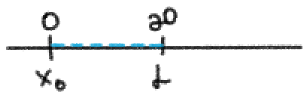
Sección 3.4

3 $m = 3\,kg$ $\quad x_0 = 0 \quad V_0 = -10\,m/s$

$L = 20\,cm$

$F = 15\,N$ $\quad A = ? \quad T = ? \quad w_0 = ?$

No hay amortiguador
por lo tanto $C = 0$



$\boxed{K = F/L}$ ⟶ Ley de Hooke

$K = 15N/0{,}2m$

$K = 75N/m$

$mx'' + \cancel{cx'} + Kx = F$

$\boxed{3x'' + 75x = 0}$

Homogénea

$w_0 = \sqrt{\frac{K}{m}}$

$T = \frac{2\pi}{w_0} = \frac{2\pi}{5} = 1{,}257\,s$

$w_0 = \sqrt{\frac{75}{3}} = \sqrt{25} = 5\,rad/s$

$x'' + w_0^2 x = 0$

$x'' + 25x = 0$

$\boxed{x(t) = A\cos w_0 t + B\,\mathrm{Sen}\,w_0 t}$

Sln General

$\boxed{x'(t) = -w_0 A\,\mathrm{Sen}\,w_0 t + w_0 B\cos w_0 t}$

$x(0) = A\,\xcancel{\cos 5(0)}^{1} + B\,\xcancel{\mathrm{Sen}\,5(0)}^{0}$

$A = 0\,m$

$x'(0) = \xcancel{-5(0)\,\mathrm{Sen}\,5(0)}^{0} + 5B\,\xcancel{\cos 5(0)}^{1}$

$-10\,m/s = 5B$

$B = -2\,m/s$

$\boxed{x(t) = -2\,\mathrm{Sen}\,5t}$ ⟶ Sln Particular

4. $m = 250g$ $\quad X_0 = 1m$ $\quad V_0 = -5 m/s$

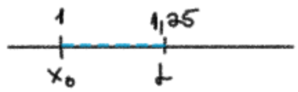
$0{,}25 Kg$

$L = 25 cm$

$0{,}25 m$

$F = 9N$

No hay amortiguador por lo tanto $C=0$



$\boxed{K = F/L}$ → Ley de Hooke

$K = 9\, N/0{,}25m$

$K = 36\, N/m$

$mx'' + \cancel{Cx'} + Kx = F$

$0{,}25x'' + 36x = 0$

Homogénea

$0{,}25r^2 + 36 = 0$

$r^2 = -144$

$r = \pm 12i$

$r = a \pm bi$

$a = 0 \quad b = 12$

$\boxed{x(t) = A\cos w_0 t + B \operatorname{Sen} w_0 t}$

Sln General

$\boxed{x'(t) = -w_0 A \operatorname{Sen} w_0 t + w_0 B \cos w_0 t}$

$T = \frac{2\pi}{12} = \frac{\pi}{6} s$

$x(t) = e^{at}(C_1 \cos bt + C_2 \operatorname{Sen} bt)$

$x(t) = \cancel{e^{0t}}^{\,1}(C_1 \cos 12t + C_2 \operatorname{Sen} 12t)$

$x(0) = C_1 \cancel{\cos 12(0)}^{\,1} + \cancel{C_2 \operatorname{Sen} 12(0)}^{\,0}$

$1 = C_1 = A$

$x'(t) = -12C_1 \operatorname{Sen} 12t + 12C_2 \cos 12t$

$x'(0) = \cancel{-12C_1 \operatorname{Sen} 12(0)}^{\,0} + 12C_2 \cancel{\cos 12(0)}^{\,1}$

