Onbepaalde integraal

www.karelappeltans.be

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1 Basis integralen

Lijst met fundamentele (basis) integralen

$$\int 0dx = c$$

$$\int 1dx = x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \; (n \in \mathbb{R}\backslash\{-1\})$$

$$\int \frac{1}{x}\, dx = ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \frac{1}{\cos^2 x}\, dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x}\, dx = -\cot x + c$$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = Arcsinx + c = -Arccosx + c$$

$$\int \frac{1}{1+x^2}\, dx = Arctanx + c$$

$$\int \frac{1}{k+x^2}\, dx = \frac{1}{\sqrt{k}} Arctan\left(\frac{x}{\sqrt{k}}\right) + c \;\; (k > 0)$$

$$\int \frac{1}{\sqrt{k-x^2}}\, dx = Arcsin\left(\frac{x}{\sqrt{k}}\right) + c \;\; (k > 0)$$

Figure 1: `https://www.geogebra.org/m/RMwPMmq7`

2 Splitsing

2.1 Voorbeelden

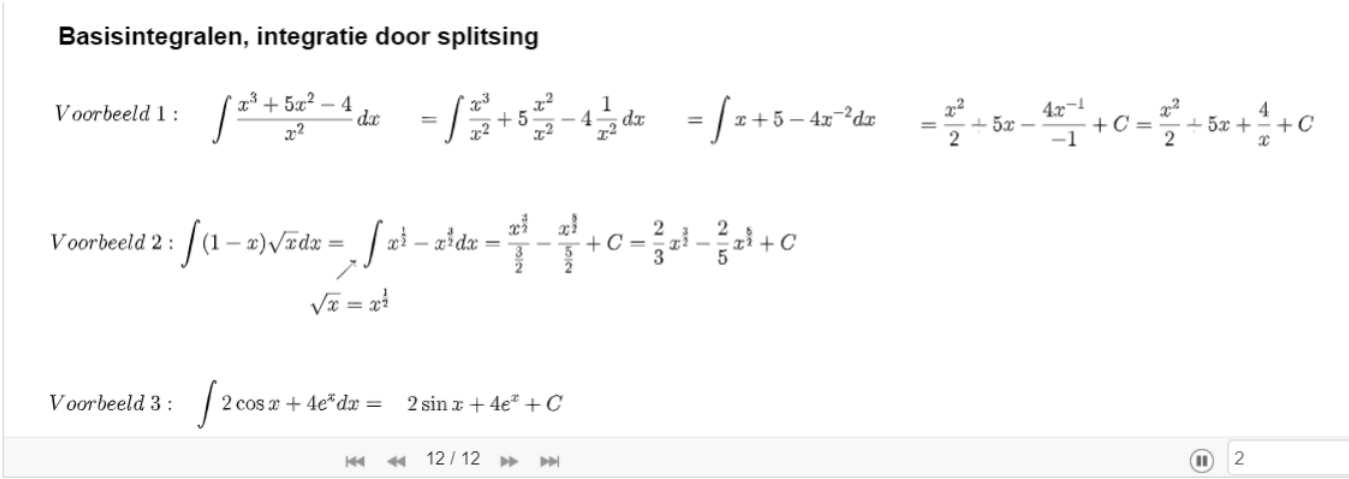


Figure 2: `https://www.geogebra.org/m/RMwPMmq7`

2.2 Oefeningen

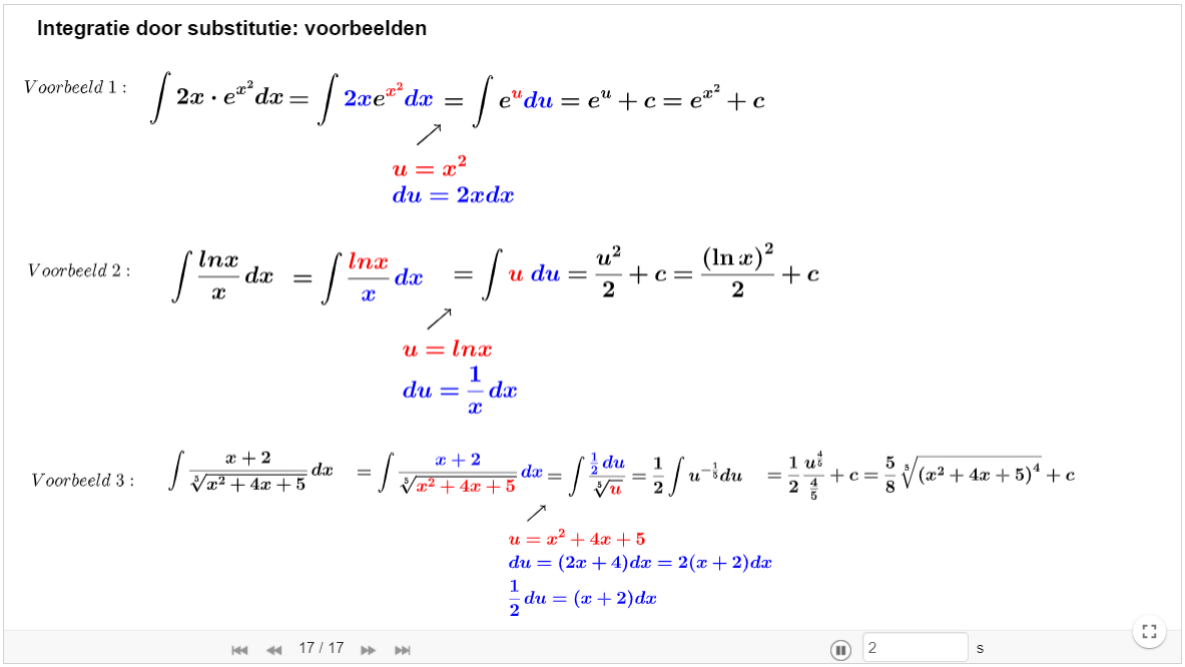
1. Los op:

1. $\int x^5 dx$
2. $\int x^{\frac{3}{4}} dx$
3. $\int \frac{1}{x^2} dx$
4. $\int 5dx$
5. $\int (x^{\frac{1}{2}} - 3x^{\frac{2}{3}} + 6)dx$
6. $\int \left(3\sqrt{x} - \frac{2}{x^3} + \frac{1}{x}\right) dx$
7. $\int \left(\frac{e^x}{2} + x\sqrt{x}\right) dx$
8. $\int \left(\sqrt{x^3} - \frac{1}{2\sqrt{x}} + \sqrt{2}\right) dx$
9. $\int \left(\frac{1}{3x} - \frac{3}{2x^2} + e^2 + \frac{\sqrt{x}}{2}\right) dx$
10. $\int \frac{x^2+2x+1}{x^2} dx$
11. $\int x^3 \left(2x + \frac{1}{x}\right) dx$
12. $\int \sqrt{x}(x^2 - 1)dx$
13. $\int x(2x+1)^2 dx$

2. los op:
   (a) $\int \cot^2 x dx$
   (b) $\int \frac{5^x}{3^{x-1}} dx$

3 Substitutie

3.1 Voorbeelden



Speciale substitutie

$$\int x\sqrt{x+3}dx$$

$$\left| \begin{array}{l} u = \sqrt{x+3} \quad \Leftrightarrow u^2 - 3 = x \\ du = \frac{1}{2\sqrt{x+3}} dx \quad \Leftrightarrow 2\sqrt{x+3}du = dx \Leftrightarrow 2udu = dx \end{array} \right.$$

$$\int (u^2-3) \cdot u \cdot 2udu = \int 2u^4 - 6u^2 du = \frac{2}{5}u^5 - 2u^3 + C = \frac{2}{5}\left(\sqrt{x+3}\right)^5 - 2\left(\sqrt{x+3}\right)^3 + C$$

$$\int \frac{x^2}{\sqrt{1-x}} dx$$

$$\left| \begin{array}{l} u = \sqrt{1-x} \Leftrightarrow x = 1 - u^2 \\ du = \frac{1}{2\sqrt{1-x}} dx \quad \Leftrightarrow 2udu = dx \end{array} \right.$$

$$\int \frac{(1-u^2)^2}{\cancel{x}} \cancel{2u}du = 2\int 1 - 2u^2 + u^4 du = 2\left(u - \frac{2}{3}u^3 + \frac{u^5}{5}\right) + C = 2\sqrt{1-x} - \frac{4}{3}(\sqrt{1-x})^3 + \frac{2(\sqrt{1-x})^5}{5} + C$$

