousands of years, and are constantly developing. Digital technologies are facilitating this expansion of ideas and provide new tools for mathematical exploration and invention. While the usefulness of mathematics for problem solving is well known, mathematics also has a fundamental role in both enabling and sustaining cultural, social, economic and technological advances and empowering individuals to become critical citizens.

The specification is underpinned by the conception of mathematics as an interconnected body of ideas and reasoning processes that students negotiate collaboratively with teachers and their peers and as independent learners. Number, measurement and geometry, statistics and probability are common aspects of most people's mathematical experiences in everyday personal, study and work situations. Equally important are the essential roles that algebra, functions and relations, logic, mathematical structure and working mathematically play in people's understanding of the natural and social worlds, and the interaction between them.

The mathematics specification builds on students' prior learning and focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, reasoning, computational thinking and problem solving. These capabilities enable students to respond to familiar and unfamiliar situations by employing mathematics to make informed decisions and solve problems efficiently.

The specification supports student learning across the whole educational system by ensuring that the links between the various components of mathematics, as well as the relationship between mathematics and other subjects, are emphasised. Mathematics is composed of multiple but interrelated and interdependent concepts and structures which students can apply beyond the mathematics classroom. For example, in science, understanding sources of error and their impact on the confidence of conclusions is vital; in geography, interpretation of data underpins the study of human populations and their physical environments; in history, students need to be able to imagine timelines and time frames to reconcile related events; and in English, deriving quantitative, logical and spatial information is an important aspect of making meaning of texts. Thus the understanding of mathematics developed through study at junior cycle can inform and support students' learning across the whole educational system.

Aim

The aim of junior cycle mathematics is to provide relevant and challenging opportunities for all students to become mathematically proficient so that they can cope with the mathematical challenges of daily life and enable them to continue their study of mathematics in senior cycle and beyond. In this specification, mathematical proficiency is conceptualised not as a one-dimensional trait but as having five interconnected and interwoven components:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
- adaptive reasoning—capacity for logical thought, reflection, explanation, justification and communication
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy.

Overview: Links

Mathematics supports a broad range of learning experiences at junior cycle. Table 1 shows how junior cycle mathematics is linked to central features of learning and teaching in junior cycle.

Table 1: Links between junior cycle mathematics and the statements of learning

STATEMENTS OF LEARNING

The statement	Examples of possible relevant learning
SOL 1: The student communicates effectively using a variety of means in a range of contexts in L1.	Students organise, consolidate and communicate numerical and mathematical thinking clearly and coherently to peers, teachers and others verbally, and in written form using diagrams, graphs, tables and mathematical symbols.
SOL 14: The student makes informed financial decisions and develops good consumer skills.	Students learn to develop their critical thinking and reasoning skills by making value-for-money calculations and judgements which will enable them to make informed financial decisions.
SOL 15: The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.	Students apply their mathematical knowledge and skills to a wide variety of problems across different subjects, including gathering, analysing, and presenting data, and using mathematics to model real-world situations.
SOL 16: The student describes, illustrates, interprets, predicts and explains patterns and relationships.	Students develop techniques to explore and understand patterns and relationships in both mathematical and non-mathematical contexts.
SOL 17: The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.	Students develop problem-solving strategies through engaging in tasks for which the solution is not immediately obvious. They reflect on their own solution strategies to such tasks and compare them to those of others as part of a collaborative learning cycle.
SOL 18: The student observes and evaluates empirical events and processes and draws valid deductions and conclusions.	Students generate and summarise data, select appropriate graphical or numerical methods to describe it, and draw conclusions from graphical and numerical summaries of the data. As part of their understanding of mathematical proof they come to appreciate the distinction between contingent deductions from particular cases, and deductions which can be proved to be universally true.
SOL 24: The student uses technology and digital media tools to learn, communicate, work and think collaboratively and creatively in a responsible and ethical manner.	Students engage with digital technology to analyse and display data numerically and graphically; to display and explore algebraic functions and their graphs; to explore shapes and solids; to investigate geometric results in a dynamic way; and to communicate and collaborate with others.

