

# Onbepaalde integraal

www.karelappeltans.be

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## 1 Basis integralen

Lijst met fundamentele (basis) integralen

$\int 0 dx = c$	$\int \frac{1}{k+x^2} dx = \frac{1}{\sqrt{k}} \operatorname{Arctan}\left(\frac{x}{\sqrt{k}}\right) + c \quad (k > 0)$
$\int 1 dx = x + c$	$\int \frac{1}{\sqrt{k-x^2}} dx = \operatorname{Arcsin}\left(\frac{x}{\sqrt{k}}\right) + c \quad (k > 0)$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \in \mathbb{R} \setminus \{-1\})$	
$\int \frac{1}{x} dx = \ln x  + c$	
$\int e^x dx = e^x + c$	
$\int a^x dx = \frac{a^x}{\ln a} + c$	
$\int \cos x dx = \sin x + c$	
$\int \sin x dx = -\cos x + c$	
$\int \frac{1}{\cos^2 x} dx = \tan x + c$	
$\int \frac{1}{\sin^2 x} dx = -\cot x + c$	
$\int \frac{1}{\sqrt{1-x^2}} dx = \operatorname{Arcsin} x + c = -\operatorname{Arccos} x + c$	
$\int \frac{1}{1+x^2} dx = \operatorname{Arctan} x + c$	

Figure 1: <https://www.geogebra.org/m/RMwPMmq7>

## 2 Splitsing

### 2.1 Voorbeelden

#### Basisintegralen, integratie door splitsing

$$\text{Voorbeeld 1: } \int \frac{x^3 + 5x^2 - 4}{x^2} dx = \int \frac{x^3}{x^2} + 5 \frac{x^2}{x^2} - 4 \frac{1}{x^2} dx = \int x + 5 - 4x^{-2} dx = \frac{x^2}{2} + 5x - \frac{4x^{-1}}{-1} + C = \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

$$\text{Voorbeeld 2: } \int (1-x)\sqrt{x} dx = \int x^{\frac{1}{2}} - x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$$

$\sqrt{x} = x^{\frac{1}{2}}$

$$\text{Voorbeeld 3: } \int 2 \cos x + 4e^x dx = 2 \sin x + 4e^x + C$$

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Figure 2: <https://www.geogebra.org/m/RMwPMmq7>

### 2.2 Oefeningen

1. Los op:

- |   |  |
|---|--|
| 1. $\int x^5 dx$  | 2. $\int x^{\frac{3}{4}} dx$                               |
| 3. $\int \frac{1}{x^2} dx$  | 4. $\int 5 dx$   |
| 5. $\int (x^{\frac{1}{2}} - 3x^{\frac{2}{3}} + 6) dx$                   | 6. $\int (3\sqrt{x} - \frac{2}{x^3} + \frac{1}{x}) dx$     |
| 7. $\int (\frac{e^x}{2} + x\sqrt{x}) dx$                                | 8. $\int (\sqrt{x^3} - \frac{1}{2\sqrt{x}} + \sqrt{2}) dx$ |
| 9. $\int (\frac{1}{3x} - \frac{3}{2x^2} + e^2 + \frac{\sqrt{x}}{2}) dx$ | 10. $\int \frac{x^2+2x+1}{x^2} dx$                         |
| 11. $\int x^3 (2x + \frac{1}{x}) dx$                                    | 12. $\int \sqrt{x}(x^2 - 1) dx$                            |
| 13. $\int x(2x + 1)^2 dx$   |  |

2. los op:

- (a)  $\int \cot^2 x dx$   
(b)  $\int \frac{5^x}{3^{x-1}} dx$

### 3 Substitutie

#### 3.1 Voorbeelden

##### Integratie door substitutie: voorbeelden

$$\text{Voorbeeld 1: } \int 2x \cdot e^{x^2} dx = \int 2x e^{x^2} dx = \int e^u du = e^u + c = e^{x^2} + c$$

$\nearrow$   
 $u = x^2$   
 $du = 2x dx$

$$\text{Voorbeeld 2: } \int \frac{\ln x}{x} dx = \int \frac{\ln x}{x} dx = \int u du = \frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$$

$\nearrow$   
 $u = \ln x$   
 $du = \frac{1}{x} dx$

$$\text{Voorbeeld 3: } \int \frac{x+2}{\sqrt[4]{x^2+4x+5}} dx = \int \frac{x+2}{\sqrt[4]{x^2+4x+5}} dx = \int \frac{\frac{1}{2} du}{\sqrt[4]{u}} = \frac{1}{2} \int u^{-\frac{1}{4}} du = \frac{1}{2} \cdot \frac{4}{5} u^{\frac{1}{4}} + c = \frac{1}{8} u^{\frac{1}{4}} + c = \frac{1}{8} \sqrt[4]{(x^2+4x+5)^4} + c$$

$\nearrow$   
 $u = x^2 + 4x + 5$   
 $du = (2x+4)dx = 2(x+2)dx$   
 $\frac{1}{2} du = (x+2)dx$

