INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 3.6]

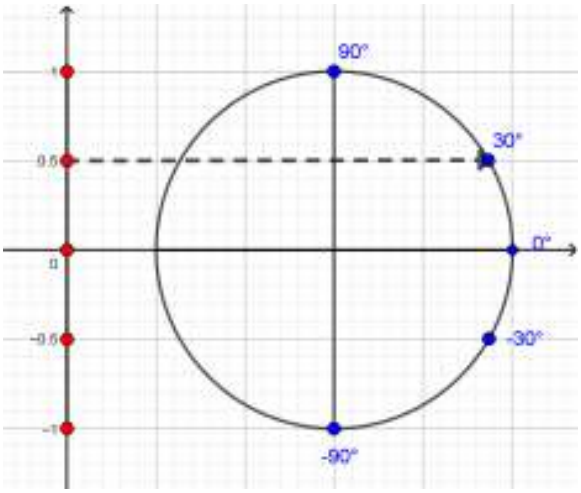
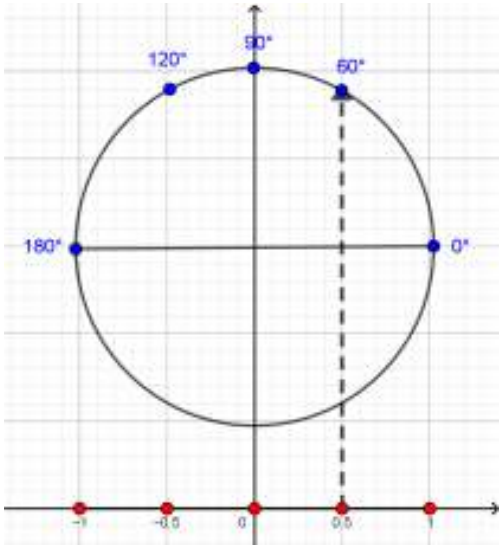
TRIGONOMETRIC EQUATIONS

Compiled by Christos Nikolaidis

O. Practice questions

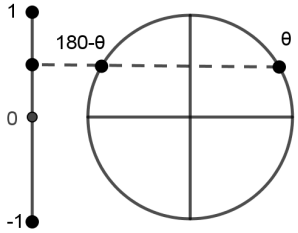
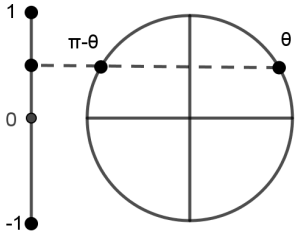
1. [Maximum mark: 8] ***[without GDC]***

Complete the following table with the values of $\sin^{-1} a$ and $\cos^{-1} a$ in **degrees**.

<table>
<tr><td rowspan="5"> 
</td><td>$\sin^{-1}(1) =$</td><td></td></tr>
<tr><td>$\sin^{-1}\left(\frac{1}{2}\right) =$</td><td>30°</td></tr>
<tr><td>$\sin^{-1}(0) =$</td><td></td></tr>
<tr><td>$\sin^{-1}\left(-\frac{1}{2}\right) =$</td><td></td></tr>
<tr><td>$\sin^{-1}(-1) =$</td><td></td></tr>
<tr><td rowspan="5"> 
</td><td>$\cos^{-1}(1) =$</td><td></td></tr>
<tr><td>$\cos^{-1}\left(\frac{1}{2}\right) =$</td><td>60°</td></tr>
<tr><td>$\cos^{-1}(0) =$</td><td></td></tr>
<tr><td>$\cos^{-1}\left(-\frac{1}{2}\right) =$</td><td></td></tr>
<tr><td>$\sin^{-1}(-1) =$</td><td></td></tr>
</table>

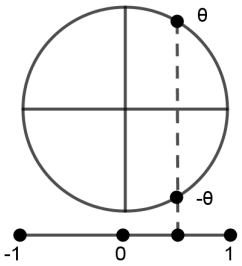
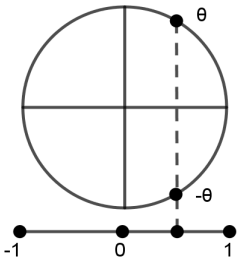
2. [Maximum mark: 28] ***[without GDC]***

