

On this diagram, chords are drawn starting from zero and joining every $7^{\text {th }}$ point: zero to seven; seven to fourteen and so on until after several times around the circle we return to our first point. The result of using an increment of 7 on a 24 point circle is a star.

The aim of this project is to figure out what increments result in a regular polygon; what increments results in a star and when it is a star, when does it use all 24 points and when does it only use some points? And of course, why?

In this investigation, we have 4 variables:

1. The number of points on the circumference of the circle. Let's use $N$ to represent this number.
2. The increment we use to draw the chords. Let's use $n$ to represent this number.
3. The number of chords we need to draw to return to the starting point. Let's use $c$ to represent this number.
4. The value of the final point, if we keep counting in our increment as we go around and around the circle. Let's use $v$ to represent this number.


Use the applets at https://ggbm.at/epewd6aw to draw all other increments, and record results on the pages that follow.

## Star Drawings Record Sheet

## 12 Point Star



9 Point Star


## 15 Point Star



## Hypothesis

When joining $N$ points on a circumference using an increment of $n$, one of three shapes occurs:

1. A regular polygon
2. A star that uses all $N$ points
3. A star that uses only some of the points but not all.

| A regular polygon occurs when | An $N$ point star occurs when | A star that has less than $N$ points occurs <br> when |
| :--- | :--- | :--- |

## Prediction for a 24 point circle

| Suppose $N=24$. The following <br> increments will yield regular polygons <br> (state the kind of polygon for each): | Suppose $N=24$. The following <br> increments will yield a 24 point star (state <br> the end value for each increment): | Suppose $N=24$. The following <br> increments will yield a star that has less <br> than 24 points (state the end value for <br> each increment): |
| :--- | :--- | :--- |

Test your prediction on the 24 point circle https://ggbm.at/mpnPshPh and record your results on the following page.

| Increment <br> $n$ | $\begin{gathered} \text { Polygon } \\ \text { (what kind?) } \\ \text { or Star? } \end{gathered}$ | Chords <br> c | End Value <br> $v$ | Example using an increment of ........ |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  | 24 point circle $\quad 23 \quad 0$ or 24 |
| 2 |  |  |  |  |
| 3 |  |  |  | $20$ |
| 4 |  |  |  | $19$ |
| 5 |  |  |  | 18 |
| 6 |  |  |  | $17 \oint$ |
| 8 |  |  |  | 15 |
| 9 |  |  |  | $13 \bigcirc$ |
| 10 |  |  |  | If the radius of the circle is 5 cm , calculate the length of one chord. |
| 11 |  |  |  |  |

## Conclusions:

There are 23 possible increments for a 24 point star. For which increments was your prediction correct?

Write down any formula that you have created that relate any two or more of the values $N, n, c, v$.

Now choose your own value $N$, between 10 and 100 . Choose an increment that will give you one of the following, and state the values required:

| $N=$ |  |  |
| :--- | :--- | :--- |
| Regular Polygon. | $N$ point star. |  |
| Let $n=$ | Let $n=$ | Star, less than $N$ points. |
| This will yield a regular polygon with ....... <br> sides. | This will yield an $N$ point star. |  |
| The number of chords $c=$ |  |  |
| The end value $v=$ | The number of chords $c=$ |  |
| The end value $v=$ | This will yield a star with ....... points. |  |
| The number of chords $c=$ |  |  |

Confirm your answers with the 'star drawings' applet: https://ggbm.at/srwmxtwt

## Finally:

Suppose there are $N$ points on the circle, and you use an increment of $n$ and the radius of the circle is 5 cm . How long is one chord, in terms of $N$ and $n$ ?