Figure 3: `https://www.geogebra.org/m/vvnmeyj8`

3.2 grenzen aanpassen bij substitutie

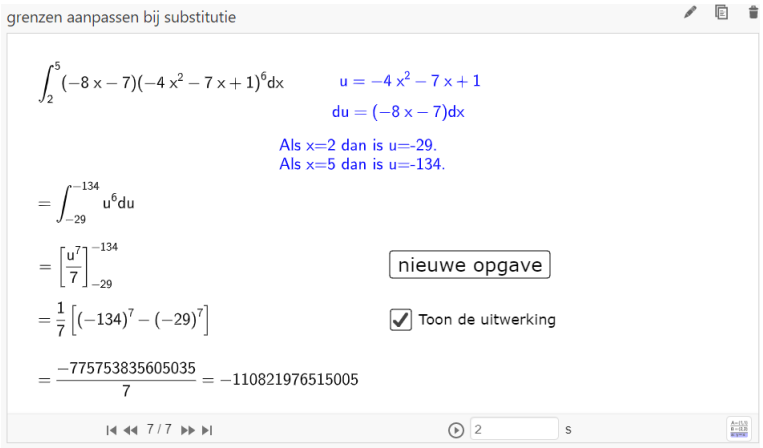


Figure 4: `https://www.geogebra.org/m/vvnmeyj8`

3.3 Oefeningen

1. Bereken volgende onbepaalde integralen

1. $\int e^{-4x}\,dx$
2. $\int_0^8 e^{-4x}\,dx$
3. $\int (x^3-2)\,(x^3-6x)^{207}\,dx$
4. $\int e^{\cos x}\sin x\,dx$
5. $\int_0^{\pi/2} e^{\cos x}\sin x\,dx$
6. $\int \frac{3x}{(x^2+1)^7}\,dx$
7. $\int \frac{12x^3}{3x^4+1}\,dx$
8. $\int (-3x+4)\,e^{-3x^2+8x}\,dx$
9. $\int_0^2 (-3x+4)\,e^{-3x^2+8x}\,dx$
10. $\int \frac{x+5}{x^2+1}\,dx$
11. $\int \frac{e^x}{e^x+1}\,dx$
12. $\int \cos x \sin^5 x\,dx$
13. $\int \frac{x}{\sqrt{x+1}}\,dx$
14. $\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx$
15. $\int_0^3 \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$
16. $\int \frac{1}{\sqrt{x}\,(\sqrt{x}+1)}\,dx$
17. $\int_0^1 \frac{x+1}{x^2+1}\,dx$
18. $\int_0^1 xe^{-3x^2}\,dx$
19. $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$
20. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
21. $\int \frac{x^3}{(x^2+2)^2}\,dx$
22. $\int \frac{1}{x^2+9}\,dx$
23. $\int \frac{1}{a^2x^2+b^2}\,dx$
24. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$
25. $\int_0^{\pi/3} \sin 2x \sin x\,dx$
26. $\int 42\cos x \sin x\,(\sin x+1)^5\,dx$
27. $\int_e^{e^3} \frac{1}{x\ln\sqrt{x}}\,dx$
28. $\int \tan x\,dx$
29. $\int \sec x\,dx$
30. $\int \frac{x^2}{\sqrt{1-(x^3-1)^2}}\,dx$
31. $\int \frac{e^x}{e^{2x}+1}\,dx$
32. $\int \frac{1}{\sqrt{16-x^2}}\,dx$
33. $\int_4^5 \frac{1}{4x\ln x\,[\ln(\ln x)]^3}dx$
34. $\int_0^{\pi/3} \sin x\,(\cos x - \cos^2 x)\,dx$
35. $\int \sin^2 x\,dx$
36. $\int_{-1}^1 \sqrt{1-x^2}\,dx$
37. $2\int_{-r}^r \sqrt{r^2-x^2}\,dx$

2. Verklaar waarom de opp tussen x-as en de grafiek van de functie $f(x) = \frac{(\sqrt{x}-1)^2}{2\sqrt{x}}$ in het interval $[4, 9]$ en deze tussen de x-as en de grafiek van de functie $f(x) = x^2$ in het interval $[1, 2]$ aan elkaar gelijk zijn.

3. Over een continue functie $f : \mathbb{R} \to \mathbb{R}$ is gegeven dat $\int_0^1 f(x)dx = 8$, $\int_1^2 f(x)dx = 2$ en $\int_2^4 f(x)dx = 4$. Dan is $\int_0^2 f(2x)dx =$

3.4 oplossingen

1. Oplossingen oefening 1

1.) $-\frac{1}{4}e^{-4x} + C$ 2.) $\frac{1}{4} - \frac{1}{4}e^{-32}$ 3.) $\frac{1}{624}\left(x^3 - 6x\right)^{208} + C$ 4.) $-e^{\cos x} + C$ 5.) $e - 1$

6.) $-\frac{1}{4\left(x^2+1\right)^6} + C$ 7.) $\ln\left(3x^4 + 1\right) + C$ 8.) $\frac{1}{2}e^{-3x^2+8x} + C$ 9.) $\frac{1}{2}e^4 - \frac{1}{2}$

10.) $5\tan^{-1} x + \frac{1}{2}\ln\left(x^2+1\right) + C$ 11.) $\ln\left(e^x + 1\right) + C$ 12.) $\frac{1}{6}\sin^6 x + C$ 13.) $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$

14.) 36 15.) $\ln\left(e^3 + \frac{1}{e^3}\right) - \ln 2$ 16.) $2\ln\left(\sqrt{x}+1\right) + C$ 17.) $\frac{\pi}{4} + \frac{\ln 2}{2}$ 18.) $\frac{1}{6} - \frac{1}{6e^3}$

19.) $\frac{4}{3}\left(1+\sqrt{x}\right)\sqrt{1+\sqrt{x}} + C$ 20.) $-2\cos\sqrt{x} + C$ 21.) $\frac{1}{2}\ln\left(x^2+2\right) + \frac{1}{x^2+2} + C$ 22.) $\frac{1}{3}\tan^{-1}\frac{1}{3}x + C$

23.) $\frac{1}{ab}\tan^{-1}\left(\frac{a}{b}x\right) + C$ 24.) $\frac{1}{2}\left(\sin^{-1} x\right)^2 + C$ 25.) $\frac{\sqrt{3}}{4}$ 26.) $6(\sin x + 1)^7 - 7(\sin x + 1)^6 + C$

27.) $2\ln 2$ 28.) $\ln|\sec x| + C$ 29.) $\ln|\sec x + \tan x| + C$ 30.) $\frac{1}{3}\sin^{-1}\left(x^3 - 1\right) + C$

31.) $\tan^{-1}\left(e^x\right) + C$ 32.) $\sin^{-1}\left(\frac{x}{4}\right) + C$ 33.) $\frac{1}{8\ln^2(\ln 4)} - \frac{1}{8\ln^2(\ln 5)}$ 34.) $\frac{9}{64}$

35.) $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ 36.) $\frac{\pi}{2}$ 37.) πr^2