Write down the **general solutions** of the equations below

$\sin x = a$	**in degrees**	**in radians**
first find $\theta = \sin^{-1} a$		
$\sin x = \frac{1}{2}$	$x = 30^\circ + 360^\circ k$ $x = 150^\circ + 360^\circ k$	$x = \frac{\pi}{6} + 2k\pi$ $x = \frac{5\pi}{6} + 2k\pi$
$\sin x = \frac{\sqrt{2}}{2}$		
$\sin x = \frac{\sqrt{3}}{2}$		
$\sin x = -\frac{1}{2}$	$x = -30^\circ + 360^\circ k$ $x = -150^\circ + 360^\circ k$	$x = -\frac{\pi}{6} + 2k\pi$ $x = -\frac{5\pi}{6} + 2k\pi$
$\sin x = -\frac{\sqrt{2}}{2}$		
$\sin x = -\frac{\sqrt{3}}{2}$		
$\sin x = 0$		
$\sin x = 1$		
$\sin x = -1$		

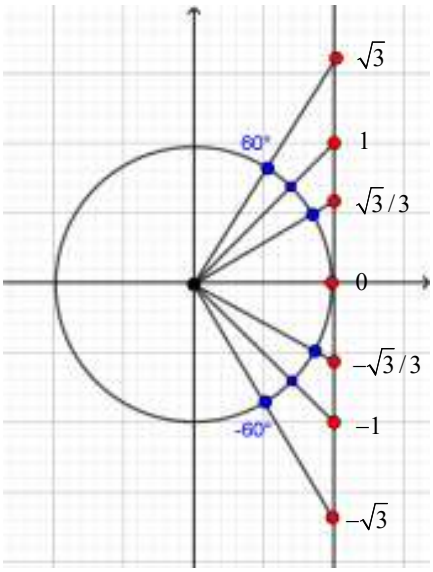
3. [Maximum mark: 28] ***[without GDC]***

Write down the **general solutions** of the equations below

$\cos x = a$	**in degrees**	**in radians**
first find $\theta = \cos^{-1} a$		
$\cos x = \frac{1}{2}$	$x = 60^\circ + 360^\circ k$ $x = -60^\circ + 360^\circ k$	$x = \frac{\pi}{3} + 2k\pi$ $x = -\frac{\pi}{3} + 2k\pi$
$\cos x = \frac{\sqrt{2}}{2}$		
$\cos x = \frac{\sqrt{3}}{2}$		
$\cos x = -\frac{1}{2}$	$x = 120^\circ + 360^\circ k$ $x = -120^\circ + 360^\circ k$	$x = \frac{2\pi}{3} + 2k\pi$ $x = -\frac{2\pi}{3} + 2k\pi$
$\cos x = -\frac{\sqrt{2}}{2}$		
$\cos x = -\frac{\sqrt{3}}{2}$		
$\cos x = 0$		
$\cos x = 1$		
$\cos x = -1$		

4. [Maximum mark: 6] ***[without GDC]***

Complete the following table with the values of $\tan^{-1} a$ in **degrees**.



$\tan^{-1}\left(\sqrt{3}\right) =$	$60°$
$\tan^{-1}(1) =$	
$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$	
$\tan^{-1} 0 =$	
$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) =$	
$\tan^{-1}(-1) =$	
$\tan^{-1}\left(-\sqrt{3}\right) =$	

5. [Maximum mark: 24] ***[without GDC]***

Write down the **general solutions** of the equations below

$\tan x = a$	**in degrees**	**in radians**
just find $\theta = \tan^{-1} a$	$x = \theta + 180^\circ k$	$x = \theta + k\pi$
$\tan x = \sqrt{3}$	$x = 60^\circ + 180^\circ k$	$x = \frac{\pi}{3} + k\pi$
$\tan x = 1$		
$\tan x = \frac{\sqrt{3}}{3}$		
$\tan x = 0$		
$\tan x = -\frac{\sqrt{3}}{3}$		
$\tan x = -1$		
$\tan x = -\sqrt{3}$		