$-5 = 12C_2$

$C_2 = -\frac{5}{12} = B$

$\alpha = 2\pi + \tan^{-1}(B/A)$

$\alpha = 2\pi + \tan^{-1}(-(5/12)/1) = 5{,}89$

$C = \sqrt{A^2 + B^2} = \sqrt{1^2 + (-5/12)^2} = \frac{13}{12}$

$x(t) = \underbrace{\frac{13}{12}}_{\text{Amplitud}} \cos(12t - 5{,}89)$

13

$m = 10 \qquad x(0) = 0$

$c = 9 \qquad x'(0) = 5$

$K = 2$

$mx'' + cx' + Kx = F$

$10x'' + 9x' + 2x = 0$

$10r^2 + 9r + 2 = 0$

$(5r+2)(2r+1) = 0$

$5r+2 = 0 \qquad 2r+1 = 0$

$r_1 = -\frac{2}{5} \qquad r_2 = -\frac{1}{2}$

$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

$x(t) = C_1 e^{-2t/5} + C_2 e^{-t/2}$

$x(0) = C_1 \cancel{e^{-2(0)/5}} + C_2 \cancel{e^{-(0)/2}}$

$0 = C_1 + C_2 \longrightarrow C_1 = -C_2$

$x'(t) = -\frac{2}{5} C_1 e^{-2t/5} - \frac{1}{2} C_2 e^{-t/2}$

$x'(0) = -\frac{2}{5} C_1 \cancel{e^{-2(0)/5}} - \frac{1}{2} C_2 \cancel{e^{-(0)/2}}$

$5 = -\frac{2}{5} C_1 - \frac{1}{2} C_2$

Reemplazar

$5 = \frac{2}{5} C_2 - \frac{1}{2} C_2$

$5 = -\frac{1}{10} C_2$

$x(t) = 50(e^{-2t/5} - e^{-t/2})$

$C_2 = -50$

$C_1 = 50$

$x'(t) = -20 e^{-2t/5} + 25 e^{-t/2}$

$5e^{-\frac{4t}{10}}(5e^{-t/10} - 4) = 0$

$\underbrace{5e^{-2t/5} = 0}_{\text{No cumple Propiedades}} \qquad 5e^{-t/10} - 4 = 0$

$e^{-t/10} = 4/5$

$-t/10 = \ln|4/5|$

$t = -10 \ln|\frac{4}{5}| = 2,23$

$x(2,23) = 50 e^{-2(2,23)/5} - 50 e^{-2,23/2}$

$x(t) = 4,096$

14

$m = 25 \qquad x(0) = 20 \qquad x'(0) = 41$

$C = 10$

$K = 226$

$mx'' + cx' + Kx = F$

$25x'' + 10x' + 226x = 0$

$25r^2 + 10r + 226 = 0$

$$r = \frac{-10 \pm \sqrt{100^2 - 4(25)(226)}}{2(25)}$$

$r = -\frac{1}{5} \pm 3i \qquad r = a \pm bi$

$a = -\frac{1}{5} \quad b = 3$

$x(t) = e^{at}(C_1 \text{Cos}\, bt + C_2 \text{Sen}\, bt)$

$x(t) = e^{-t/5}(C_1 \text{Cos}\, 3t + C_2 \text{Sen}\, 3t)$

$x(0) = \cancelto{1}{e^{-(0)/5}}(C_1 \cancelto{1}{\text{Cos}\, 3(0)} + C_2 \cancelto{0}{\text{Sen}\, 3(0)})$

$20 = C_1$

$x'(t) = -\frac{1}{5}e^{-t/5}(C_1 \text{Cos}\, 3t + C_2 \text{Sen}\, 3t) + e^{-t/5}(-3C_1 \text{Sen}\, 3t + 3C_2 \text{Cos}\, 3t)$

$x'(t) = e^{-t/5}\left(-\frac{1}{5}20 \text{Cos}\, 3t - \frac{1}{5}C_2 \text{Sen}\, 3t - 3(20)\text{Sen}\, 3t + 3C_2 \text{Cos}\, 3t\right)$

$41 = \cancelto{1}{e^{-0/5}}\left(-\frac{1}{5}20 \cancelto{1}{\text{Cos}\, 3(0)} - \cancelto{0}{\frac{1}{5}C_2 \text{Sen}\, 3(0)} - \cancelto{0}{3(20)\text{Sen}\, 3(0)} + 3C_2 \cancelto{1}{\text{Cos}\, 3(0)}\right)$