2. uitgewerkte voorbeelden

Figure 5: `https://www.geogebra.org/m/vvnmeyj8`

4 Partiële integratie

4.1 Voorbeelden

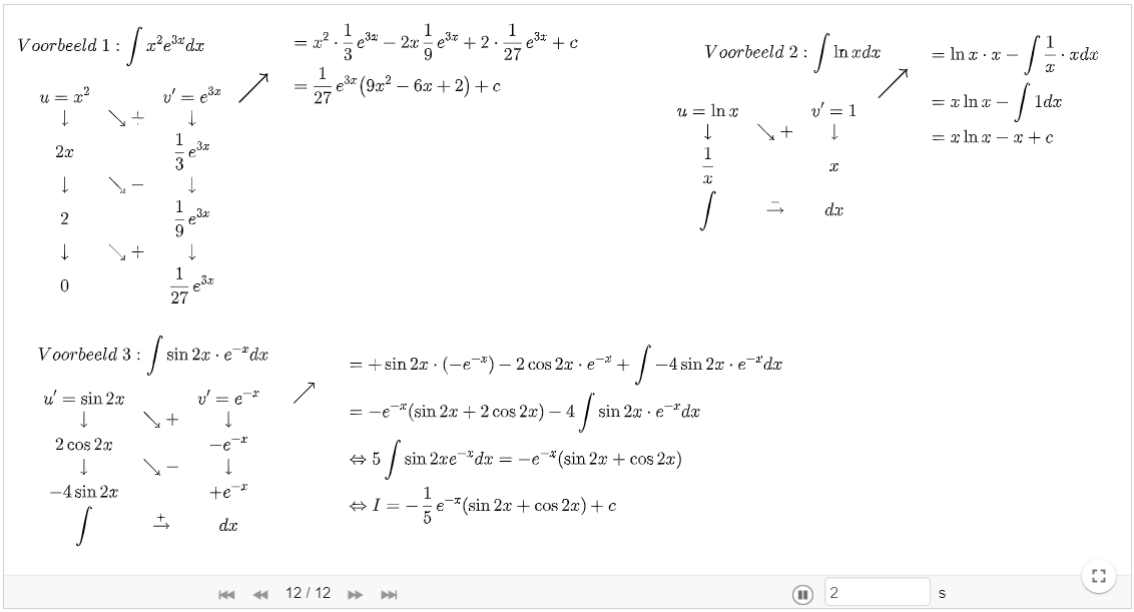


Figure 6: `https://www.geogebra.org/m/b4XpwQYH`

4.2 Oefeningen

1. Los op

1. $\int xe^{2x}\ dx$
2. $\int xe^{-3x}\ dx$
3. $\int_0^{\ln 2} xe^{-3x}\ dx$
4. $\int \cos^{-1} x\ dx$
5. $\int x2^x\ dx$
6. $\int x^2 2^x\ dx$
7. $\int x \cos x\ dx$
8. $\int x^2 \cos x\ dx$
9. $\int x \ln x\ dx$
10. $\int x^5 \ln x\ dx$
11. $\int x \sin 10x\ dx$
12. $\int_1^9 \frac{\ln x}{\sqrt{x}}\ dx$
13. $\int e^x \sin 2x\ dx$
14. $\int_0^{\pi/4} x \sin 2x\ dx$
15. $\int \frac{x^3}{(x^2+2)^2}\ dx$
16. $\int \frac{\ln x}{x^7}\ dx$
17. $\int e^{5x} \cos 3x\ dx$

4.3 Oplossingen

1. Oplossingen oef 1

1.) $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$ 2.) $-\frac{1}{9}e^{-3x} - \frac{1}{3}xe^{-3x} + C$ 3.) $\frac{7}{72} - \frac{1}{24}\ln 2$ 4.) $x\cos^{-1}x - \sqrt{1-x^2} + C$

5.) $\frac{2^x}{\ln 2}\left(x - \frac{1}{\ln 2}\right) + C$ 6.) $\frac{2^x}{\ln 2}\left(x^2 - \frac{2x}{\ln 2} + \frac{2}{\ln^2 2}\right) + C$ 7.) $x\sin x + \cos x + C$

8.) $x^2\sin x + 2x\cos x - 2\sin x + C$ 9.) $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$ 10.) $\frac{1}{6}x^6\ln x - \frac{1}{36}x^6 + C$

11.) $\frac{1}{100}\sin 10x - \frac{1}{10}x\cos 10x + C$ 12.) $6\ln 9 - 8$ 13.) $\frac{1}{5}e^x\sin 2x - \frac{2}{5}e^x\cos 2x + C$ 14) $\frac{1}{4}$

15.) $\frac{1}{2}\ln\left(x^2+2\right) + \frac{1}{x^2+2} + C$ 16.) $-\frac{1}{36x^6} - \frac{1}{6x^6}\ln x + C$ 17.) $\frac{5}{34}(\cos 3x)\,e^{5x} + \frac{3}{34}(\sin 3x)\,e^{5x} + C$