6. [Maximum mark: 24] ***[with / without GDC]***

Solve the following equations, within the given domain.

	in degrees $-180° \le x \le 180°$	*in radians* $-\pi \le x \le \pi$
$\sin x = \frac{1}{2}$	$x = 30^\circ$, $x = 150^\circ$	$x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$
$\sin x = \frac{\sqrt{2}}{2}$		
$\sin x = \frac{\sqrt{3}}{2}$		
$\sin x = -\frac{1}{2}$	$x = -30^\circ$, $x = -150^\circ$	$x = -\frac{\pi}{6}$, $x = -\frac{5\pi}{6}$
$\sin x = -\frac{\sqrt{2}}{2}$		
$\sin x = -\frac{\sqrt{3}}{2}$		

7. [Maximum mark: 24] ***[with / without GDC]***

Solve the following equations, within the given domain.

	in degrees $-180° \le x \le 180°$	*in radians* $-\pi \le x \le \pi$
$\cos x = \frac{1}{2}$	$x = 60^\circ$, $x = -60^\circ$	$x = \frac{\pi}{3}$, $x = -\frac{\pi}{3}$
$\cos x = \frac{\sqrt{2}}{2}$		
$\cos x = \frac{\sqrt{3}}{2}$		
$\cos x = -\frac{1}{2}$	$x = 120^\circ$, $x = -120^\circ$	$x = \frac{2\pi}{3}$, $x = -\frac{2\pi}{3}$
$\cos x = -\frac{\sqrt{2}}{2}$		
$\cos x = -\frac{\sqrt{3}}{2}$		

8. [Maximum mark: 24] ***[with / without GDC]***

Solve the following equations, within the given domain.

	in degrees $0° \le x \le 360°$	*in radians* $0 \le x \le 2\pi$
$\sin x = 0$		
$\sin x = 1$		
$\sin x = -1$		
$\cos x = 0$		
$\cos x = 1$		
$\cos x = -1$		

9. [Maximum mark: 24] ***[with / without GDC]***

Solve the following equations, within the given domain.

	in degrees $-180° \le x \le 180°$	*in radians* $-\pi \le x \le \pi$
$\sin x = 0$		
$\sin x = 1$		
$\sin x = -1$		
$\cos x = 0$		
$\cos x = 1$		
$\cos x = -1$		

10. [Maximum mark: 15] ***[with / without GDC]***

Solve the following equations (in degrees).

(a) $\sin x = \frac{1}{2}, \quad 0° \le x \le 720°$ [5]

(b) $\sin 2x = \frac{1}{2}, \quad 0° \le x \le 360°$ [5]

(c) $\sin 3x = \frac{1}{2}, \quad 0° \le x \le 360°$ [5]

11. [Maximum mark: 15] ***[with / without GDC]***

Solve the following equations (in radians).

(a) $\sin x = \frac{1}{2}, \quad 0 \le x \le 4\pi$ [5]

(b) $\sin 2x = \frac{1}{2}, \quad 0 \le x \le 2\pi$ [5]

(c) $\sin 3x = \frac{1}{2}, \quad 0 \le x \le 2\pi$ [5]

12. [Maximum mark: 15] ***[with / without GDC]***

Solve the following equations (in degrees).

(a) $\cos x = \frac{1}{2}, \quad 0° \le x \le 720°$ [5]

(b) $\cos 2x = \frac{1}{2}, \quad 0° \le x \le 360°$ [5]

(c) $\cos 3x = \frac{1}{2}, \quad 0° \le x \le 360°$ [5]

13. [Maximum mark: 15] ***[with / without GDC]***

Solve the following equations (in radians).