Key Skills

In addition to their specific content and knowledge, the subjects and short courses of junior cycle provide students with opportunities to develop a range of key skills. There are opportunities to support all key skills in this course but some are particularly significant.

The junior cycle curriculum focuses on eight key skills:

Figure 1: Key skills of junior cycle

KEY SKILL ELEMENTS RELATING TO MATHEMATICS

The examples below identify some of the elements that are related to learning activities in mathematics. Teachers can also build many of the other elements of key skills into their classroom planning. The eight key skills are set out in detail in Key Skills of Junior Cycle.

Table 2: Links between junior cycle mathematics and key skills

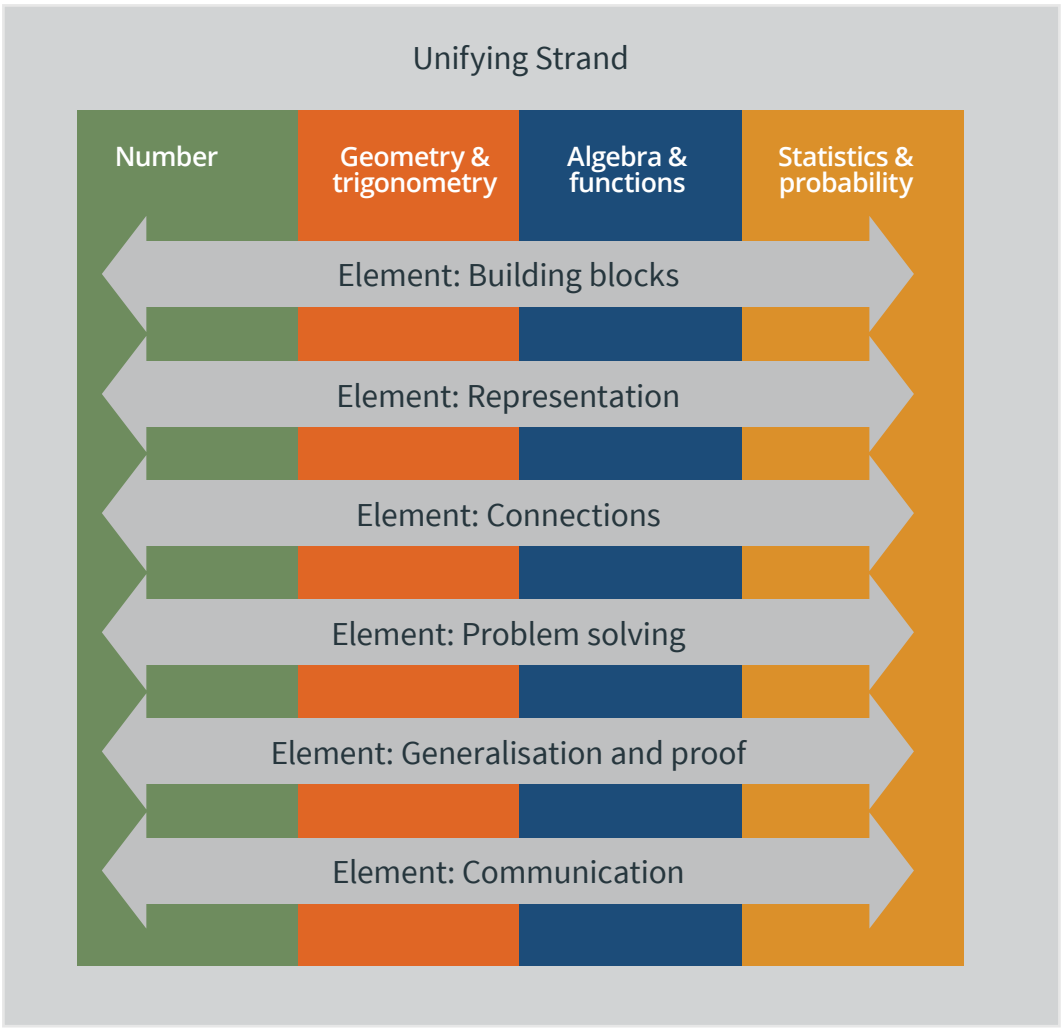
Key skill	Key skill element	Examples of possible student learning activities
Being creative	Exploring options and alternatives	As students engage in a task for which the solution is not immediately obvious, they ask questions, explore ideas and alternatives, evaluate ideas and actions and take more responsibility for their learning.
Being literate	Expressing ideas clearly and accurately	Students explain their thinking and justify their reasoning, using mathematical terminology appropriately and accurately.
Being numerate	Using digital technology to develop numeracy skills and understanding	Students use digital technology to analyse and display data numerically and graphically; to display and explore algebraic functions and their graphs; to explore shapes and solids; to investigate geometric results in a dynamic way; and to communicate and collaborate with others.
Communicating	Using numbers	Students use numbers to describe or summarise a situation; to support their reasoning and conclusions; and to convey and explain patterns and relationships.
Managing information and thinking	Thinking creatively and critically	Students engage in rich tasks which require them to use their mathematical knowledge and skills in novel ways. They reflect on their own approaches to such tasks and compare them to those of others, evaluating the strengths and weaknesses of different possible approaches.
Managing myself	Being able to reflect on my own learning	Students reflect on which learning activities they find most effective, using this knowledge to help further their learning in mathematics.
Staying well	Being confident	Students enjoy frequent opportunities to experience success in mathematics. They experience a positive approach to learning in which different approaches are valued and they are encouraged to learn from mistakes.
Working with others	Learning with others	Students work on collaborative tasks with peers in which they develop both their mathematical and their interpersonal skills, offering mutual support and feedback throughout the process.