2. Uitgewerkte oefeningen

Figure 7: `https://www.geogebra.org/m/b4XpwQYH`

5 Partieelbreuken

5.1 Voorbeelden

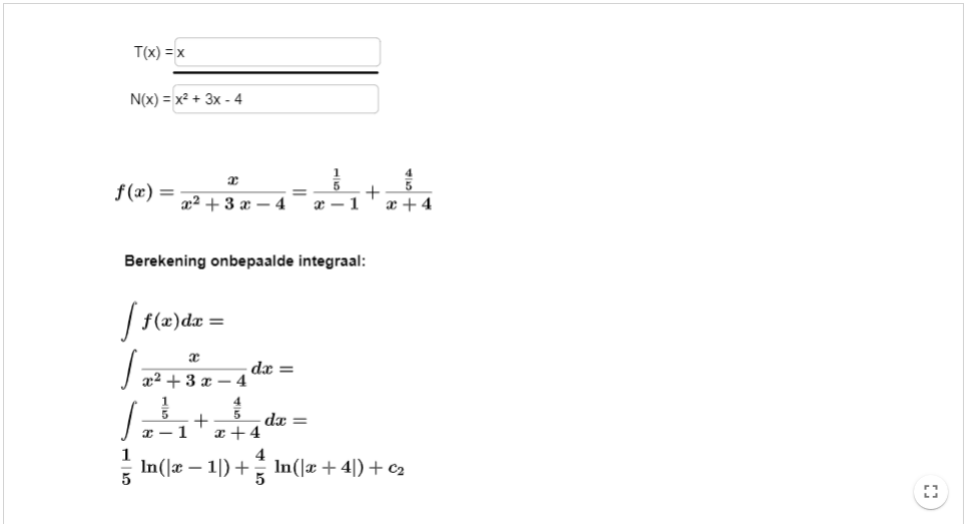


Figure 8: `https://www.geogebra.org/m/j5JTq5y8`

5.2 Oefeningen

1. Bereken:

1. $\displaystyle\int \frac{1}{x^2+3x}\,dx$

2. $\displaystyle\int \frac{x-5}{x^2-2x-8}\,dx$

3. $\displaystyle\int \frac{1}{x^2-a^2}\,dx$

4. $\displaystyle\int \frac{x-1}{x^2-4}\,dx$

5. $\displaystyle\int \frac{x-1}{x^2+4}\,dx$

6. $\displaystyle\int \frac{x^2}{x^2+2x-3}\,dx$

7. $\displaystyle\int \frac{2x^3-x^2-10x-4}{x^2-4}\,dx$

8. $\displaystyle\int \frac{5x-17}{x^2-6x+9}\,dx$

9. $\displaystyle\int \frac{2x^2+7x+3}{x^2+1}\,dx$

10. $\displaystyle\int \frac{2x^2-x+20}{(x-2)(x^2+9)}\,dx$

11. $\displaystyle\int \frac{x^4}{x^4-16}\,dx$

12. $\displaystyle\int \frac{2x+1}{x^2+1}\,dx$

13. $\displaystyle\int \frac{x^2+2}{x(x^2+6)}\,dx$

14. $\displaystyle\int \frac{-x+6}{(x+3)^2}\,dx$

15. $\displaystyle\int \frac{2x-3}{x^2+9}\,dx$

16. $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$

5.3 Oplossingen

1. Oplossingen oef 1

1.) $\frac{1}{3}\ln|x| - \frac{1}{3}\ln|x+3| + C$ 2.) $\frac{7}{6}\ln|x+2| - \frac{1}{6}\ln|x-4| + C$ 3.) $\frac{1}{2a}\ln|x-a| - \frac{1}{2a}\ln|x+a| + C$

4.) $\frac{1}{4}\ln|x-2| + \frac{3}{4}\ln|x+2| + C$ 5.) $\frac{1}{2}\ln\left(x^2+4\right) - \frac{1}{2}\tan^{-1}\frac{1}{2}x + C$

6.) $x + \frac{1}{4}\ln|x-1| - \frac{9}{4}\ln|x+3| + C$ 7.) $x^2 - x - 3\ln|x-2| + \ln|x+2| + C$

8.) $5\ln|x-3| + \frac{2}{x-3} + C$ 9.) $2x + \frac{7}{2}\ln\left(x^2+1\right) + \tan^{-1}x + C$

10.) $2\ln|x-2| - \frac{1}{3}\tan^{-1}\frac{x}{3} + C$ 11.) $x + \frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| - \tan^{-1}\frac{x}{2} + C$

12.) $\tan^{-1}x + \ln\left(x^2+1\right) + C$ 13.) $\frac{1}{3}\ln\left|x^3+6x\right| + C$ 14.) $-\ln|x+3| - \frac{9}{x+3} + C$

15.) $\ln\left(x^2+9\right) - \tan^{-1}\frac{x}{3} + C$ 16.) $\ln|x| + \ln|x-1| - \ln|x+1| + C$

2. Uitgewerkte oefeningen

oefeningen partieelbreuken

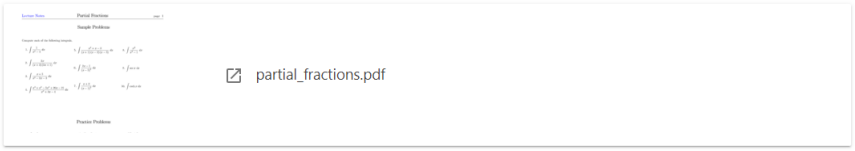


Figure 9: `https://www.geogebra.org/m/j5JTq5y8`

6 Alle technieken door elkaar

1–80 ■ Evaluate the integral.

