Exercícios de compreensão:

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14.1

3- $\int_1^5\int_2^4 (3x^2-2xy+y^2)\,dx\,dy = \int_{\square}^{\square} \underline{\qquad}\,dy$

$\int_2^4 (3x^2-2xy+y^2)\,dx = \frac{3x^3}{3} - \frac{2x^2}{2}y + y^2 \Big|_2^4$

$\Rightarrow x^3 - x^2y + y^2 \Big|_2^4 = 4^3 - 4^2y + y^2 - (2^3 - 2^2y + y^2)$

$= 64 - 16y + \cancel{y^2} - 8 + 4y - \cancel{y^2}$

$= -12y + 56_{//}$

$\Rightarrow \int_1^5 -12y + 56\,dy_{//}$

4. $z = \frac{x}{y}$ e o retângulo $0 \leq x \leq 4$, $1 \leq y \leq e^2$ do plano xy

$\int_1^{e^2}\int_0^4 \frac{x}{y}\,dx\,dy =$

$\int_0^4 \frac{x}{y}\,dx = \frac{x^2}{2y}\Big|_0^4 = \frac{4^2}{2y} - \cancel{\frac{0^2}{2y}}^{\,0} = \boxed{\frac{8}{y}}$

$\int_1^{e^2} \frac{8}{y}\,dy = 8\int_1^{e^2} \frac{1}{y}\,dy = 8\ln|y|\Big|_1^{e^2} = 8\ln|e^2| - 8\ln|1|$

$8 \cdot 2 - 0 = 16_{//}$

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1- Calcule as integrais iteradas

a) $\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\,dy\,dx$

$\int_0^{\ln 2} e^{x+y}\,dy = \int_0^{\ln 2} e^x . e^y\,dy = e^x . e^y \Big|_0^{\ln 2} = (e^x . e^{\ln 2}) - (e^x . e^0) = 2e^x - e^x = \boxed{e^x}$

$\int_0^{\ln 3} e^x\,dx = e^x \Big|_0^{\ln 3} = e^{\ln 3} - e^0 = 3 - 1 = \boxed{2}_{//}$

b) $\int_0^2\int_0^1 y\,\text{sen}x\,dy\,dx$

$\int_0^1 y.\text{sen}x\,dy = \frac{y^2}{2}\text{sen}x \Big|_0^1 = \frac{1^2}{2}\text{sen}x - \frac{0^2}{2} = \frac{\text{sen}x}{2} + C_{//}$

$\int_0^2 \frac{\text{sen}x}{2}\,dx = \frac{1}{2}\int_0^2 \text{sen}x\,dx = \frac{1}{2} - \cos x \Big|_0^2$

$\Rightarrow -\frac{\cos x}{2}\Big|_0^2 = -\frac{1}{2}\left[\cos(2) - \cos(0)\right] = \frac{1 - \cos(2)}{2} + 1$

$= \left(-\frac{\cos(2)}{2} + \frac{1}{2}\right)$

c) $\int_4^6\int_{-3}^7 dy\,dx =$

$\int_{-3}^7 1.\,dy = y\Big|_{-3}^7 = 7 - (-3) = 10_{//}$

$\int_4^6 10\,dx = 10x\Big|_4^6 = 10.6 - 10.4 = 20_{//}$

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d) $\int_{3}^{4}\int_{1}^{2} \frac{1}{(x+y)^2}\,dy\,dx$

$$\int_{1}^{2} \frac{1}{(x+y)^2}\,dy = \int_{1}^{2} \frac{1}{u^2}\,du = \int_{1}^{2} u^{-2}\,du$$