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#### Speciale substitutie

$$\int x \sqrt{x+3} dx$$

$$\left| \begin{array}{l} u = \sqrt{x+3} \Leftrightarrow u^2 - 3 = x \\ du = \frac{1}{2\sqrt{x+3}} dx \Leftrightarrow 2\sqrt{x+3} du = dx \Leftrightarrow 2udu = dx \end{array} \right.$$

$$\int (u^2 - 3) \cdot u \cdot 2udu = \int 2u^4 - 6u^2 du = \frac{2}{5}u^5 - 2u^3 + C = \frac{2}{5}(\sqrt{x+3})^5 - 2(\sqrt{x+3})^3 + C$$

$$\int \frac{x^2}{\sqrt{1-x}} dx$$

$\left| \begin{array}{l} u = \sqrt{1-x} \Leftrightarrow x = 1 - u^2 \\ du = \frac{1}{2\sqrt{1-x}} dx \Leftrightarrow 2udu = dx \end{array} \right.$

$$\int \frac{(1-u^2)^2}{x} 2udu = 2 \int 1 - 2u^2 + u^4 du = 2 \left( u - \frac{2}{3}u^3 + \frac{u^5}{5} \right) + C = 2\sqrt{1-x} - \frac{4}{3}(\sqrt{1-x})^3 + \frac{2(\sqrt{1-x})^5}{5} + C$$

Figure 3: <https://www.geogebra.org/m/vvnmeyej8>

### 3.2 grenzen aanpassen bij substitutie

grenzen aanpassen bij substitutie

The screenshot shows a step-by-step solution for the integral  $\int_2^5 (-8x - 7)(-4x^2 - 7x + 1)^6 dx$ . The solution uses the substitution  $u = -4x^2 - 7x + 1$  and  $du = (-8x - 7)dx$ . It notes that when  $x=2$ ,  $u=-29$ , and when  $x=5$ ,  $u=-134$ . The integral is then transformed into  $\int_{-29}^{-134} u^6 du$ , evaluated as  $\left[ \frac{u^7}{7} \right]_{-29}^{-134}$ , which simplifies to  $\frac{1}{7} [(-134)^7 - (-29)^7]$ . The final result is  $= \frac{-775753835605035}{7} = -110821976515005$ .

**nieuwe opgave**

Toon de uitwerking

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Figure 4: <https://www.geogebra.org/m/vvnmeij8>

### 3.3 Oefeningen

1. Bereken volgende onbepaalde integralen

1.  $\int e^{-4x} dx$
2.  $\int_0^8 e^{-4x} dx$
3.  $\int (x^2 - 2)(x^2 - 6x)^{207} dx$
4.  $\int e^{\cos x} \sin x dx$
5.  $\int_0^{\pi/2} e^{\cos x} \sin x dx$
6.  $\int \frac{3x}{(x^2 + 1)^7} dx$
7.  $\int \frac{12x^3}{3x^4 + 1} dx$
8.  $\int (-3x + 4)e^{-3x^2 + 8x} dx$
9.  $\int_0^2 (-3x + 4)e^{-3x^2 + 8x} dx$
10.  $\int \frac{x+5}{x^2+1} dx$
11.  $\int \frac{e^x}{e^x + 1} dx$
12.  $\int \cos x \sin^5 x dx$
13.  $\int \frac{x}{\sqrt{x+1}} dx$
14.  $\int_0^{15} \frac{x}{\sqrt{x+1}} dx$
15.  $\int_0^3 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$
16.  $\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)} dx$
17.  $\int_0^1 \frac{x+1}{x^2+1} dx$
18.  $\int_0^1 x e^{-3x^2} dx$
19.  $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$
20.  $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$
21.  $\int \frac{x^2}{(x^2+2)^2} dx$
22.  $\int \frac{1}{x^2+9} dx$
23.  $\int \frac{1}{a^2x^2+b^2} dx$
24.  $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$
25.  $\int_0^{\pi/3} \sin 2x \sin x dx$
26.  $\int 42 \cos x \sin x (\sin x + 1)^5 dx$
27.  $\int_e^{e^2} \frac{1}{x \ln \sqrt{x}} dx$
28.  $\int \tan x dx$
29.  $\int \sec x dx$
30.  $\int \frac{x^2}{\sqrt{1-(x^2-1)^2}} dx$
31.  $\int \frac{e^x}{e^{2x}+1} dx$
32.  $\int \frac{1}{\sqrt{16-x^2}} dx$
33.  $\int_4^5 \frac{1}{4x \ln x [\ln(\ln x)]^3} dx$
34.  $\int_0^{\pi/3} \sin x (\cos x - \cos^3 x) dx$
35.  $\int \sin^2 x dx$
36.  $\int_{-1}^1 \sqrt{1-x^2} dx$
37.  $2 \int_{-r}^r \sqrt{r^2 - x^2} dx$