1. $\int \frac{2x+5}{x-3}\,dx$
2. $\int e^{x+e^x}\,dx$
3. $\int \sin^2 x \cos^3 x\,dx$
4. $\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx$
5. $\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx$
6. $\int_1^2 x^3 \ln x\,dx$
7. $\int \frac{\sqrt{x-2}}{x+2}\,dx$
8. $\int \frac{x}{(x+2)^2}\,dx$
9. $\int \ln(1+x^2)\,dx$
10. $\int \frac{\sqrt{1+\ln x}}{x \ln x}\,dx$
11. $\int_0^1 (1+\sqrt{x})^8\,dx$
12. $\int_0^{\pi/4} \tan^3 x \sec^4 x\,dx$
13. $\int \frac{x}{x^2-2x+2}\,dx$
14. $\int x \sin^{-1} x\,dx$
15. $\int \frac{\sqrt{9-x^2}}{x}\,dx$
16. $\int \frac{x}{x^2+3x+2}\,dx$
17. $\int x^2 \cosh x\,dx$
18. $\int \frac{x^3+x+1}{x^4+2x^2+4x}\,dx$
19. $\int \frac{\cos x}{1+\sin^2 x}\,dx$
20. $\int \cos\sqrt{x}\,dx$
21. $\int_0^1 \cos \pi x \tan \pi x\,dx$
22. $\int \frac{e^{2x}}{1+e^x}\,dx$
23. $\int e^{3x} \cos 5x\,dx$
24. $\int \cos 3x \cos 5x\,dx$
25. $\int \frac{dx}{x^3+x^2+x+1}$
26. $\int x^2 \ln(1+x)\,dx$
27. $\int x^5 e^{-x^3}\,dx$
28. $\int \tan^2 4x\,dx$
29. $\int \frac{1}{\sqrt{9x^2+12x-5}}\,dx$
30. $\int x^2 \tan^{-1} x\,dx$
31. $\int \sqrt[3]{x}(1-\sqrt{x})\,dx$
32. $\int \frac{dx}{e^x - e^{-x}}$
33. $\int \frac{x}{x^4+2x^2+10}\,dx$
34. $\int \frac{1}{x+\sqrt[3]{x}}\,dx$
35. $\int \sin^2 x \cos^4 x\,dx$
36. $\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$
37. $\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx$
38. $\int \frac{1+\cos x}{\sin x}\,dx$
39. $\int \frac{e^x}{e^{2x}-1}\,dx$
40. $\int \frac{1}{x^3-8}\,dx$
41. $\int_{-1}^1 x^5 \cosh x\,dx$
42. $\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\sin x \cos x}\,dx$
43. $\int_{-3}^3 |x^3+x^2-2x|\,dx$
44. $\int_0^{\pi/4} \cos^5\theta\,d\theta$
45. $\int \cot x \ln(\sin x)\,dx$
46. $\int \frac{1+e^x}{1-e^x}\,dx$
47. $\int \frac{x}{(x^2+1)(x^2+4)}\,dx$
48. $\int \frac{dx}{4-5\sin x}$
49. $\int x\sqrt[3]{x+c}\,dx$
50. $\int e^{\sqrt[3]{x}}\,dx$
51. $\int \frac{1}{x+4+4\sqrt{x+1}}\,dx$
52. $\int \frac{x^3+1}{x^3-x^2}\,dx$
53. $\int (x^2+4x-3)\sin 2x\,dx$
54. $\int \sin x \cos(\cos x)\,dx$
55. $\int \frac{x}{\sqrt{16-x^4}}\,dx$
56. $\int \frac{x^3}{(x+1)^{10}}\,dx$
57. $\int \cot^3 2x \csc^3 2x\,dx$
58. $\int (x+\sin x)^2\,dx$
59. $\int \frac{e^{\arctan x}}{1+x^2}\,dx$
60. $\int \frac{dx}{x(x^4+1)}$
61. $\int t^3 e^{-2t}\,dt$
62. $\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt$
63. $\int \sin x \sin 2x \sin 3x\,dx$
64. $\int_1^3 |\ln(x/2)|\,dx$
65. $\int \sqrt{\frac{1+x}{1-x}}\,dx$
66. $\int \frac{x \ln x}{\sqrt{x^2-1}}\,dx$
67. $\int \frac{x+a}{x^2+a^2}\,dx$
68. $\int \sqrt{1+x-x^2}\,dx$
69. $\int \frac{x^4}{x^{10}+16}\,dx$
70. $\int \frac{x+2}{x^2+x+2}\,dx$
71. $\int x \sec x \tan x\,dx$
72. $\int \frac{x}{x^4-a^4}\,dx$
73. $\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$
74. $\int \frac{1}{1+2e^x-e^{-x}}\,dx$
75. $\int \frac{\arctan\sqrt{x}}{\sqrt{x}}\,dx$
76. $\int \frac{\ln(x+1)}{x^2}\,dx$
77. $\int \frac{1}{e^{3x}-e^x}\,dx$
78. $\int \frac{1+\cos^2 x}{1-\cos^2 x}\,dx$
79. $\int \frac{dx}{x\sqrt{2x-25}}$
80. $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}\,dx$