$$41 = -4 + 3C_2$$

$$C_2 = \frac{45}{3} = 15$$

$$x(t) = e^{-t/5}(20 \operatorname{Cos} 3t + 15 \operatorname{Sen} 3t)$$

$$x(t) = A \operatorname{Cos} w_0 t + B \operatorname{Sen} w_0 t$$

$$A = e^{-t/5} 20 \qquad B = e^{-t/5} 15 \qquad C^2 = A^2 + B^2$$

$$C^2 = e^{-2t/5}(20)^2 + e^{-2t/5}(15)^2$$

$$C = \sqrt{e^{-\cancel{2}t/5}\ 625} = 25\, e^{-t/5}$$

$$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$$

$$\alpha = \tan^{-1}\left(\frac{15}{20}\right) = 0{,}644 \text{ rad}$$

$$x(t) = C \operatorname{Cos}(w_0 t - \alpha)$$

$$x(t) = 25 e^{-t/5} \operatorname{Cos}(3t - 0{,}644)$$

22

$w = 12\,lb$ $\qquad$ $x(0) = 1\,ft \quad x'(0) = 0$

$L = 6\,in = 0{,}5\,ft$ $\qquad$ $w_0 = ?$

$C = 3\,lb$ $\qquad$ $A = ?$

$\alpha = ?$

$m = w/g$ $\qquad$ $K = w/L$

$m = \frac{12\,lb}{32\,ft/s^2} = 0{,}375\,slugs$ $\qquad$ $K = 12\,lb/0{,}5\,ft = 24\,lb/ft$

$$mx'' + Cx' + Kx = 0$$

$$0{,}375x'' + 3x' + 24x = 0$$
$$0{,}375r^2 + 3r + 24 = 0$$
$$r^2 + 8r + 64 = 0$$

$$r = \frac{-8 \pm \sqrt{8^2 - 4(64)}}{2} \qquad r = -4 \pm 4\sqrt{3}\,i \qquad r = a \pm bi$$

$$a = -4 \qquad b = 4\sqrt{3} = w_0$$

$$x(t) = e^{at}(C_1 \cos bt + C_2 \operatorname{Sen} bt)$$

$$x(t) = e^{-4t}(C_1 \cos 4\sqrt{3}\,t + C_2 \operatorname{Sen} 4\sqrt{3}\,t)$$

$$x(0) = \cancelto{1}{e^{-4(0)}}(C_1 \cancelto{1}{\cos 4\sqrt{3}(0)} + C_2 \cancelto{0}{\operatorname{Sen} 4\sqrt{3}(0)})$$

$C_1 = 1\,ft$ $\qquad$ $A = e^{-4t}$

$$x'(t) = -4e^{-4t}(C_1 \cos 4\sqrt{3}\,t + C_2 \operatorname{Sen} 4\sqrt{3}\,t) + e^{-4t}(-4\sqrt{3}\,C_1 \operatorname{Sen} 4\sqrt{3}\,t + 4\sqrt{3}\,C_2 \cos 4\sqrt{3}\,t)$$

$$x'(0) = -4\cancelto{1}{e^{-4(0)}}(C_1 \cancelto{1}{\cos 4\sqrt{3}(0)} + C_2 \cancelto{0}{\operatorname{Sen} 4\sqrt{3}(0)}) + \cancelto{1}{e^{-4(0)}}(\cancelto{0}{-4\sqrt{3}\,C_1 \operatorname{Sen} 4\sqrt{3}(0)} + 4\sqrt{3}\,C_2 \cancelto{1}{\cos 4\sqrt{3}(0)})$$

$$0 = -4 + 4\sqrt{3}\,C_2$$

$C_2 = \frac{\sqrt{3}}{3}$ $\qquad$ $B = e^{-4t}\frac{\sqrt{3}}{3}$

$$\alpha = \tan^{-1}(B/A) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$C = \sqrt{A^2 + B^2} = e^{-4t}\frac{2}{3}\sqrt{3}$$

$$x(t) = e^{-4t}\frac{2}{3}\sqrt{3}\cos\left(4\sqrt{3}\,t - \frac{\pi}{6}\right)$$

(23) $W = 3200\ lb$

$$m = \frac{W}{g} = \frac{3200\,lb}{32\,ft/s^2} = 100\ slugs$$

(a) $f_0 = 80\ \text{ciclos/min} = \frac{4}{3}\ Hz$

$$w_0 = f_0 2\pi = \frac{4}{3} 2\pi\ rad/s$$

$$w_0 = \sqrt{\frac{K}{m}} \longrightarrow K = m w_0^2 = 100 \cdot \left(\frac{8\pi}{3}\right)^2 = 7018\ lb/ft$$

