Opt. I Maths MARKING SCHEME

Q.No.	Description of Questions	Marks
	Group- A [10x1=10]	
1.(a)	A function which is expressed in the form of y=f(x)=c , where c is constant. Example: {(a, 1), (b, 1), (c, 1), (d, 1)}	1
1. (b)	In G.P., $\frac{2^{nd}\ term}{1^{st}\ term}=\frac{4^{th}\ term}{3^{rd}\ term}$ $\frac{a}{3}=\frac{81}{27}$ $a=9$	1
2. (a)	The point of discontinuity of the function f(x) is 1 as: f(1)= $\frac{1+1}{1-1}=\frac{2}{0}=\infty$	1
2. (b)	4×5-(1+k) $\times6$= 2×3-$4\times$k 20-6-6k= 6-4k k= 4	1
3. (a)	Slope of $a_1x+b_1y+c_1=0$: $m_1=\frac{-a_1}{b_1}$ Slope of $a_2x+b_2y+c_2=0$: $m_2=\frac{-a_2}{b_2}$ Condition for parallelism: $m_1=m_2$ $\frac{-a_1}{b_1}=\frac{-a_2}{b_2}$ $a_1b_2=a_2b_1$	1
3. (b)	Hyperbola	1
4. (a)	2sinA.sinB= cos(A-B)- cos(A+B)	1
4. (b)	$3\tan^2\theta=1$ $\tan\theta=\sqrt{\frac{1}{3}}$ $\tan\theta=\tan30°,\tan(180°+30°)$ $\theta=30°,210°$	1
5. (a)	$\vec{a}.\vec{b}=(\vec{i}-2\vec{j}).(-6\vec{i}-3\vec{j})$ $\vec{a}.\vec{b}$ =-6+6 $\vec{a}.\vec{b}$=0	1
5. (b)	As, A(x,y) $\xrightarrow{y=-x}$ A`(-y, -x) So, P(-4, 3a-5) $\xrightarrow{y=-x}$ P`(-3a+5, 4) On equating the co-ordinates of image formed with the image provided: -3a+5= 7-a $\therefore$ a= -1	1
	Group- B [13x2=26]	
6. (a)	g(x+5)= x+20 g(x+5-5)= x-5+20 g(x)= x+15 Now, $g_og(x)$ = g[g(x)] = g(x+15) = x+15+15 $g_og(x)$= x+ 30	1 1
6. (b)	Comparing (x- $\sqrt{2}$) with x-b, we get: b= $\sqrt{2}$ Using factor theorem: f($\sqrt{2}$)= 0 or, a($\sqrt{2}$)^3-6 $\sqrt{2}$ +2 $\sqrt{2}$ = 0 or, 2 $\sqrt{2}$ a-4 $\sqrt{2}$ = 0 $\therefore$ a= 2	1 1
6. (c)	First term(a)= 100 $S_{15}=\frac{n}{2}[2a+(n-1)d]$ 450= $\frac{15}{2}[2\times100+(15-1)d]$ 900= 15(200+14d) 60 = 200+14d $\therefore$ d = -10	1 1

7. (a)

$$|P| = \begin{vmatrix} 2 & -1 \\ -4 & 5 \end{vmatrix} = 10 - 4 = 6$$

$$P' = \frac{1}{|P|}.Adjoint\ of P$$

$$P' = \frac{1}{6}\begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$ 1

Here, R= P'Q

$$R = \frac{1}{6}\begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -15+3 \\ -12+6 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -12 \\ -6 \end{bmatrix}$$ 1

$$R = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

7. (b)

Coefficient of x	Coefficient of y	Constant
1	1	5
1	-1	3

Here,

$$D_x = \begin{vmatrix} 5 & 1 \\ 3 & -1 \end{vmatrix} = -5 - 3 = -8$$ 1

$$D_y = \begin{vmatrix} 1 & 5 \\ 1 & 3 \end{vmatrix} = 3 - 5 = -2$$ 1

8. (a)

Slope of 2x-y+6=0:

$$m_1 = \frac{-2}{-1} = 2$$

Slope of 3x+ky+4=0:

$$m_2 = \frac{-3}{k}$$ 1

Angle between the lines:

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan 45° = \pm\frac{2 - \frac{3}{k}}{1 + \frac{6}{k}}$$

$$1 = \pm\frac{2k-3}{k+6}$$ 1

$$k + 6 = \pm(2k - 3)$$

Taking positive sign:

k+6= 2k-3

k= 9

8. (b)

$$\text{Slope}(m_1) = \frac{-a}{b} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$ 1

$$\text{Slope}(m_2) = \frac{-a}{b} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$ 1

$$m_1 m_2 = -\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$$

9. (a)

L.H.S.= cot2A+ tan$A$

$$= \frac{\cos 2A}{\sin 2A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos 2A.\cos A + \sin 2A.\sin A}{\sin 2A.\cos A}$$ 1

$$= \frac{\cos(2A - A)}{\sin 2A.\cos A}$$

$$= \frac{\cos A}{\sin 2A.\cos A}$$ 1

$$= \cos ec 2A$$

9. (b)

L.H.S.= $2\cos 105°.\cos 15° + \frac{1}{2}$

1

	$=\cos(105^\circ+15^\circ)+\cos(105^\circ-15^\circ)+\frac{1}{2}$ $=\cos120^\circ+\cos90^\circ+\frac{1}{2}$ $=-\frac{1}{2}+0+\frac{1}{2}$ $=0$	1
9. (c)	$\left(\tan\frac{\theta}{3}\right)^2-2.\tan\frac{\theta}{3}.\frac{1}{\sqrt{3}}+\left(\frac{1}{\sqrt{3}}\right)^2=0$ $\left(\tan\frac{\theta}{3}-\frac{1}{\sqrt{3}}\right)^2=0$ $\tan\frac{\theta}{3}=\frac{1}{\sqrt{3}}$ $\tan\frac{\theta}{3}=\tan 30^\circ$ $\theta=90^\circ$	1 1
10.(a)	$\vec{a}+\vec{b}=-\vec{c}$ Squaring on both sides, we get: $(\vec{a}+\vec{b})^2=(-\vec{c})^2$ $(\vec{a})^2+2\vec{a}.\vec{b}+(\vec{b})^2=(\vec{c})^2$ $(6)^2+2\vec{a}.\vec{b}+(7)^2=\left(\sqrt{127}\right)^2$ $2\vec{a}.\vec{b}=42$ $\vec{a}.\vec{b}=21$ Angle between $\vec{a}$ and $\vec{b}$: $\cos\theta=\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert.\lvert\vec{b}\rvert}=\frac{21}{6\times7}$ $\cos\theta=\frac{1}{2}$ $\cos\theta=\cos60^\circ$ $\theta=60^\circ$	1 1
10.(b)	$\overrightarrow{OG}=\frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{3}$ $3\vec{i}+4\vec{j}=\frac{2\vec{i}+3\vec{i}+\vec{i}-2\vec{j}+\overrightarrow{OC}}{3}$ $9\vec{i}+12\vec{j}=3\vec{i}+\vec{j}+\overrightarrow{OC}$ $\overrightarrow{OC}=6\vec{i}+11\vec{j}$	1 1
10 (c)	Q.D. = $\frac{Q_3-Q_1}{2}$ $20=\frac{Q_3-17.5}{2}$ $Q_3=57.5$ Co-efficient of Q.D. = $\frac{Q_3-Q_1}{Q_3+Q_1}=\frac{57.5-17.5}{57.5+17.5}=0.533$	1 1
11.	The factors of 16 are ±1,± 2, ±4, ±8, ±16 When x=-1 p(x)=0 ∴ x+1 is one factor. Now, $3x^3+3x^2-16x^2-16x+16x+16=0$ Or, $3x^2(x+1)-16x(x+1)+16(x+1)=0$ Or, $(x+1)(3x^2-16x+16)=0$ Or, (x+1) (x-4) (3x-4)=0 $\therefore x=-1,4,\frac{4}{3}$	1 1 1 1
12.	The three numbers in A.P. be (a-d), a and (a+d) such that: a-d+a+a+d= 18 $\therefore a=6$ When 1,2 and 7 are added to the numbers respectively, they become a-d+1, a+2 and a+d+7 i.e. 7-d, 8 and 13+d which are in G.P. So, $\frac{8}{7-d}=\frac{13+d}{8}$ $or, 64=91-13d+7d-d^2$ $or, d^2+6d-27=0$ $or, (d+9)(d-3)=0$ $Either, d=-9$ $Or, d=3$ When d=-9, the numbers are: a-d= 6+9= 15 a= 6 a+d= 6-9= -3 When d=3, the numbers are: a-d= 6-3= 3 a= 6	1 1 1 1