Mat104 Fall 2002, Integration Problems From Old Exams

Compute the following integrals

(1) $\displaystyle\int \frac{\sin^5 x}{\cos x}\,dx$

(2) $\displaystyle\int \frac{dx}{(4+x^2)^{5/2}}$

(3) $\displaystyle\int \sin(\sqrt{1+x})\,dx$

(4) $\displaystyle\int \arctan(x)\,dx$

(5) $\displaystyle\int \cos^4 x\,dx$

(6) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{4-\sin^2 x}\,dx$

(7) $\displaystyle\int \frac{\ln(1+\ln x)}{x}\,dx$

(8) $\displaystyle\int x^2 \arctan x\,dx$

(9) $\displaystyle\int_{-1}^{2} \frac{dx}{(4+2x+x^2)^{5/2}}$

(10) $\displaystyle\int x\sin(x^2)e^{x^2}\,dx$

(11) $\displaystyle\int \frac{dx}{\sqrt{x^2+25}}$

(12) $\displaystyle\int \frac{2+x}{\sqrt[3]{x+2}+x}\,dx$

(13) $\displaystyle\int \frac{3x^2}{x^2+x-2}\,dx$

(14) $\displaystyle\int \frac{\cos\sqrt[3]{x}}{\sqrt[3]{x}}\,dx$

(15) $\displaystyle\int \frac{dx}{\sqrt{x^2+2x}}$

(16) $\displaystyle\int \frac{x^2+3x-3}{(x+1)(x^2+6x+10)}\,dx$

(17) $\displaystyle\int \frac{dx}{x\sqrt{1-x^2}}$

(18) $\displaystyle\int x^3 e^{x^2}\,dx$

(19) $\displaystyle\int x^2 \ln x\,dx$

(20) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

(21) $\displaystyle\int \tan^4\theta\,d\theta$

(22) $\displaystyle\int \frac{x+1}{x^2+4x+13}\,dx$

(23) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sin^2 x+5\sin x+6}\,dx$

(24) $\displaystyle\int \frac{e^{x/2}}{1+e^x}\,dx$

(25) $\displaystyle\int \frac{2x^2+5x+10}{x^3+2x^2+10x}\,dx$

(26) $\displaystyle\int (x-2)\sqrt{9-x^2}\,dx$

(27) $\displaystyle\int_1^{\sqrt{e}} \frac{\arcsin(\ln x)}{x}\,dx$

(28) $\displaystyle\int_0^1 xe^{-x}\,dx$

(29) $\displaystyle\int (\ln x)^2\,dx$

(30) $\displaystyle\int \frac{\sin x}{\sqrt{1+\cos x}}\,dx$

(31) $\displaystyle\int \frac{x^2}{x^6-1}\,dx$

(Hint: Try a substitution first.)

(32) $\displaystyle\int \sin^5 x\cos^2 x\,dx$

(33) $\displaystyle\int \frac{1+e^x}{1-e^x}\,dx$

(34) $\displaystyle\int_1^e \sin(\ln x)\,dx$

(35) $\displaystyle\int e^{\sqrt{x}}\,dx$

(36) $\displaystyle\int \frac{dx}{(4-x^2)^{3/2}}$

(37) $\displaystyle\int_2^3 \frac{e^{1/x}}{x^2}\,dx$

(38) $\displaystyle\int \frac{\sqrt{x^2-4}}{x^3}\,dx$

(39) $\displaystyle\int \frac{x-1}{x^3+x}\,dx$

(40) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2+4}}$

(41) $\displaystyle\int \sin(\sqrt{x})\,dx$

(42) $\displaystyle\int \frac{dx}{x(1-x)^2}$

(43) $\displaystyle\int \frac{(x-5)(\sqrt{x-1}+3)}{\sqrt{x-1}+2}\,dx$

(44) $\displaystyle\int (2x+3)\ln x\,dx$

(45) $\displaystyle\int \frac{\sqrt{9+x^2}}{x^2}\,dx$

(46) $\displaystyle\int \frac{x}{(x^2+1)(x+1)}\,dx$

(47) $\displaystyle\int_0^1 (e^x+1)^{20}e^x\,dx$

(48) $\displaystyle\int \frac{x^2}{x^2+4x+5}\,dx$

(49) $\displaystyle\int \frac{x+1}{x^2+2x+3}\,dx$

Mat104 Fall 2002, Integration Problems From Old Exams

Warning: Many of these integrals can be done several different ways. If you choose a different method than I did, your answer may look quite different from the answer given here. The two different-looking answers may simply differ by a constant or perhaps they can be seen to be the same through the clever use of identities. If you believe that your answer is correct, but it does not match the one given here, consult your instructor! If you find errors, please let me know (jmjohnso@math.princeton.edu).