Overview: Course

The specification for junior cycle mathematics focuses on developing students' ability to think logically, strategically, critically, and creatively through the **Unifying strand** and the four **contextual strands: Number; Geometry and trigonometry; Algebra and functions;** and **Statistics and probability.**

The specification has been designed for a minimum of 240 hours timetabled student engagement across the three years of junior cycle. This is a minimum and schools should be aware that there are students who would benefit from an engagement of more than 240 hours to realise the national improvement targets set out in the Literacy and Numeracy strategy (DES,2011).

Figure 2: The structure of the specification for junior cycle mathematics



Unifying strand

This strand permeates all of the contextual strands and is composed of the six elements of the specification, which are shown below.

There is no specific content linked to this strand; rather, its learning outcomes underpin the rest of the specification. Each learning outcome in this strand is applicable to all of the activities and content of the other four strands—for example, students should be able to draw on all of their mathematical knowledge and skills to solve a problem or to communicate mathematics.

Furthermore, the elements of this strand are interdependent, so that students should develop the different skills associated with each element in tandem rather than in isolation – for example, engaging in problem-solving can help students improve their understanding of building blocks and their ability to make connections within mathematics.

The elements

Elements	
Building blocks	Students should understand and recall the concepts that underpin each strand, and be able to carry out the resulting procedures accurately, effectively, and appropriately.
Representation	Students should be able to represent a mathematical situation in a variety of different ways and translate flexibly between them.
Connections	Students should be able to make connections within strands and between strands, as well as connections between mathematics and the real world.
Problem solving	Students should be able to investigate patterns, formulate conjectures, and engage in tasks in which the solution is not immediately obvious, in familiar and unfamiliar contexts.
Generalisation and proof	Students should be able to move from specific instances to general mathematical statements, and to present and evaluate mathematical arguments and proofs.
Communication	Students should be able to communicate mathematics effectively in verbal and written form.

Number

This strand focuses on different aspects of number, laying the groundwork for the transition from arithmetic to algebra. Learners explore different representations of numbers and the connections between them, as well as the properties and relationships of binary operations. They investigate number patterns, and use ratio and proportionality to solve a variety of problems in numerous contexts. Learners are expected to be able to use calculators appropriately and accurately, as well as to carry out calculations by hand and mentally. They appreciate when it is appropriate to use estimation and approximation, including to check the reasonableness of results.

Geometry and trigonometry

This strand focuses on analysing characteristics and properties of two- and three-dimensional geometric shapes. Learners use geometry and trigonometry to model and solve problems involving area, length, volume, and angle measure. They develop mathematical arguments about geometric relationships and explore the concept of formal proof, using deduction to establish the validity of certain geometric conjectures and critiquing the arguments of others.