(b) $f_1 = 78\ \text{ciclos/min} = 1{,}3\ Hz$

$$w_1 = f_1 2\pi = 2{,}6\pi\ rad/s$$

$$w_1 = \sqrt{\frac{K}{m}} \longrightarrow K = m w_1^2 = 100(2{,}6\pi)^2 = 6671\ lb/ft$$

$$w_1 = \sqrt{w_0^2 - \rho^2} \qquad \rho = \frac{c}{2m}$$

$$w_1^2 = w_0^2 - \frac{c^2}{4m^2}$$

$$(2{,}6\pi)^2 = \left(\frac{8}{3}\pi\right)^2 - \frac{c^2}{40000}$$

$$\sqrt{c^2} = \sqrt{\left(\left(\frac{8}{3}\pi\right)^2 - (2{,}6\pi)^2\right) 40000} \approx 372{,}31\ \frac{lb}{ft/s}$$

$$\rho = \frac{372{,}31}{2(100)} = 1{,}862$$

$$e^{-\rho t} = 1\%$$

$$-\rho t = \ln|0{,}01|$$

$$t = \frac{\ln|0{,}01|}{-1{,}862} = 2{,}473\ s$$

Sección 3.5

5 $y'' + y' + y = \text{Sen}^2 x$

$y'' + y' + y = 0$ Homogenea

$r^2 + r + 1 = 0$

$r = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$r = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$r = a \pm bi$

$a = -\frac{1}{2} \quad b = \frac{\sqrt{3}}{2}$

$y_h = e^{ax}(A\text{Cos}\, bx + B\text{Sen}\, bx)$

$y_h = e^{-x/2}(A\text{Cos}\frac{\sqrt{3}}{2}x + B\text{Sen}\frac{\sqrt{3}}{2}x)$

$r(x) = \text{Sen}^2 x$

$r'(x) = 2\text{Sen}x\,\text{Cos}x = \text{Sen}\, 2x$

$r''(x) = 2\text{Cos}\, 2x$

$y_p = A\text{Sen}\, 2x + B\,\text{Cos}\, 2x + C$

$y_p' = 2A\,\text{Cos}\, 2x - 2B\text{Sen}\, 2x$

$y_p'' = -4A\text{Sen}\, 2x - 4B\text{Cos}\, 2x$

$-4A\text{Sen}\, 2x - 4B\text{Cos}\, 2x + 2A\text{Cos}\, 2x - 2B\text{Sen}\, 2x + A\text{Sen}\, 2x + B\text{Cos}\, 2x + C = \text{Sen}^2 x$

$-3A\text{Sen}\, 2x - 3B\text{Cos}\, 2x + 2A\text{Cos}\, 2x - 2B\text{Sen}\, 2x + C = \text{Sen}^2 x$

$(2A-3B)\text{Cos}\, 2x - (3A+2B)\text{Sen}\, 2x + C = \text{Sen}^2 x$

$(2A-3B) - (2A-3B)2\text{Sen}^2 x - (3A+2B)\text{Sen}\, 2x + C = \text{Sen}^2 x$

$2A - 3B + C = 0 \longrightarrow 2(-\frac{1}{13}) - 3(\frac{3}{26}) = -C \longrightarrow C = \frac{1}{2}$

$4A - 6B = -1 \longrightarrow -\frac{8}{3}B - 6B = -1 \longrightarrow B = \frac{3}{26}$