(1) $\cos^2 x - \dfrac{\cos^4 x}{4} + \ln|\sec x| + C$

(2) $\dfrac{1}{16}\dfrac{x}{\sqrt{4+x^2}} - \dfrac{1}{48}\cdot\dfrac{x^3}{(\sqrt{4+x^2})^3} + C$

(3) $-2\sqrt{1+x}\cos\sqrt{1+x} + 2\sin\sqrt{1+x} + C$

(4) $x\arctan x - \ln\sqrt{1+x^2} + C$

(5) $\dfrac{3x}{8} + \dfrac{\sin 2x}{4} + \dfrac{\sin 4x}{32} + C$ or, equivalently, $\dfrac{3x}{8} + \dfrac{\cos^3 x \sin x}{4} + \dfrac{3}{8}\cos x \sin x + C$

(6) $\dfrac{1}{4}\ln 3$

(7) $\ln(1+\ln x) + (\ln x)\ln(1+\ln x) - \ln x + C$

(8) $\dfrac{x^3\arctan x}{3} - \dfrac{x^2}{6} + \dfrac{\ln\sqrt{1+x^2}}{3} + C$

(9) $\dfrac{1}{8\sqrt{3}}$

(10) $\dfrac{e^{x^2}}{4}(\sin x^2 - \cos x^2) + C$

(11) $\ln|x + \sqrt{x^2+25}| + C$

(12) $u^3 - 3u + \dfrac{3}{4}\ln|u-1| + \dfrac{21}{8}\ln|u^2+u+2| + \dfrac{39}{4\sqrt{7}}\arctan\left(\dfrac{2u+1}{\sqrt{7}}\right) + C$ where $u = \sqrt[3]{x+2}$

(13) $3x - 4\ln|x+2| + \ln|x-1| + C$

(14) $3\sqrt[3]{x}\sin\sqrt[3]{x} + 3\cos\sqrt[3]{x} + C$

(15) Assume that $x > 0$ for simplicity. In that case, the answer is $\ln|x+1+\sqrt{x^2+2x}| + C$

(16) $\ln(x^2+6x+10) + \arctan(x+3) - \ln|x+1| + C$

(17) $-\ln\left|\dfrac{1}{x} + \dfrac{\sqrt{1-x^2}}{x}\right| + C$ (if we use $-\ln|\csc\theta + \cot\theta|$ as our antiderivative for $\csc\theta$.) Alternatively, we might use $\ln|\csc\theta - \cot\theta|$ as an antiderivative for $\csc\theta$ and this would give $\ln\left|\dfrac{1}{x} - \dfrac{\sqrt{1-x^2}}{x}\right| + C$ instead.

(18) $\frac{1}{2}(x^2e^{x^2} - e^{x^2}) + C.$

(19) $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

(20) $\frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2} + C$

(21) $\frac{\tan^3 \theta}{3} - \tan\theta + \theta + C$

(22) $\ln\sqrt{x^2+4x+13} - \frac{\arctan((x+2)/3)}{3} + C$

(23) $\ln(9/8)$

(24) $2\arctan(e^{x/2}) + C$

(25) $\ln|x| + \ln\sqrt{x^2+2x+10} + \frac{2}{3}\arctan\left(\frac{x+1}{3}\right) + C$

(26) $-\frac{(\sqrt{9-x^2})^3}{3} - 9\arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C$

(27) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

(28) $1 - \frac{2}{e}$

(29) $x(\ln x)^2 - 2x\ln x + 2x + C$

(30) $-2\sqrt{1+\cos x} + C$

(31) $\frac{1}{6}\ln\left|\frac{x^3-1}{x^3+1}\right| + C$

(32) $\frac{2}{5}\cos^5 x - \frac{\cos^7 x}{7} - \frac{\cos^3 x}{3} + C$

(33) $x - 2\ln|1-e^x| + C$

(34) $\frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}$

(35) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

(36) $\frac{x}{4\sqrt{4-x^2}} + C$

(37) $\sqrt{e} - \sqrt[3]{e}$

(38) $\frac{\operatorname{arcsec}(x/2)}{4} - \frac{\sqrt{x^2-4}}{2x^2} + C$

(39) $-\ln|x| + \ln\sqrt{x^2+1} + \arctan x + C$

(40) $-\dfrac{1}{4}\cdot\dfrac{\sqrt{x^2+4}}{x} + C$

(41) $-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

(42) $\ln|x| - \ln|x-1| - \dfrac{1}{x-1} + C$

(43) $\dfrac{(\sqrt{x-1})^4}{2} + \dfrac{2(\sqrt{x-1})^3}{3} - 6(\sqrt{x-1})^2 + C$

which can be simplified to $\dfrac{(x-1)^2}{2} + \dfrac{2}{3}(x-1)^{3/2} - 6(x-1) + C$

or, simplifying further, $\dfrac{x^2}{2} - 7x + \dfrac{2}{3}(x-1)^{3/2} + C$.

(44) $(x^2+3x)\ln x - \dfrac{x^2}{2} - 3x + C$

(45) $-\dfrac{\sqrt{9+x^2}}{x} + \ln\left|\dfrac{\sqrt{9+x^2}}{3} + \dfrac{x}{3}\right| + C$

(46) $\dfrac{1}{4}\ln(x^2+1) + \dfrac{1}{2}\arctan x - \dfrac{1}{2}\ln|x+1| + C$

(47) $\dfrac{(1+e)^{21}}{21} - \dfrac{2^{21}}{21}$

(48) $x - 2\ln(x^2+4x+5) + 3\arctan(x+2) + C$

(49) $\dfrac{1}{2}\ln(x^2+2x+3) + C$