Algebra and functions

This strand focuses on representing and analysing patterns and relationships found in numbers. Building on their work in the Number strand, learners generalise their observations, expressing, interpreting, and justifying general mathematical statements in words and in symbolic notation. They use the idea of equality to form and interpret equations, and the syntactic rules of algebra to transform expressions and solve equations. Learners explore and analyse the relationships between tables, diagrams, graphs, words, and algebraic expressions as representations of functions.

Statistics and probability

This strand focuses on determining probability from random events and generating and investigating data. Students explore the relationship between experimental and theoretical probability as well as completing a data investigation; from formulating a question and designing the investigation through to interpreting their results in context and communicating their findings. Learners use graphical and numerical tools, including summary statistics and the concepts and processes of probability, to explore and analyse patterns in data. Through these activities, learners gain an understanding of data anaysis as a tool for learning about the world.

Progression from early childhood to senior cycle

EARLY CHILDHOOD

Aistear, the early childhood curriculum framework, celebrates early childhood as a time of wellbeing and enjoyment where children learn from experiences as they unfold. Children's interests and play should be the source of their first mathematical experiences. These experiences can become mathematical as they are represented and explored. Young children represent their ideas by talking, but also through models and graphics. From the motoric and sing-song beginnings of rhymes and geometric patterns built from unit blocks stem the gradual generalisation and abstraction of patterns throughout the child's day.

PRIMARY SCHOOL

The mathematics curriculum at primary school aims to provide children with a language and a system through which to analyse, describe, illustrate and explain a wide range of experiences, make predictions, and solve problems. Mathematics education seeks to enable learners to think and communicate quantitatively and spatially, solve problems, recognise situations where mathematics can be applied, and use appropriate technology to support such applications. The junior cycle mathematics specification consolidates and develops students' learning from primary school and as such experience of the learning outcomes in the Primary School Mathematics Curriculum is assumed.

SENIOR CYCLE

The junior cycle mathematics specification is developed to align with Leaving Certificate Mathematics to allow for the effective transfer of knowledge, understanding, and skills from junior to senior cycle. While certain aspects of the strands have been adapted to specifically suit junior cycle—for example, having four rather than five strands—it is nonetheless clear from the structure of this specification how students' learning in junior cycle mathematics should be developed in senior cycle. A good understanding of the knowledge and skills outlined in this specification will lay the foundations for successful engagement with senior cycle mathematics.

Expectations for Students

Expectations for students is an umbrella term that links learning outcomes with annotated examples of student work in the subject specification. When teachers, students or parents looking at the online specification scroll over the learning outcomes, a link will sometimes be available to examples of work associated with a specific learning outcome or with a group of learning outcomes. The examples of student work will have been selected to illustrate expectations and will have been annotated by teachers and will be made available alongside this specification. The examples will include work that is:

- exceptional
- above expectations
- in line with expectations.

The purpose of the examples of student work is to show the extent to which the learning outcomes are being realised in actual cases.

Learning outcomes

Learning outcomes are statements that describe what knowledge, understanding, skills and values students should be able to demonstrate having studied mathematics in junior cycle. Junior cycle mathematics is offered at Ordinary and Higher level. The majority of the learning outcomes set out in the following tables apply to all students. Additional learning outcomes for those students who take the Higher-level mathematics examination are highlighted in bold. As set out here the learning outcomes represent outcomes for students at the end of their three years of study. The specification stresses that the learning outcomes are for three years and therefore the learning outcomes focused on at a point in time will not have been 'completed', but will continue to support students' learning of mathematics up to the end of junior cycle.

The outcomes are numbered within each strand. The numbering is intended to support teacher planning in the first instance and does not imply any hierarchy of importance across the outcomes themselves. The examples of student work linked to learning outcomes will offer commentary and insights that support different standards of student work.

