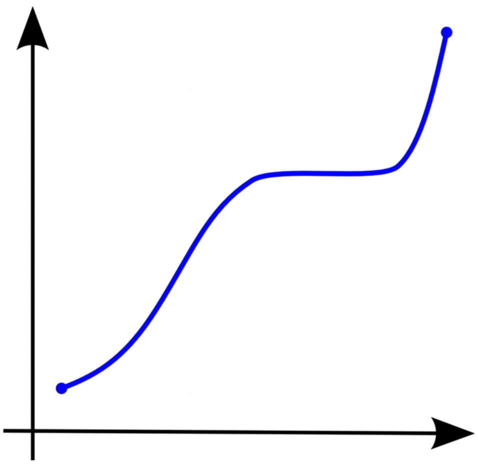
Monotonicity - *Monotonie*

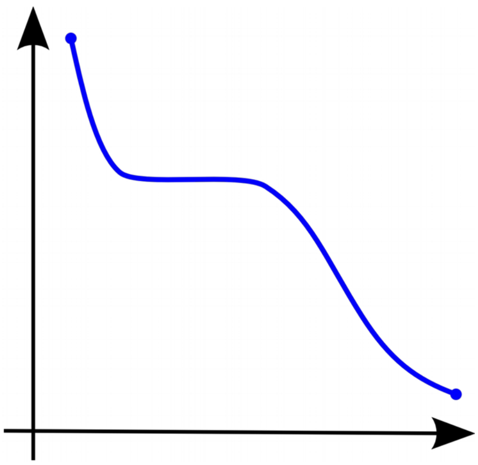
Definition: The function f is defined in an interval I.
If for all $x_{1,} x_2 \in I$ where $x_1 < x_2$ such that following holds:

$f(x_1) \leqslant f(x_2)$, then the function f in I is **monotonically increasing**.



If there is no equality, then the function f in I is **strictly increasing.**

$f(x_1) \geqslant f(x_2)$, then the function f in I is **monotonically decreasing**.



If there is no equality, then the function f in I is **strictly decreasing.**

Theorem (*Monotoniesatz*):
Let the function f be differentiable in the interval I. If for all x in I following holds:

$f'(x) > 0$ then f is strictly increasing in I.

$f'(x) < 0$ then f is strictly decreasing in I.

Example 1: Investigate the monotonicity of the function $f(x) = 2^x$.

Solution: If x_1 , x_2 are real numbers, where $x_1 < x_2$, then $x_2 = x_1 + d$ where $d > 0$. For the following values of the function we have: $f(x_2) = 2^{x_2} = 2^{x_1+d} = 2^x_1 \cdot 2^d$. And, as $d > 0$ then $2^d > 1$. So, $f(x_2) = 2^x_1 \cdot 2^d > 2^x_1 = f(x_1)$. Therefore, f is strictly increasing ("*monoton wachsend*").

Example 2: (Applying the theorem on monotonicity)
Investigate the monotonicity of the function $f(x) = \frac{1}{3}x^3 - x$

Solution: $f'(x) = x^2 - 1$. The inequality $x^2 - 1 > 0$ is satisfied for $x < -1$ and $x > 1$ and the inequality $x^2 - 1 < 0$ is satisfied for $-1 < x < 1$.
Hence, the function f is strictly decreasing for $-1 \leqslant x \leqslant 1$ and strictly increasing for $x \leqslant -1$ and $x \geqslant 1$.

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Exercises