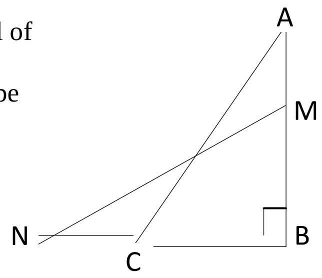
	a+d= 6+3= 9	
13.	(a) f(1.99)= 1.99+2= 3.99 (b) f(2.01)= 3$\times$2.01-2= 4.03 $(c) \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x+2 = 2+2 = 4$ $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x-2 = 3\times 2-2 = 4$ $Yes, \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$ (d) Functional value at x=2: f(2)= 3 $\times$2-2= 4 As $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$ =functional value at 2, the given function f(x) is continuous at x=2.	1 1 1 1
14.	Representing the equations in matrix form: $\begin{bmatrix} 4 & -9 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} x \\ \frac{1}{y} \end{bmatrix} = \begin{bmatrix} -11 \\ 8 \end{bmatrix}$ $Let, A = \begin{bmatrix} 4 & -9 \\ -3 & 6 \end{bmatrix}, B = \begin{bmatrix} -11 \\ 8 \end{bmatrix}, X = \begin{bmatrix} x \\ \frac{1}{y} \end{bmatrix}$ $\lvert A \rvert = \begin{vmatrix} 4 & -9 \\ -3 & 6 \end{vmatrix} = 24-27 = -3$ $A^{-1} = \frac{1}{\lvert A \rvert}.Adjoint\ of A$ $A^{-1} = \frac{1}{-3}\begin{bmatrix} 6 & 9 \\ 3 & 4 \end{bmatrix}$ $Here, X = A^{-1}.B$ $X = \frac{1}{-3}\begin{bmatrix} 6 & 9 \\ 3 & 4 \end{bmatrix}.\begin{bmatrix} -11 \\ 8 \end{bmatrix}$ $\begin{bmatrix} x \\ \frac{1}{y} \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -66+72 \\ -33+32 \end{bmatrix}$ $\begin{bmatrix} x \\ \frac{1}{y} \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{1}{3} \end{bmatrix}$ $\therefore x = -2$ $And, \frac{1}{y} = \frac{1}{3}$ $\therefore y = 3$	 1 1 1 1

15.	The given equation of pair of straight lines: $x^2 - xy - 2y^2 = 0$ $(x-2y)(x+y) = 0$ $Either, x-2y = 0$ $Or, x+y = 0$ From x-2y=0: Slope(m_1)= $\frac{1}{2}$ Let slope of required line be m_1^{-1} such that: $m_1 . m_1^{-1}$ = -1 So, m_1^{-1} = -2 Equation of the line: $y-y_1 = m_1^{-1}(x-x_1)$ y+1= -2(x-3) $\therefore$ 2x+y-5=0 Similarly, from x+y=0: Slope(m_2)= -1 Let slope of required line be m_2^{-1} such that: $m_2 . m_2^{-1}$ = -1 m_2^{-1} =1 Equation of the line: $y-y_1 = m_2^{-1}(x-x_1)$ y+1= 1(x-3) $\therefore$ x-y-4= 0 Finally, the equation of pair of straight lines: (2x+y-5).(x-y-4)= 0 $\therefore$ $2x^2-y^2$-xy-13x+y+20= 0	1 1 1 1

16.	L.H.S= $\sin^4\frac{\pi^c}{8}+\sin^4\frac{3\pi^c}{8}+\sin^4\frac{5\pi^c}{8}+\sin^4\frac{7\pi^c}{8}$ $=\sin^4\frac{\pi^c}{8}+\sin^4\frac{3\pi^c}{8}+\sin^4\left(\pi^c-\frac{3\pi^c}{8}\right)+\sin^4\left(\pi^c-\frac{\pi^c}{8}\right)$ $=2\sin^4\frac{\pi^c}{8}+2\sin^4\frac{3\pi^c}{8}$ $=2\left[\sin^4\frac{\pi^c}{8}+\sin^4\left(\frac{\pi^c}{2}-\frac{\pi^c}{8}\right)\right]$ $=2\left[\sin^4\frac{\pi^c}{8}+\cos^4\frac{\pi^c}{8}\right]$ $=2\left[\left(\sin^2\frac{\pi^c}{8}+\cos^2\frac{\pi^c}{8}\right)^2-2\sin^2\frac{\pi^c}{8}.\cos^2\frac{\pi^c}{8}\right]$ $=2\left[1-\frac{1}{2}\left(\sin 2.\frac{\pi^c}{8}\right)^2\right]$ $=2\left[1-\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2\right]$ $=\frac{3}{2}$	1 1 1 1

