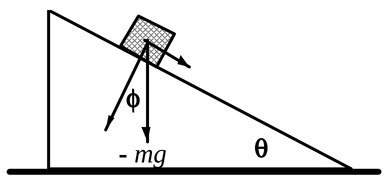
EQUATION SUMMARY: INCLINED PLANE; FORCE PUSHING; FORCE PULLING

INCLINED PLANE

This situation amounts to a rotated coordinate system, compared to the usual horizontal *x* and vertical *y*. That is, we can choose our coordinate system to be along the "ramp" (the inclined plane), so that *x* is now measured along the ramp and *y* is perpendicular to it. These problems can be solved using our usual horizontal/vertical system, but the math becomes complicated. So we can simplify things by a good choice of coordinate system- that is, along the ramp.

The key is resolving the gravitational force into components along (parallel) and normal to (perpendicular) the ramp surface. Gravitation always acts in a direction toward the center of the earth, so that when an object sits on an inclined plane, the vector representing the weight of the object is tilted with respect to what is now our x-axis (the ramp).



The force that is directed **down** the ramp is *mgsin(θ)*, and the force **into** the ramp is *-mgcos(θ)*. This can be proved in several ways.

(1) Common sense: if the angle *θ* is zero, the ramp is horizontal, and there will be no force acting to move the object along the ramp. As the angle increases toward 90 degrees, there will be an increasing force until at 90 degrees all of the weight acts straight downward. The trig function that is zero at zero degrees and is unity at 90 degrees is the sine, so the force along the ramp must be *mgsin(θ)*.

(2) Geometry: a careful drawing of the angles will reveal that the angle ϕ between the gravitational (weight) vector and the vector component into the ramp is the same as the ramp angle *θ*. The sine and cosine components along and into the ramp follow directly from this equality.

(3) Rotation matrix: a more advanced math technique, this shows that the forces can be written

$$\begin{pmatrix} F_{\text{along ramp}} \\ F_{\text{into ramp}} \end{pmatrix} = \begin{pmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix} \begin{pmatrix} 0 \\ -mg \end{pmatrix} = \begin{pmatrix} mg\sin(\theta) \\ -mg\cos(\theta) \end{pmatrix} \tag{1}$$

The last proof starts with zero force in the "x" direction (which is our usual true horizontal) and the negative weight *-mg* in the vertical direction, and the rotation creates the indicated forces along and into (normal to) the ramp. The ramp coordinate system is rotated *downward* from the horizontal, thus the -*θ*.

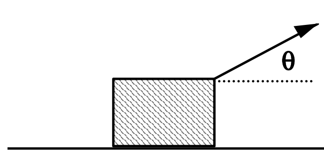
Now we can write the following expressions (remember, *x* is along the ramp, *y* is normal to it):

Force along the ramp:	$m\,g\,\sin(\theta)$
Force into the ramp:	$-m\,g\,\cos(\theta)$
Normal force:	$m\,g\,\cos(\theta)$
Friction force:	$-\mu\,m\,g\,\cos(\theta)$

$$\sum F_y = 0 \qquad \sum F_x = \pm F_{\text{applied}} + m\,g\sin(\theta) - \mu\,m\,g\cos(\theta) = m\,a_x \tag{2}$$

The applied force could be in either direction, but is parallel to the ramp; usually we take the direction of motion to be positive down the ramp, so that the gravity term is positive.

FORCE PULLING AT AN ANGLE



In these problems we have an object on a horizontal surface, and a force *F* is applied at an angle, pulling (away from) the object. We usually take this force as acting to the right, which will be the positive x direction. The key here is that there will be a component of this applied force that acts vertically, so that we have

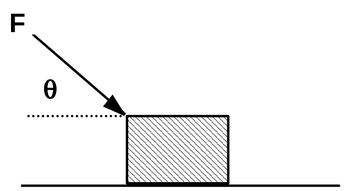
$$\sum F_y = -mg + F_{\text{Normal}} + F\sin(\theta) = m\,a_y = 0 \tag{3}$$

If we solve this for the normal force then we can write for the x-direction

$$\sum F_x = F\cos(\theta) - \mu F_{\text{Normal}} = F\cos(\theta) - \mu\left[mg - F\sin(\theta)\right] = m\,a_x \tag{4}$$

Note that the vertical component of *F* has in effect reduced the normal and friction forces. In the limit of a pull equal in magnitude to the weight *mg* at 90 degrees, the friction force is zero, as is the normal force. In that case, the horizontal acceleration is zero!

FORCE PUSHING AT AN ANGLE



These problems are formulated in the same way, except that the force *F* is pushing downward (that is, it is directed toward the object being moved). Then we have:

$$\sum F_y = -mg + F_{\text{Normal}} - F\sin(\theta) = m\,a_y = 0 \tag{5}$$

and so

$$\sum F_x = F\cos(\theta) - \mu F_{\text{Normal}} = F\cos(\theta) - \mu\left[mg + F\sin(\theta)\right] = m\,a_x \tag{6}$$

Now the force pushing downward has in effect increased the normal and friction forces.

SUMMARY

$$\sum F_x = \pm F_{\text{applied}} + m\,g\sin(\theta) - \mu\,m\,g\cos(\theta) = m\,a_x$$ **inclined plane**

$$\sum F_x = F\cos(\theta) - \mu\left[mg - F\sin(\theta)\right] = m\,a_x$$ **force pulling**

$$\sum F_x = F\cos(\theta) - \mu\left[mg + F\sin(\theta)\right] = m\,a_x$$ **force pushing**