Al-Khwarizmi

By A. B. ARNDT, 6519 Dalroy Lane, Bethesda, MD 20817

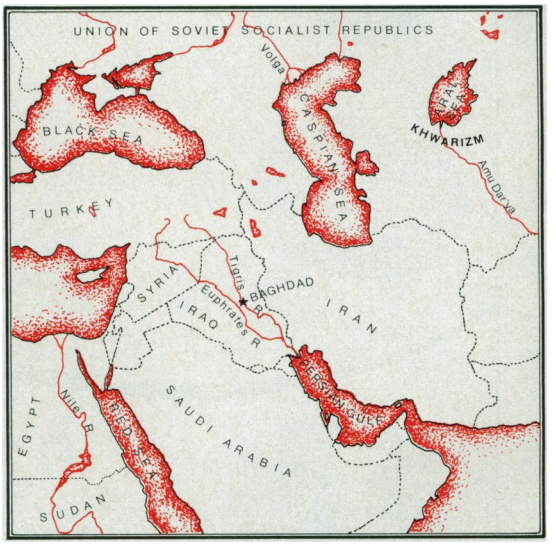
I enjoy introducing my mathematics classes to the history of mathematics and science. It is a subject they don't seem to encounter anywhere else, not even in their history courses, and it is one of the few topics that span that wide and culturally divisive gap between science and art in our society. In these brief historical excursions, I point out that our word *algorithm* is derived from the name of the ninth-century Arab mathematician Al-Khwarizmi. A teacher is always looking for some fact on the fringes of a student's knowledge to use as a peg on which to hang new information.

When I taught at an industrial training school in Saudi Arabia, I naturally tried the same trick, but it became immediately apparent that my students there were far more familiar with the great Al-Khwarizmi than I. "Oh yes, Abu-Abdullah Muhammed ibn-Musa," they responded, smiling.

"Uhh, right," I said, checking my notes quickly. I resolved on the spot to find out more about the man. My students had found a peg on the fringes of my information, and I was eager to load it with as many facts as possible.

This task wasn't as easy as I had expected. Many books about mathematicians or the history of mathematics, especially the older ones, simply fail to mention Al-Khwarizmi or any of his Arab colleagues at all. For example, H. W. Turnbull's *The Great Mathematicians* (1929), introduced as a "biographical history of mathematics," makes a quantum leap from Diophantus, who worked in Alexandria in the fourth century A.D., to the Renaissance Europeans Napier and Kepler. More recent historians, however, have begun to abandon the erroneous old notion that the Arabs had merely preserved Greek science for a few centuries before handing it on to Europe and to recognize, instead, original Arab contributions to science, particularly those of Al-Khwarizmi.

Abu-Abdullah Muhammed ibn-Musa Al-Kwarizmi (AH-boo ab-DULL-ah moo-HAM-mud IB-ben mū-sah al-Kwār-IZ-mee), that is, Muhammed, the father of Abdullah and the son of Moses, worked in the ninth century under the patronage of the Caliph Al-Ma'mun. He was one of many scholars gathered together by the caliph in his capital city of Baghdad, situated beside the Tigris river in present-day Iraq. His surname indicates that either he or his family originally came from Khwarizm, a region located east of the Caspian Sea and now incorporated into the U.S.S.R. (see map).



We know little about Al-Khwarizmi's personal life. Some sources give his birth date as 780 A.D.; it is thought that he died sometime between the years 830 A.D. and 850 A.D. Some difficulty in searching for the facts about Al-Khwarizmi is caused by another man, also named Muhammed, who worked in Baghdad under a later Abbasid caliph. Since this man was also an astronomer and mathematician and his father,

too, was called Musa, this Muhammed ibn-Musa has been frequently confused, even by major reference works today, with our scientist; but he is Abu-Jaffer (the father of Jaffer), not Abu-Abdullah, and he is *not* Al-Khwarizmi!

However little we know of Al-Khwarizmi himself, his scientific work was widely known and well documented. Al-Khwarizmi was the author of many books—in addition to his best-known works, he wrote about the astrolabe and the sundial and elaborated on the geometry of Ptolemy. He probably also took part in a project, sponsored by Al-Ma'mun, to measure the length of one degree of the earth's circumference.

Early in his career, Al-Khwarizmi prepared a practical abridgment of the Hindu astronomical tables known in Arabic as the *Sindhind*. This work made him famous almost instantly throughout the Arab world. Caliph Al-Ma'mun then asked him for a popular work on the science of equations, and he produced *Al-Kitab Al-jabr wa'l muqabalah*.

The words *jabr* (JAH-ber) and *muqabalah* (moo-KAH-ba-lah), which do not appear earlier in mathematics, were used by Al-Khwarizmi to designate the two basic operations in solving equations. The title of his work has been variously translated along the lines of "The Book of Completion and Cancellation" and "The Book of Restoration and Balancing."

Jabr is employed in the step in which an equation such as

$$x - 4 = 10$$

becomes

$$x = 14,$$

where the left-hand side of the first equation, in which x is diminished by 4, is "restored" or "completed" back to x. In Spain and Portugal today, a somewhat obsolete name for a "restorer" of bones—a bonesetter—is *algebrista*.

The operation Al-Khwarizmi called *muqabalah* leads us from

$$x^2 + x = x^2 + 4$$

to

$$x = 4$$

by "canceling" or "balancing" the two sides of the equation. In time, the word *muqabalah* was dropped, and the name of this branch of mathematics became, in a great many languages, simply *algebra*.