17.	A+B+C= π^c Sin (A+B)= sin (π^c -C)= sin C Cos (A+B)= cos (π^c -C)= -cos C L.H.S.= $\frac{\cos A}{\sin B.\sin C}+\frac{\cos B}{\sin C.\sin A}+\frac{\cos C}{\sin A.\sin B}$ $=\frac{2(\cos A\sin A+\cos B\sin B+\cos C\sin C)}{2\sin A.\sin B.\sin C}$ $=\frac{\sin 2A+\sin 2B+\sin 2C}{2\sin A.\sin B.\sin C}$ $=\frac{2\sin(A+B)\cos(A-B)+2\sin C.\cos C}{2\sin A.\sin B.\sin C}$ $=\frac{2\sin C[\cos(A-B)+\cos C]}{2\sin A.\sin B.\sin C}$ $=\frac{2\sin C[\cos(A-B)-\cos(A+B)]}{2\sin A.\sin B.\sin C}$ $=\frac{2\sin C.(2\sin A.\sin B)}{2\sin A.\sin B.\sin C}$ $=2$	1 1 1 1

18. Let, AC be the height of a ladder leaning at point A against the wall of height AB. The angle made by base of the ladder on the ground be $\angle ACB$. When the ladder slides to the point M on the wall, let the new point at which the base of the ladder rests be N. Then,



In Δ ABC: 2

$\sin \angle ACB = \dfrac{AB}{AC}$

$\sin 60° = \dfrac{AM + MB}{20}$

$\dfrac{\sqrt{3}}{2} = \dfrac{7.32 + MB}{20}$ 1

$MB = 10\, feet$

Now, In Δ MBN:

$\sin \angle MNB = \dfrac{MB}{MN}$

$\sin \angle MNB = \dfrac{10}{20}$

$\sin \angle MNB = \sin 30°$ 1

$\angle MNB = 30°$

19. Here the given point A(5,4)

equation of circle $x^2 + y^2 - 6x - 4y + 9 = 0$. 1

i) Comparing eq (i) with $x^2+y^2+2gx+2fy+c = 0$ we get

entre (h, k) = (3, 2) and r = 2 units 1

ii) $x' = h + \dfrac{r^2(x-h)}{(x-h)^2+(y-k)^2} = 4$ 1

iii) $y' = k + \dfrac{r^2(y-k)}{(x-h)^2+(y-k)^2} = 3$ 1

iv) (x' , y') = (4, 3) 1

20. Computation of Mean Deviation and its coefficient:

Marks	Mid Value (m)	Frequency(f)	c.f.	$\lvert m - Md \rvert$	$f\lvert m - Md \rvert$
0-10	5	5	5	18.33	91.65
10-20	15	2	7	8.33	16.66
20-30	25	9	16	1.67	15.03
30-40	35	2	18	11.67	23.34
40-50	45	2	20	21.67	43.34
Total		N=20			$\sum f\lvert m - Md \rvert$ =187.02

2

Here,

Position of Median= $\left(\dfrac{N}{2}\right)^{th} term$

$= \left(\dfrac{20}{2}\right)^{th} term$

$= 10^{th}\, term$ 1

Median Class= 20-30

Median Value(Md)= $L + \dfrac{\frac{N}{2} - cf}{f} \times i$

$= 20 + \dfrac{10 - 7}{9} \times 10$

Median Value(Md) = 23.33

Now, Mean Deviation $= \dfrac{\sum f\lvert m - Md \rvert}{N} = \dfrac{187.02}{20} = 9.351$ 1

Coefficient of Mean Deviation=

$\dfrac{MeanDeviation}{Median} = \dfrac{9.351}{23.33} = 0.4008$

21. Computation of Standard Deviation and its coeffecient:

Marks	Mid Value (m)	Frequency (f)	m^2	fm	fm^2
0-10	5	9	25	45	225
10-20	15	6	225	90	1350
20-30	25	4	625	100	2500

30-40	35	12	1225	420	14700
40-50	45	9	2025	405	18225
Total		N=40		$\sum fm = 1060$	$\sum fm^2 =$ 37000

