## Normal Form of a Line

Objectives: 1. Define and graph the general form of a line and the vector form of a line.
2. Show the line's relationship to it's general, vector, and normal forms.

Equations of a line in $\mathbb{R}^{2}$
Algebraic Forms: $\vec{p}$ is the vector form of the point $p=\left(p_{1}, p_{2}\right) ; \quad \vec{x}$ is the vector form of any point $x=\left(x_{1}, x_{2}\right)$

| General form | Vector form | Normal Form |
| :---: | :---: | :---: |
| $\boldsymbol{a x}+\boldsymbol{b y}=\boldsymbol{c}$ | $\vec{x}=\overrightarrow{\boldsymbol{p}}+\boldsymbol{t} \cdot \overrightarrow{\boldsymbol{d}},-\infty<\boldsymbol{t}<\infty$ | $\vec{n} \bullet \vec{x}=\vec{n} \bullet \vec{p}$ |
|  | $\left[\begin{array}{l}x_{1} \\ x_{2}\end{array}\right]=\left[\begin{array}{l}p_{1} \\ p_{1}\end{array}\right]+\boldsymbol{t} \cdot\left[\begin{array}{l}d_{1} \\ d_{2}\end{array}\right]$ | $\vec{n} \bullet(\vec{x}-\vec{p})$ |

## Review: Line - Vector Form

1. On the coordinate grid, plot and label the points $(1,1)$ and $(2,-1)$. Graph the line $\mathbf{L}$ defined by these points. Determine the equation of $\mathbf{L}$ and put it into the general form of a line.

2. Define the vector form of this line
$\overrightarrow{\boldsymbol{x}}=\overrightarrow{\boldsymbol{p}}+\boldsymbol{t} \cdot \overrightarrow{\boldsymbol{d}},-\infty<\boldsymbol{t}<\infty$,
For line $\mathbf{L}, \overrightarrow{\boldsymbol{p}}=$

$$
\vec{d}=
$$

Draw and label these vectors.

State the vector form of line $\mathbf{L}$. $\qquad$

Calculate the vector $\vec{x}$, draw and label it for the following $\boldsymbol{t}$ values:

$$
t=-\mathbf{1} \rightarrow \vec{x}=
$$

$\qquad$

$$
\begin{aligned}
& t=1 \rightarrow \vec{x}= \\
& t=3 \rightarrow \vec{x}= \\
& \hline
\end{aligned}
$$

$$
t=-2 \rightarrow \vec{x}=
$$

$\qquad$

State the vector form of line $\mathbf{L}$ determined above. $\qquad$ Here: $\overrightarrow{\boldsymbol{p}}=\quad \overrightarrow{\boldsymbol{d}}=$

Normal Form of a Line - two forms
Normal Form
$\overrightarrow{\boldsymbol{n}} \bullet \overrightarrow{\boldsymbol{x}}=\overrightarrow{\boldsymbol{n}} \bullet \overrightarrow{\boldsymbol{p}}$
$\overrightarrow{\boldsymbol{n}} \bullet(\overrightarrow{\boldsymbol{x}}-\overrightarrow{\boldsymbol{p}})=\mathbf{0}$

If $\overrightarrow{\boldsymbol{n}}$ is a normal vector to $\overrightarrow{\boldsymbol{d}}$, then $\overrightarrow{\boldsymbol{n}}=$

Using $\overrightarrow{\boldsymbol{n}}$ state each Normal Form of $\boldsymbol{L}$.
$\overrightarrow{\boldsymbol{n}} \bullet \overrightarrow{\boldsymbol{x}}=\overrightarrow{\boldsymbol{n}} \bullet \overrightarrow{\boldsymbol{p}}$ becomes $\qquad$ $\overrightarrow{\boldsymbol{n}} \bullet(\overrightarrow{\boldsymbol{x}}-\overrightarrow{\boldsymbol{p}})=\mathbf{0}$ becomes

Draw the line, graphing and labeling $\overrightarrow{\boldsymbol{n}}, \overrightarrow{\boldsymbol{x}}$, and $\overrightarrow{\boldsymbol{p}}$

Explain how your results relate to the
a) General Form of $\boldsymbol{L}$.
b) Vector Form of $\boldsymbol{L}$.