$3A + 2B = 0 \longrightarrow A = -\frac{2}{3}B$

$= -\frac{2}{\cancel{3}}(\frac{3}{26}) = -\frac{1}{13}$

$y_p = -\frac{1}{13}\text{Sen}\, 2x + \frac{3}{26}\text{Cos}\, 2x + \frac{1}{2}$

19 $y^{(5)} + 2y^{(3)} + 2y'' = 3x^2 - 1$

$r^5 + 2r^3 + 2r^2 = 0$

$r^2(r^3 + 2r + 2) = 0$

$r_1 = 0 \quad r_2 = 0$

$r^3 + 2r + 2 = 0$

$r_3 = -0{,}77$

$y_h = (c_1 + c_2 x)\cancel{e^{0x}}^{\,1} + c_3 e^{-0{,}77x}$

$= c_1 + c_2 x + c_3 e^{-0{,}77x}$

$y_p = (A + Bx + Cx^2)x^2 = Ax^2 + Bx^3 + Cx^4$

$y_p' = 2Ax + 3Bx^2 + 4Cx^3$

$y_p'' = 2A + 6Bx + 12Cx^2$

$y_p^{(3)} = 6B + 24Cx$

$y_p^{(4)} = 24C$

$y_p^{(5)} = 0$

$(0) + 2(6B + 24Cx) + 2(2A + 6Bx + 12Cx^2) = 3x^2 - 1$

$12B + 48Cx + 4A + 12Bx + 24Cx^2 = 3x^2 - 1$

$24Cx^2 + (48C + 12B)x + (12B + 4A) = 3x^2 - 1$

$24C = 3 \longrightarrow C = \frac{1}{8}$

$48C + 12B = 0 \longrightarrow B = -\frac{1}{2}$

$12B + 4A = -1 \longrightarrow A = -\frac{5}{2}$

$y_p = \frac{5}{4}x^2 - \frac{1}{2}x^3 + \frac{1}{8}x^4$

25 $y'' + 3y' + 2y = x(e^{-x} - e^{-2x})$

$r^2 + 3r + 2 = 0$

$(r+2)(r+1) = 0$

$r_1 = -2 \quad r_2 = -1$

$y_h = c_1 e^{-2x} + c_2 e^{-x}$

$y_p = Ae^{-2x} + Bxe^{-2x} + Ce^{-x} + Dxe^{-x}$

$y_p = Axe^{-2x} + Bx^2e^{-2x} + Cxe^{-x} + Dx^2e^{-x}$

(39) $y^{(3)} + y'' = x + e^{-x}$; $y(0) = 1$, $y'(0) = 0$, $y''(0) = 1$

$r^3 + r^2 = 0$

$r^2(r+1) = 0$

$r_1 = 0 \quad r_2 = 0 \quad r_3 = -1$

$y_h = (C_1 + C_2 x)\cancel{e^{0x}}^{1} + C_3 e^{-x}$

$y_h = C_1 + C_2 x + C_3 e^{-x}$

$y_p = A + Bx + Ce^{-x}$

$y_p = Ax^2 + Bx^3 + Cxe^{-x}$

$y_p' = 2Ax + 3Bx^2 + Ce^{-x} - Cxe^{-x}$

$y_p'' = 2A + 6Bx - Ce^{-x} - Ce^{-x} + Cxe^{-x}$

$y_p^{(3)} = 6B + 2Ce^{-x} + Ce^{-x} - Cxe^{-x}$

$6B + \cancel{3}^{1}Ce^{-x} \cancel{- Cxe^{-x}} + 2A + 6Bx \cancel{- 2Ce^{-x}} + \cancel{Cxe^{-x}} = x + e^{-x}$

$(6B + 2A) + 6Bx + Ce^{-x} = x + e^{-x}$

$6B + 2A = 0$

$6B = 1$

$C = 1$

$A = -\frac{1}{2}$

$B = \frac{1}{6}$

$C = 1$

$y_p = -\frac{1}{2}x^2 + \frac{1}{6}x^3 + xe^{-x}$

$$y = C_1 + C_2x + C_3e^{-x} - \frac{1}{2}x^2 + \frac{1}{6}x^3 + xe^{-x}$$

$$1 = C_1 + \cancel{C_2(0)} + C_3\cancel{e^0}^{1} - \cancel{\frac{1}{2}(0)} + \cancel{\frac{1}{6}(0)} + \cancel{(0)e^0}$$

$$1 = C_1 + C_3$$

$$y' = C_2 - C_3e^{-x} - x + \frac{1}{2}x^2 + e^{-x} - xe^{-x}$$

$$0 = C_2 - C_3\cancel{e^0}^{1} - 0 + \cancel{\frac{1}{2}(0)} + \cancel{e^0}^{1} - \cancel{(0)e^0}$$

$$0 = C_2 - C_3 + 1 \longrightarrow -1 = C_2 - C_3$$

$$y'' = C_3e^{-x} - 1 + x - e^{-x} - e^{-x} + xe^{-x}$$

$$1 = C_3\cancel{e^0}^{0} - 1 + 0 - \cancel{e^0}^{1} - \cancel{e^0}^{1} + \cancel{(0)e^0}$$

$$1 = C_3 - 1 - 1 - 1 \longrightarrow C_3 = 4$$

$C_1 = -3$

$C_2 = 3$

$C_3 = 4$

$$y = -3 + 3x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + 4e^{-x} + xe^{-x}$$