2

Here,

Standard Deviation (σ) = $\sqrt{\frac{\sum fm^2}{N} - \left(\frac{\sum fm}{N}\right)^2}$

$= \sqrt{\frac{3700}{40} - \left(\frac{1060}{40}\right)^2}$ 1

$= \sqrt{925 - (26.5)^2}$

$= 14.9248$ 1

Also, Coefficient of Standard Deviation=

$\frac{\sigma}{Mean} = \frac{14.9248}{26.5} = 0.5632$

Group- D [4x5 =20]

22 The corresponding boundary line equations of the inequalities are respectively:

x+y= 6 ---------(i)

x-y= -2 ---------(ii)

x=0 ------------(iii)

y=2 ------------(iv)

From equation (i), y= 6-x:

x	0	6	3
y	6	0	3

1

Boundary line (i) passes through (0, 6), (6, 0) abd (3, 3).

On testing, when x=0 and y=0:

x+y$\leq$6

0$\leq$6, which is true, so the solution region contains the origin.

From equation (ii), y= x+2

x	0	-2	2
y	2	0	4

1

Boundary line (ii) passes through (0, 2), (-2, 0) and (2, 4). On testing, when x=0 and y=0:

x-y$\geq$-2

22. 0$\geq$-2, which is true, so the solution region contains the origin. x=0 and y=2 represent Y-axis and X-axis respectively. The solution region of x$\geq$0 is the right plane of Y-axis and that of y$\geq$2 is the upper half plane of X-axis. 1

The common soultion region found after plotting the boundary lines in graph is (0, 2), (2, 4) and (4, 2). 1

Graph 1

Vertices	F= 6x+5y	Result
(0, 2)	10	Minimum
(2, 4)	32	
(4, 2)	34	Maximum

23. The equation of given circle is:

$x^2+y^2-2x+4y-4= 0$

or, $(x-1)^2+(y+2)^2= 9$ 2

where, on comparing the equation with $(x-h)^2+(y-k)^2 = r^2$ the centre obtained is (1, -2).

Thus required circle passes through (1, -2) with radius as: 1

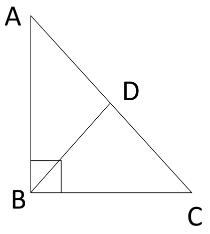
Radius= $\sqrt{(3-1)^2 + (2+2)^2} = \sqrt{20} units$

Therefore the required equation of the circle with centre (3, 2) is: 1

$(x-h)^2+ (y-k)^2 = r^2$

$(x-3)^2+ (y-2)^2= (\sqrt{20})^2$

$\therefore x^2+y^2-6x-4y-7= 0$ 1

24.	Let ΔABC be a right angled triangle with $\angle ABC = 90^o$. D is the mid-point of hypotenuse AC i.e. AD= DC To prove: AD= BD= CD Proof: In ΔABD, $\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$ (By triangle law of vector addition)  In ΔBDC, $\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC} = \overrightarrow{BD} + \overrightarrow{AD} = \overrightarrow{AD} + \overrightarrow{BD} (\Theta\ \overrightarrow{AD} = \overrightarrow{DC})$ Since, $\angle ABC = 90^o$, $\overrightarrow{AB}.\overrightarrow{BC} = 0$ $or, (\overrightarrow{AD} + \overrightarrow{DB}).(\overrightarrow{AD} + \overrightarrow{BD}) = 0$ $or, (\overrightarrow{AD} - \overrightarrow{DB}).(\overrightarrow{AD} + \overrightarrow{BD}) = 0$ $or, (\overrightarrow{AD})^2 = (\overrightarrow{BD})^2$ $or, AD^2 = BD^2$ $\therefore AD = BD$ D is the mid-point of AC so, AD=DC Therefore, AD= BD= CD.	1 1 1 1 1
25.	A(x, y) $\xrightarrow{y=-x(reflection)}$ A\`(-y, -x) A\`(-y, -x) $\xrightarrow{(0,0),k=2(enlargement)}$ A\`\`(-2y, -2x) So, A(x, y) $\xrightarrow{E_oR}$ A\`\`(-2y, -2x) Therefore: P (-4, 6) $\xrightarrow{E_oR}$ P\`\`(-12, 8) Q (-6, -10) $\xrightarrow{E_oR}$ Q\`\`(20,12) R (12, -8) $\xrightarrow{E_oR}$ R\`\`(16, -24) *Graph*	1 1 1 2