(a) $\cos x = \frac{1}{2}, \quad 0 \le x \le 4\pi$ [5]

(b) $\cos 2x = \frac{1}{2}, \quad 0 \le x \le 2\pi$ [5]

(c) $\cos 3x = \frac{1}{2}, \quad 0 \le x \le 2\pi$ [5]

14. [Maximum mark: 36] ***[with / without GDC]***

Write down the solutions of the following equations, within the given domain.

	in degrees $0° \le x \le 360°$	*in radians* $0 \le x \le 2\pi$
$\tan x = \sqrt{3}$	$x = 60^\circ$, $x = 240^\circ$	$x = \frac{\pi}{3}$, $x = \frac{4\pi}{3}$
$\tan x = 1$		
$\tan x = \frac{\sqrt{3}}{3}$		
$\tan x = 0$		
$\tan x = -\frac{\sqrt{3}}{3}$		
$\tan x = -1$		
$\tan x = -\sqrt{3}$		

15. [Maximum mark: 36] ***[with / without GDC]***

Solve the following equations, within the given domain.

	in degrees $-180° \le x \le 180°$	*in radians* $-\pi \le x \le \pi$
$\tan x = \sqrt{3}$	$x = 60^\circ$, $x = -120^\circ$	$x = \frac{\pi}{3}$, $x = -\frac{2\pi}{3}$
$\tan x = 1$		
$\tan x = \frac{\sqrt{3}}{3}$		
$\tan x = 0$		
$\tan x = -\frac{\sqrt{3}}{3}$		
$\tan x = -1$		
$\tan x = -\sqrt{3}$		

16. [Maximum mark: 16] ***[with / without GDC]***

Solve the following equations (in degrees).

(a) $\tan 2x = 0$, $0° \le x \le 180°$ [4]

(b) $\tan 3x = 0$, $-90° \le x \le 90°$ [4]

(c) $\tan 3x = 1$, $-90° \le x \le 90°$ [4]

(d) $\tan 3x = -1$, $-90° \le x \le 90°$ [4]

17. [Maximum mark: 16] ***[with / without GDC]***

Solve the following equations (in radians).

(a) $\tan 2x = 0$, $0 \le x \le \pi$ [4]

(b) $\tan 3x = 0$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ [4]

(c) $\tan 3x = 1$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ [4]

(d) $\tan 3x = -1$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ [4]

18. [Maximum mark: 6] ***[with / without GDC]***

Solve the equation $\tan^2 x = 3$, for $-\pi \leq x \leq \pi$

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19. [Maximum mark: 5] ***[with / without GDC]***

Solve the equation $3\sin x = \sqrt{3}\cos x$, for $-\pi \leq x \leq \pi$

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20. [Maximum mark: 10] ***[without GDC]***

. Solve the equation, $2\sin^2 x = \sin x$,

(a) for $0° \le x \le 360°$ (in degrees). [5]

(b) for $0 \le x \le 2\pi$ (in radians). [5]

21*. [Maximum mark: 15] ***[without GDC]***

Solve the equation $3\sin^2 x = \cos^2 x$. $0° \le x \le 180°$ in three different ways

(a) by transforming to an equivalent expression of the form $\sin^2 x = a$. [5]

(b) by transforming to an equivalent expression of the form $\cos^2 x = b$. [5]

(c) by transforming to an equivalent expression of the form $\tan^2 x = c$. [5]

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A. Exam style questions (SHORT)

22. [Maximum mark: 6] ***[without GDC]***

Solve the equation $2\cos x = \sin 2x$, for $0 \le x \le 3\pi$.

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23. [Maximum mark: 6] ***[with / without GDC]***

Solve the equation $2\cos^2 x = \sin 2x$ for $0 \le x \le \pi$, giving your answers in terms of π.

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24. [Maximum mark: 5] ***[without GDC]***

Solve $2\sin x = \tan x$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

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25. [Maximum mark: 4] ***[without GDC]***

Solve the equation $\sqrt{3}\cos x = \sin x$, for x in the interval $0° \le x \le 360°$.

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26. [Maximum mark: 6] ***[without GDC]***

Let $f(x) = \sqrt{3}\mathrm{e}^{2x}\sin x + \mathrm{e}^{2x}\cos x$, for $0 \le x \le \pi$. Given that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

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27. [Maximum mark: 6] ***[without GDC]***

Solve $\tan^2 2\theta = 1$, in the interval $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

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28. [Maximum mark: 6] ***[without GDC]***

Solve the equation $\sin 2x = 2\sqrt{3}\cos^2 x$, in the interval $0 \le x \le 2\pi$.