Unifying strand

Elements	*Students should be able to:*	
Building blocks	U.1	recall and demonstrate understanding of the fundamental concepts and procedures that underpin each strand
	U.2	apply the procedures associated with each strand accurately, effectively, and appropriately
	U.3.	recognise that equality is a relationship in which two mathematical expressions have the same value
Representation	U.4	represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations
Connections	U.5	make connections within and between strands
	U.6	make connections between mathematics and the real world
Problem solving	U.7	make sense of a given problem, and if necessary mathematise a situation
	U.8	apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions
	U.9	interpret their solution to a problem in terms of the original question
	U.10	evaluate different possible solutions to a problem, including evaluating the reasonableness of the solutions, and exploring possible improvements and/or limitations of the solutions (if any)
Generalisation and proof	U.11	generate general mathematical statements or conjectures based on specific instances
	U.12	generate and evaluate mathematical arguments and proofs
Communication	U.13	communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely

Number strand

Students should be able to:

N.1 investigate the representation of numbers and arithmetic operations so that they can:

a. represent the operations of addition, subtraction, multiplication, and division in $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ using models including the number line, decomposition, and accumulating groups of equal size

b. perform the operations of addition, subtraction, multiplication, and division and understand the relationship between these operations and the properties: commutative, associative and distributive in $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ **and in $\mathbb{R}\backslash\mathbb{Q}$, including operating on surds**

c. explore numbers written as a^b (in index form) so that they can:

I. flexibly translate between whole numbers and index representation of numbers

II. use and apply generalisations such as $a^p\, a^q = a^{p+q}$; $(a^p)/(a^q) = a^{p-q}$; $(a^p)^q = a^{pq}$; and $n^{1/2} = \sqrt{n}$, for $a \in \mathbb{Z}$, and $p, q, p-q, \sqrt{n} \in \mathbb{N}$
and for $a, b, \sqrt{n} \in \mathbb{R}$, and $p, q \in \mathbb{Q}$

III. use and apply generalisations such as $a^0 = 1$; $a^{p/q} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$; $a^{-r} = 1/(a^r)$; $(ab)^r = a^r\, b^r$; and $(a/b)^r = (a^r)/(b^r)$, for $a, b \in \mathbb{R}$; $p, q \in \mathbb{Z}$; and $r \in \mathbb{Q}$

IV. generalise numerical relationships involving operations involving numbers written in index form

V. correctly use the order of arithmetic and index operations including the use of brackets

d. calculate and interpret factors (including the highest common factor), multiples (including the lowest common multiple), and prime numbers

e. present numerical answers to the degree of accuracy specified, for example, correct to the nearest hundred, to two decimal places, or to three significant figures

f. convert the number p in decimal form to the form $a \times 10^n$, where $1 \le a < 10$, $n \in \mathbb{Z}$, $p \in \mathbb{Q}$, and $p \ge 1$ **and $0 < p < 1$**

N.2 investigate equivalent representations of rational numbers so that they can:

a. flexibly convert between fractions, decimals, and percentages

b. use and understand ratio and proportion

c. solve money-related problems including those involving bills, VAT, profit or loss, % profit or loss (on the cost price), cost price, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts), value for money calculations and judgements, **mark up (profit as a % of cost price), margin (profit as a % of selling price), compound interest, income tax and net pay (including other deductions)**

N.3 investigate situations involving proportionality so that they can:

a. use absolute and relative comparison where appropriate

b. solve problems involving proportionality including those involving currency conversion and those involving average speed, distance, and time

N.4 analyse numerical patterns in different ways, including making out tables and graphs, and continue such patterns

N.5 explore the concept of a set so that they can:

a. understand the concept of a set as a well-defined collection of elements, and that set equality is a relationship where two sets have the same elements

b. define sets by listing their elements, if finite (including in a 2-set or **3-set** Venn diagram), or by generating rules that define them

c. use and understand suitable set notation and terminology, including null set, $\emptyset$, subset, $\subset$, complement, element, $\in$, universal set, cardinal number, #, intersection, $\cap$, union, $\cup$, set difference, \ , $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{R}\backslash\mathbb{Q}$

d. perform the operations of intersection and union on 2 sets **and on 3 sets**, set difference, and complement, including the use of brackets to define the order of operations

e. investigate whether the set operations of intersection, union, and difference are commutative and/or associative