

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi (8)^2 \left(\frac{3\pi}{128}\right) = 6$$

Prepa Tec
Calculus I 3rd partial
Quiz # 1A

$$\frac{dV}{dt} = 6 \text{ cm}^3$$

$$\frac{dr}{dt} = \frac{3\pi}{128} \text{ cm}$$

$$r = 8 \text{ cm}$$

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70

I. Determine if true or false for each of the following statements (10 points each)

1. True The second derivative of $y = 2e^{x^3}$ is $\frac{d^2y}{dx^2} = 6xe^{x^3}(2+3x^3)$

2. False The derivative of $6x - 4x^2y = 2y^2 + 1$ is $\frac{dy}{dx} = \frac{-6}{8x+4y}$ $y' = \frac{8xy-6}{-4x^2y-4y}$

3. False The derivative of $y = x^{2x}$ is $y' = 2x^{2x}(\ln(x)+1) \rightarrow y' = \frac{2y}{x} + 2\ln(x) \cdot (x^{2x})$

4. True A spherical balloon is being inflated with a gas at a rate of 6 cm^3 per second. Then the rate at which its radius is changing when its radius measures 8 cm is $\frac{dr}{dt} = \frac{3\pi}{128} \left[\frac{\text{cm}}{\text{sec}}\right]$. (Hint: $V = \frac{4}{3}\pi r^3$)

$$\frac{dV}{dt} = 6 \text{ cm}^3$$

$$6 = 4\pi (8)^2 \frac{dr}{dt}$$

$$\frac{6}{4\pi(64)} = \frac{dr}{dt} = \frac{3\pi}{128}$$

II. Answer the following problem. (10 points each letter)

A dynamite charge blows a rock up with a velocity of 160 ft/s . The height of the rock is given by the function $h(t) = 160t - 16t^2$ where "h" is measured in feet and "t" in seconds. Find the following:

a) The equation that gives the velocity of the rock at any time.

$$\text{velocity} = h'(t) = 160 - 32t$$

b) The time when velocity is zero (that is the time to reach the maximum height)

$$0 = 160 - 32t$$

$$-160 = -32t$$

$$t = -160 / -32$$

$$t = 5$$

$$t = 5 \text{ seconds}$$

c) The maximum height of the rock (that is when velocity is zero)

$$t = 5$$

$$h(5) = 160t - 16t^2$$

$$h(5) = 160(5) - 16(5)^2 \rightarrow 800 - 16(25)$$

$$800 - 400 = 400$$

$$\text{maximum height} : 400 \text{ ft}$$

d) The times (on the way up and on the way down) when the height is at 256 feet.

$$256 = 160t - 16t^2$$

$$256 = 160(2) - 16(2)^2$$

$$t = 2 \text{ seconds}$$

on the way up
and on the way
down, it's the same to both conditions

$$\text{times on the way up} = 2 \text{ seconds}$$

$$\text{times on the way down} = 2 \text{ seconds}$$

e) The velocities of the rock when the height is 256 feet.

$$\text{velocity} = h'(t) = 160 - 32t$$

$$h'(2) = 96$$

the same time

$$h'(2) = 160 - 32(2)$$

$$160 - 64$$

velocities when the height is 256

$$\text{is : } v = 96 \text{ ft/s}$$

f) The equation that gives the acceleration of the rock at any time.

$$\text{acceleration } h''(t) = -32$$

①

$$y = 2e^{x^3}$$

$$u = 2 \quad u' = 0$$

$$v = e^{x^3}$$

$$v' = 3x^2 e^{x^3}$$

$$y' = 6x^2 e^{x^3}$$

$$6x^2 (3x^2 e^{x^3}) + 12x e^{x^3}$$

$$u = 6x^2 \quad u' = 12x$$

$$18x^4 e^{x^3} + 12x e^{x^3}$$

$$v = e^{x^3} \quad v' = 3x^2 e^{x^3}$$

$$y'' = 6x e^{x^3} (2 + 3x^3)$$

$$\textcircled{2} \quad 6x - 4x^2 y' = 2y^2 + 1$$

$$6 - 4x^2 y' - 8xy - 4yy'$$

$$6x - 4x^2 y' - 2y^2 - 1 = 0$$

$$-4x^2 y' - 4yy' = 8xy - 6$$

$$6 - (4x^2 y' + 8xy) - 4yy'$$

$$y' = (-4x^2 - 4y) = 8xy - 6$$

$$4x^2 y$$

$$y' = \frac{8xy - 6}{-4x^2 - 4y}$$

$$u = 4x^2 \quad u' = 8x \rightarrow 4x^2 y' + 8xy$$

$$v = y \quad v' = y'$$

$$\textcircled{3} \quad y = x^{2x}$$

$$\ln y = \ln(x^{2x})$$

$$u = 2x \quad u' = 2$$

$$\ln y = 2x \ln(x)$$

$$v = \ln x \quad v' = \frac{1}{x}$$

$$\frac{y'}{y} = 2x \cdot \frac{1}{x} + 2 \ln x$$

$$\frac{y'}{y} = \frac{2x}{x} + 2 \ln x$$

$$\frac{y'}{y} = 2 + 2 \ln x \quad (x^{2x})$$