$u = x+y . dy$
$du = 0+1\,dy$
$du = dy$

$$\frac{u^{-1}}{-1} + C \Big|_{1}^{2} = -\frac{1}{u}\Big|_{1}^{2} = \frac{-1}{(x+y)}\Big|_{1}^{2}$$

$$\Rightarrow -\frac{1}{(x+2)} + \frac{1}{(x+1)} \;//$$

$$\int_{3}^{4} -\frac{1}{(x+2)} + \frac{1}{(x+1)}\,dx = \int_{3}^{4} -\frac{1}{(x+2)}\,dx + \int_{3}^{4} \frac{1}{(x+1)}\,dx$$

$$-\int_{3}^{4} \frac{1}{(x+2)}\,dx = -\int_{3}^{4} \frac{1}{u}\,du = -\ln(u)\Big|_{3}^{4}$$

$u = x+2\,dx$
$du = 1\,dx$

$$= -\ln(4+2) + \ln(3+2)$$
$$= -\ln(6) + \ln(5)$$

$$\int_{3}^{4} \frac{1}{(x+1)}\,dx = \int_{3}^{4} \frac{1}{u}\,dx = \ln(x+1)\Big|_{3}^{4}$$

$u = x+1\,dx$
$du = dx$

$$= \ln(4+1) - \ln(3+1) = \ln(5) - \ln(4)$$

$$\Rightarrow -\ln(6) + 2\ln(5) - \ln(4) \;//$$

2) Calcule:

a) $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\, dA; \quad R=\{(x,y): 0 \leq x \leq 1 ; 0 \leq y \leq 1\}$

$$\int_0^1 \underbrace{\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}} dx}\, dy$$

$$\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}} dx = y \int_0^1 \frac{x}{\sqrt{x^2+y^2+1}} dx =$$

$u = x^2+y^2+1 \; dx$

$du = 2x + 0 + 0 \, dx$

$du = 2x . dx$

$\frac{du}{2} = x . dx$

$$= y \int_0^1 \frac{\frac{du}{2}}{\sqrt{u}} = y \int_0^1 \frac{du}{2} \cdot \frac{1}{\sqrt{u}}$$

$$= y \int_0^1 \frac{1}{2\sqrt{u}} du = \frac{y}{2} \int_0^1 \frac{1}{\sqrt{u}} =$$

$$\frac{1}{\sqrt{u}} = \frac{1}{u^{1/2}} \qquad = \frac{y}{2} \int_0^1 u^{-1/2} du$$

$$= \frac{y}{2} \cdot \frac{u^{1/2}}{1/2} = \frac{y}{\cancel{2}} \cdot \cancel{2} u^{1/2} = y u^{1/2} + C$$

$$\Rightarrow y(x^2+y^2+1)^{1/2} \Big|_0^1 = y(1^2+y^2+1)^{1/2} - y(0^2+y^2+1)^{1/2}$$

$$= y(2+y^2)^{1/2} - y(1+y^2)^{1/2}$$

$$= y\sqrt{y^2+2} - y\sqrt{y^2+1}$$

$$= y(\sqrt{y^2+2} - \sqrt{y^2+1})$$

$$\int_0^1 y(\sqrt{y^2+2} - \sqrt{y^2+1}) dy$$

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$y\int_0^1 \sqrt{y^2+2}\, dy - y\int_0^1 \sqrt{y^2+1}\, dy$

$*$ ♡

$y\int_0^1 \sqrt{y^2+2}\, dy = \int_0^1 \sqrt{u}\, du = \int_0^1 u^{1/2}\, du$

$u = y^2+2\, dy$
$du = 2y\, dy$ $\Rightarrow \frac{u^{3/2}}{3/2} = \frac{2}{3}\sqrt{u^3}$
$\frac{du}{2} = y\, dy$

$= \boxed{\frac{2}{3}\sqrt{(y^2+2)^3}\Big|_0^1 =}$

♡ $-y\int_0^1 \sqrt{y^2+1}\, dy = -\int_0^1 \sqrt{u}\, du =$

$u = y^2+1\, dy$ $= -\frac{u^{3/2}}{3/2} = \boxed{-\frac{2}{3}\sqrt{(y^2+1)^3}\Big|_0^1}$
$du = 2y\, dy$
$\frac{du}{2} = y\, dy$

