1084. b) $133 \mid \underbrace{11^{n+2} + 12^{2n+1}}_{a_n} za\ n \geq 0$

1) n=0

$a_n = 11^2 + 12 = 133$

$\rightarrow 133 \mid a_0 = 133$

2) $\underbrace{133 \mid a_n = 11^{n+2} + 12^{2n+1}}$

indukcijska hipoteza

$\rightarrow 133 \mid a_{n+1} = 11^{n+3} + 12^{2n+3}$

3) $a_{n+1} = 11^{n+2} \cdot 11 + 12^{2n+1} \cdot 12^2 = 11 \cdot 11^{n+2} + 144 \cdot 12^{2n+1} =$

$= 11 \cdot 11^{n+2} + 11 \cdot 12^{2n+1} + 133 \cdot 12^{2n+1} = 11 \underbrace{(11^{n+2} + 12^{2n+1})} + 133 \cdot 12^{2n+1}$

po indukcijskoj hipotezi

$\rightarrow 133 \mid a_{n+1}$

1084. d) $17 \mid \underbrace{6^{2n} + 19^n - 2^{n+1}}_{a_n} za\ n \geq 0$

1) $n = 0$

$a_0 = 1 + 1 - 2 = 0$

$\rightarrow a_0 = 0$

2) $\underbrace{17 \mid a_n = 6^{2n} + 19^n - 2^{n+1}}$

indukcijska hipoteza

$\rightarrow 17 \mid a_{n+1} = 6^{2n+2} + 19^{n+1} - 2^{n+2}$

3) $a_{n+1} = 36 \cdot 6^{2n} + 19 \cdot 19^n - 2 \cdot 2^{n+1}$

$a_{n+1} = 2 \cdot 6^{2n} + 34 \cdot 6^{2n} + 2 \cdot 19^n + 17 \cdot 19^n - 2 \cdot 2^{n+1}$

$= 2 \underbrace{(6^{2n} + 19^n - 2^{n+1})} + 17(2 \cdot 6^{2n} + 19^n)$

po indukcijskoj hipotezi

$\rightarrow 17 \mid a_{n+1}$

1084. đ) $11 \mid \underbrace{30^n + 4^n(3^n - 2^n) - 1}_{a_n}$

$\rightarrow a_n = 30^n + 12^n - 8^n - 1$

1) n=0

$a_0 = 1 + 1 - 1 - 1 = 0$

$\rightarrow 11 \mid a_0 = 0$

2) $\underbrace{11 \mid a_n = 30^n + 12^n - 8^n - 1}_{\text{indukcijska hipoteza}}$

$\rightarrow 11 \mid a_{n+1} = 30^{n+1} + 12^{n+1} - 8^{n+1} - 1$

3) $a_{n+1} = 30 \cdot 30^n + 12 + 12^n - 8 \cdot 8^n - 1 =$

$= 30^n + 29 \cdot 30^n + 12^n + 11 \cdot 12^n - 8^n - 7 \cdot 8^n - 1 =$

$= \underbrace{(30^n + 12^n - 8^n - 1)}_{\text{po indukcijskoj hipotezi}} + 29 \cdot 30^n + \mathbf{11 \cdot 12^n} - 7 \cdot 8^n$

DELJIVO JE

$\rightarrow 11 \mid a_{n+1}$

- potrebno je dokazati da je $11 \mid 29 \cdot 30 \cdot 30^n - 7 \cdot 8^n \rightarrow a`_0 = 29 \cdot 30^n - 7 \cdot 8^n$

  1) n=0

  $a`_0 = 29 - 7 = 22$

  $\rightarrow 11 \mid a`_0 = 22$

  2) $\underbrace{11 \mid a`_0 = 29 \cdot 30^n - 7 \cdot 8^n}_{\text{indukcijska hipoteza}}$

  $\rightarrow 11 \mid a`_{n+1} = 29 \cdot 30^{n+1} - 7 \cdot 8^{n+1}$

  3) $a`_{n+1} = 29 \cdot 30 \cdot 30^n - 7 \cdot 8 \cdot 8^n =$

  $= 29 \cdot 8 \cdot 30^n + 29 \cdot 22 \cdot 30^n - 7 \cdot 8 \cdot 8^n =$

  $= 8 \underbrace{(29 \cdot 30^n - 7 \cdot 8^n)}_{\text{po indukcijskoj hipotezi}} + \mathbf{29 \cdot 22 \cdot 30^n}$

  DELJIVO JE