Al-Khwarizmi's popularizing treatise on algebra was extremely successful, and its fame and influence lasted for centuries. Numerous copies of the Arabic work were made, many in the West, and during the early Middle Ages, several translations were made into Latin as well. Later on, as the use of Latin waned, the work appeared in different European languages; the first treatise on algebra written in German, dated 1461, was a translation of a portion of Al-Khwarizmi's book. Even in this century—because so many of the book's examples are drawn from the field of Islamic law—students of law in Cairo and Mecca have been advised to study *Al-Kitab Al-jabr wa'l muqabalah*.

It is in looking at mathematical texts over the years, rather than in histories, that we can see Al-Khwarizmi's profound effect on mathematics. We find his influence on all who followed him. Over the centuries, writers on elementary algebra have borrowed his format, his terminology, and his classification of the types of linear and quadratic equations; often they have even used the very same numerical examples he did in the original *Algebra*.

Al-Khwarizmi's *Algebra*, including its pious introduction, was rendered into modern English in 1831 by Frederic Rosen. Somehow it came as a shock to me, when I looked at this translation of *Al-Kitab Al-jabr wa'l muqabalah*, that this granddaddy algebra textbook, written over eleven hundred years ago, was among the most lucid and useful I'd ever seen. It is straightforward and practical, full of examples, and, as Al-Khwarizmi himself stated, deals with "what is easiest and most useful in arithmetic." It is a textbook that I would have no difficulty teaching from today.

Al-Khwarizmi wrote with a genuine sympathy for the readers' difficulties. After

explaining several ways to find the circumference of a circle, for example, he cautions:

> This is an approximation, not the exact truth itself: nobody can ascertain the exact truth of this and find the real circumference, except the Omniscient: for the line is not straight so that its exact length might be found.... The best method here given is that you multiply the diameter by three and one-seventh: for it is the easiest and quickest. God knows best! (Rosen 1831, p. 200.)

All numbers in the *Algebra,* as in the quote above, are expressed by words rather than numerals—*three and one-seventh* instead of 3 1/7. The numerals we use today had at that time been developed in India only a couple of centuries earlier and were just beginning to make their way westward.

Al-Khwarizmi's next great work was a treatise on arithmetic, in which the Hindu-Arabic numerals were presented and the place-value system was explained. This textbook was the earliest written on the decimal system. It represents a milestone in the development of mathematics and science. Al-Khwarizmi demonstrated, in the *Arithmetic,* the basic operations of addition, subtraction, multiplication, and division and showed how to work with fractions and how to extract square roots. All these operations were greatly simplified by the new system.

This work, too, was enormously popular; it was avidly studied in the West and was instrumental in effecting Europe's conversion from the cumbersome Roman numerals to the present-day system. Unfortunately, no copies of Al-Khwarizmi's original Arabic version of the *Arithmetic* are extant, but we have several early Latin translations. Along with many other European scholars, the Englishman Adelard of Bath traveled to Spain to study Arabian mathematics. He produced, in 1126, what may be the earliest Latin translation of this work.

Through his *Arithmetic,* Al-Khwarizmi added another word to the vocabulary of the English language. One well-read Latin translation begins with the words *Dixit algorizmi,* or "Algorithm says," and follows with instructions for making various computations. Thus *algorithm,* a Latinized version of the mathematician's name, has come to its present meaning of a general computational procedure.

Issac Newton once said that he was able to see so far because he stood on the shoulders of giants, and surely Al-Khwarizmi stands firm in that tall pyramid of the world's great scientists. Perhaps the best assessment of the man can be found in his own quiet words from the introduction to his *Algebra:*

> The learned in times which have passed away, and among nations which have ceased to exist, were constantly employed in writing books on the several departments of science and on the various branches of knowledge....
>
> Some applied themselves to obtain information which was not known before them, and left it to posterity; others commented upon the difficulties in the works left by their predecessors, and defined the best method [of study], or rendered the access [to science] easier or placed it more within reach; others again discovered mistakes in preceding works, and arranged that which was confused, or adjusted what was irregular, and corrected the faults of their fellow-laborers, without arrogance towards them, or taking pride in what they did themselves. (Rosen 1831, pp. 2–3)

All the achievements and all the services that he mentions were accomplishments of Al-Khwarizmi himself, although he modestly attributes them to others. These works of a learned man, from a nation that has ceased to exist and in a time that passed away more than a millenium ago, are still benefiting us today.

BIBLIOGRAPHY

Boyer, Carl B. *A History of Mathematics.* New York: John Wiley & Sons, 1968.

Karpinski, Louis Charles, ed. and trans. *Robert of Chester's Latin Translation of the Algebra of Al-Khowarizmi.* New York: Macmillan Co., 1915.

Newton, Issac. *Correspondence.* Vol. I, 1661–75. Cambridge: Published for the Royal Society at the University Press, 1959.

Rosen, Frederic, ed. and trans. *The Algebra of Mohammed Ben Musa.* London: Oriental Translation Fund, 1831.

Scott, J. F., *A History of Mathematics.* London: Taylor & Francis, 1958.

Struik, Dirk J. *A Concise History of Mathematics.* New York: Dover Publications, 1967.

Turnbull, H. W. *The Great Mathematicians.* London: Methuen & Co., 1929.

Vogel, Kurt, ed. and trans. *Mohammed Ibn Musa Alchwarizmi's Algorismus; Das fruehste Lehrbuch zum Rechnen mit indischen Ziffern.* Aalen, W. Germany: Otto Zeller Verlagsbuchhandlung, 1963. ■