$\frac{2}{3}\left(\sqrt{(y^2+2)^3} - 1\sqrt{(y^2+1)^3}\right)\Big|_0^1$

$\Rightarrow \frac{2}{3}\left(\sqrt{(1^2+2)^3} - \sqrt{(1^2+1)^3}\right) - \left(\sqrt{(0^2+2)^3} - \sqrt{(0^2+1)^3}\right)$

$\Rightarrow \frac{2}{3}\left(3\sqrt{3} - 2\sqrt{2}\right) - 2\sqrt{2} + 1)$

$\frac{2}{3}\left(3\sqrt{3} - 2\sqrt{2} - 2\sqrt{2} + 1\right)$

$\frac{2(3\sqrt{3} - 4\sqrt{2} + 1)}{3}$ //

obs: no gabarito não tem o 2 na frente.

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15) $\iint_R x\sqrt{1-x^2}\,dA$; $R=\{(x,y): 0\le x\le 1 ;\ 2\le y\le 3\}$

$$\int_2^3\int_0^1 x\sqrt{1-x^2}\,dx\,dy$$

$$\int_0^1 x\sqrt{1-x^2}\,dx = x\int_0^1 \sqrt{1-x^2}\,dx$$

$u = -x^2+1\ dx$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$$= \int_0^1 \sqrt{u} \cdot -\frac{du}{2} = -\frac{1}{2}\int_0^1 u^{1/2}\,du$$

$$\Rightarrow -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C_1 = -\frac{1}{3}(-x^2+1)^{1/2} = -\frac{1}{3}\sqrt{1-x^2}\Big|_0^1$$

$$-\frac{1}{3}\sqrt{1-1^2} - \left(-\frac{1}{3}\sqrt{1-0^2}\right) = -\frac{1}{3}\cdot 0 + \frac{1}{3} = \frac{1}{3}$$

29) O volume sob o plano $z=2x+y$ e acima do retângulo $R=\{(x,y): 3\le x\le 5 ;\ 1\le y\le 2\}$

$$\int_1^2\int_3^5 2x+y\,dx\,dy$$

$$\int_3^5 2x+y\,dx = \frac{2x^2}{2}+xy\Big|_3^5 = (5^2+5y)-(3^2+3y)$$

$25+5y-9-3y$

$2y+16$

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$$\int_1^2 2y+16\,dy = \frac{\cancel{2}y^2}{\cancel{2}} + 16y \Big|_1^2 = (2^2+16\cdot 2)-(1^2+16\cdot 1)$$

$$= 36 - 17 = 19_{//}$$

31) O volume do sólido compreendido pela superfície $z = x^2$ e os planos $x=0$, $x=2$, $y=3$, $y=0$ e $z=0$.

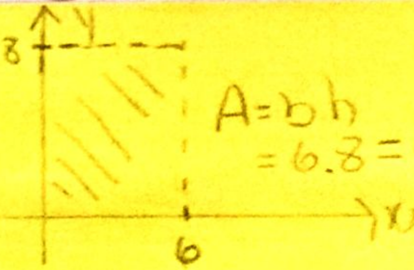
$$\int_0^3 \int_0^2 x^2\,dx\,dy$$

$$\int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \boxed{\frac{8}{3}}$$

$$\int_0^3 \frac{8}{3}\,dy = \frac{8}{3}y\Big|_0^3 = \frac{8\cdot\cancel{3}}{\cancel{3}} - \frac{8\cdot 0}{3} = 8_{//}$$

35) Encontre o valor médio de [...] retângulo $[0,8]\times[0,6]$

$$V.M = \frac{\iint_Q f(x,y)\,dA}{\text{área}} \Rightarrow \int_0^6 \int_0^8 x \ldots$$