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29. [Maximum mark: 6] ***[without GDC]***

(a) Express $2\cos^2 x + \sin x$ in terms of $\sin x$ only. [2]

(b) Solve the equation $2\cos^2 x + \sin x = 2$ for x in the interval $0 \le x \le \pi$. [4]

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30. [Maximum mark: 6] ***[without GDC]***

Consider the trigonometric equation $2\sin^2 x = 1 + \cos x$.

(a) Write this equation in the form $f(x) = 0$, where $f(x) = a\sin^2 x + b\sin x + c$, and $a, b, c \in \mathbb{Z}$. [2]

(b) Factorize $f(x)$ [1]

(c) Solve $f(x) = 0$ for $0° \le x \le 360°$. [3]

31. [Maximum mark: 7] ***[with / without GDC]***

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

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32. [Maximum mark: 7] ***[without GDC]***

(a) Show that $4 - \cos 2\theta + 5\sin\theta \equiv 2\sin^2\theta + 5\sin\theta + 3$. [2]

(b) **Hence**, solve the equation $4 - \cos 2\theta + 5\sin\theta = 0$ for $0 \le \theta \le 2\pi$. [5]

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33. [Maximum mark: 6] ***[without GDC]***

It is given that $\tan^{-1} 0.6 = 31°$ to the nearest degree. Solve the equation

$$3\cos x = 5\sin x, \text{ for } x \text{ in the interval } 0° \le x \le 360°,$$

writing the solutions to the nearest degree.

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34. [Maximum mark: 6] ***[without GDC]***

Given that $\sin^{-1}\frac{2}{3} = 42°$ to the nearest degree, solve the equation

$$3\sin^2 x - 11\sin x + 6 = 0 \quad \text{for } 0° \le x \le 180°.$$

writing the solutions to the nearest degree.

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35. [Maximum mark: 6] ***[with GDC]***

(a) Write the expression $3\sin^2 x + 4\cos x$ in the form $a\cos^2 x + b\cos x + c$. [2]

(b) Find the two values of $\cos x$ that satisfy the equation $3\sin^2 x + 4\cos x - 4 = 0$. [2]

(c) Hence or otherwise, solve the equation

$$3\sin^2 x + 4\cos x - 4 = 0,\ 0° \le x \le 90°$$ [2]

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36. [Maximum mark: 6] ***[with GDC]***

Solve the equations

(a) $\cos 3x = \cos(0.5x)$, for $0 \le x \le \pi$. [2]

(b) $3\sin^2 x - 11\sin x + 6 = 0$ for $0° \le x \le 180°$. [2]

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B. Exam style questions (LONG)

37. [Maximum mark: 15] ***[without GDC]***

Solve the following equations for x in the interval $[0, 2\pi]$.

(a) $\sin^3 x = \sin x$. [5]

(b) $2\sin^3 x = \sin x$. [5]

(c) $4\cos^3 x = \cos x$. [5]

38*. [Maximum mark: 15] ***[without GDC]***

Consider the functions $f(x) = \cos 2x + \cos x + 1$ and $g(x) = \sin 2x + \sin x$, both defined in the interval $0 \le x \le \pi$.

(a) Find the x-intercepts of $f(x)$. [5]

(b) Find the x-intercepts of $g(x)$. [4]

(c) Find the points of intersection of the two functions. [6]

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39. [Maximum mark: 10] ***[without GDC]***

Consider the equation $3\cos 2x + \sin x = 1$.

(a) Write this equation in the form $f(x) = 0$, where $f(x) = p\sin^2 x + q\sin x + r$, and $p, q, r \in \mathbb{Z}$. [2]

(b) Factorize $f(x)$. [3]

(c) Write down the number of solutions of $f(x) = 0$, for $0 \le x \le 2\pi$. [2]

(d) Find the two solutions in the 3rd and 4th quadrant, that is for $\pi \le x \le 2\pi$ [3]