$A = b.h$
$= 6.8 = 48$

$$V.M = \frac{2304}{48} = 48_{//}$$

$$\int_0^8 xy^2\,dx = \frac{x^2}{2}y^2\Big|_0^8 = \frac{8^2}{2}y^2 \ldots$$

$$\int_0^6 32y^2\,dy = 32\frac{y^3}{3}\Big|_0^6 = 32\cdot\frac{6^3}{3} - 0 = 2304$$

37) Calcule o valor médio de
retângulo $[0,1] \times [0, \pi/2]$

$$\int_0^{\pi/2}\int_0^1 y\,\mathrm{sen}(xy)\,dx\,dy$$

$$\int_0^1 y\,\mathrm{sen}(xy)\,dx = y\int_0^1 \mathrm{sen}(xy)\,dx$$

$u = xy\,dx$
$du = y\,dx$
$\frac{du}{y} = dx$

$$\Rightarrow y\int_0^1 \mathrm{sen}(u)\frac{du}{y}$$

$$\frac{y}{y}\int_0^1 \mathrm{sen}(u)\,du \quad = -\cos(u) + C$$

$$\Rightarrow -\cos(xy)\Big|_0^1$$

$$\Rightarrow -\cos(y) + \cos(0) = -\cos(y) + 1$$

$$\int_0^{\pi/2} -\cos(y) + 1\,dy = -\int_0^{\pi/2} \cos(y) + 1\,dy$$

$$-\mathrm{sen}(y) + y\Big|_0^{\pi/2} = \left(-\mathrm{sen}\left(\frac{\pi}{2}\right) + \frac{\pi}{2}\right) - \left(-\mathrm{sen}(0) + 0\right)$$

$$-1 + \frac{\pi}{2} + 0 = -1 + \frac{\pi}{2}$$

$$V.M = \frac{-1 + \frac{\pi}{2}}{\frac{\pi}{2}} = \left(-1 + \frac{\pi}{2}\right) \cdot \frac{2}{\pi} = \boxed{\frac{-2}{\pi} + 1}$$

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39- Suponha que a temperatura em graus Celsius, num ponto (x,y) de uma chapa metálica plana seja $T(x,y) = 10 - 8x^2 - 2y^2$, onde x e y estão em metros. Calcule a temp média da porção retangular da chapa dada por $0 \leq x \leq 1$ e $0 \leq y \leq 2$.

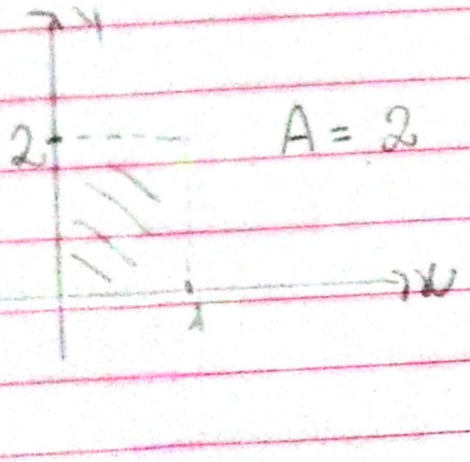
$$\int_0^2 \underbrace{\int_0^1 10 - 8x^2 - 2y^2 \, dx}\, dy$$

$$\int_0^1 10 - 8x^2 - 2y^2 \, dx = 10x - \frac{8x^3}{3} - 2y^2 x \Big|_0^1 =$$

$$\Rightarrow 10.1 - \frac{8.1^3}{3} - 2y^2.1 - 0 = 10 - \frac{8}{3} - 2y^2 = \frac{22}{3} - 2y^2$$

$$\Rightarrow \int_0^2 \frac{22}{3} - 2y^2 \, dy = \frac{22}{3}.y - \frac{2y^3}{3} \Big|_0^2 =$$

$$= \frac{22.2}{3} - \frac{2.2^3}{3} - 0 = \frac{44}{3} - \frac{16}{3} = \frac{28}{3}$$



$A = 2$

$$V.M = \frac{\frac{28}{3}}{2} = \boxed{\frac{14}{3} C°}$$