2. Verklaar waarom de opp tussen x-as en de grafiek van de functie  $f(x) = \frac{(\sqrt{x}-1)^2}{2\sqrt{x}}$  in het interval  $[4, 9]$  en deze tussen de x-as en de grafiek van de functie  $f(x) = x^2$  in het interval  $[1, 2]$  aan elkaar gelijk zijn.
3. Over een continue functie  $f : \mathbb{R} \rightarrow \mathbb{R}$  is gegeven dat  $\int_0^1 f(x)dx = 8$ ,  $\int_1^2 f(x)dx = 2$  en  $\int_2^4 f(x)dx = 4$ . Dan is  $\int_0^2 f(2x)dx =$

### 3.4 oplossingen

1. Oplossingen oefening 1

$$\begin{aligned}
 1.) & -\frac{1}{4}e^{-4x} + C & 2.) & \frac{1}{4} - \frac{1}{4}e^{-32} & 3.) & \frac{1}{624}(x^3 - 6x)^{208} + C & 4.) & -e^{\cos x} + C & 5.) & e - 1 \\
 6.) & -\frac{1}{4(x^2 + 1)^6} + C & 7.) & \ln(3x^4 + 1) + C & 8.) & \frac{1}{2}e^{-3x^2+8x} + C & 9.) & \frac{1}{2}e^4 - \frac{1}{2} \\
 10.) & 5\tan^{-1}x + \frac{1}{2}\ln(x^2 + 1) + C & 11.) & \ln(e^x + 1) + C & 12.) & \frac{1}{6}\sin^6x + C & 13.) & \frac{2}{3}(x + 1)^{3/2} - 2(x + 1)^{1/2} + C \\
 14.) & 36 & 15.) & \ln\left(e^3 + \frac{1}{e^3}\right) - \ln 2 & 16.) & 2\ln(\sqrt{x} + 1) + C & 17.) & \frac{\pi}{4} + \frac{\ln 2}{2} & 18.) & \frac{1}{6} - \frac{1}{6e^3} \\
 19.) & \frac{4}{3}(1 + \sqrt{x})\sqrt{1 + \sqrt{x}} + C & 20.) & -2\cos\sqrt{x} + C & 21.) & \frac{1}{2}\ln(x^2 + 2) + \frac{1}{x^2 + 2} + C & 22.) & \frac{1}{3}\tan^{-1}\frac{1}{3}x + C \\
 23.) & \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}x\right) + C & 24.) & \frac{1}{2}(\sin^{-1}x)^2 + C & 25.) & \frac{\sqrt{3}}{4} & 26.) & 6(\sin x + 1)^7 - 7(\sin x + 1)^6 + C \\
 27.) & 2\ln 2 & 28.) & \ln|\sec x| + C & 29.) & \ln|\sec x + \tan x| + C & 30.) & \frac{1}{3}\sin^{-1}(x^3 - 1) + C \\
 31.) & \tan^{-1}(e^x) + C & 32.) & \sin^{-1}\left(\frac{x}{4}\right) + C & 33.) & \frac{1}{8\ln^2(\ln 4)} - \frac{1}{8\ln^2(\ln 5)} & 34.) & \frac{9}{64} \\
 35.) & \frac{1}{2}x - \frac{1}{4}\sin 2x + C & 36.) & \frac{\pi}{2} & 37.) & \pi r^2
 \end{aligned}$$

2. uitgewerkte voorbeelden

uitgewerkte voorbeelden substitutie

Oorspronkelijke formule	Substitutie
$f(x)=a$	$\Rightarrow f(g(u))=a$
$f(x)=x$	$\Rightarrow f(g(u))=u$
$f(x)=x^2$	$\Rightarrow f(g(u))=u^2$
$f(x)=\sin x$	$\Rightarrow f(g(u))=\sin u$
$f(x)=\cos x$	$\Rightarrow f(g(u))=\cos u$
$f(x)=\tan x$	$\Rightarrow f(g(u))=\tan u$
$f(x)=\frac{1}{x}$	$\Rightarrow f(g(u))=\frac{1}{g(u)}$
$f(x)=\frac{1}{x^2}$	$\Rightarrow f(g(u))=\frac{1}{g(u)^2}$
$f(x)=\frac{1}{x^3}$	$\Rightarrow f(g(u))=\frac{1}{g(u)^3}$

substitution.pdf

Figure 5: <https://www.geogebra.org/m/vvnmeyej8>

## 4 Partiële integratie

### 4.1 Voorbeelden

*Voorbeeld 1:*  $\int x^2 e^{3x} dx$

$$\begin{array}{l}
 u = x^2 \quad v' = e^{3x} \\
 \downarrow \quad \searrow + \\
 2x \quad \frac{1}{3} e^{3x} \\
 \downarrow \quad \searrow - \\
 2 \quad \frac{1}{9} e^{3x} \\
 \downarrow \quad \searrow + \\
 0 \quad \frac{1}{27} e^{3x}
 \end{array}
 \quad
 \begin{array}{l}
 = x^2 \cdot \frac{1}{3} e^{3x} - 2x \frac{1}{9} e^{3x} + 2 \cdot \frac{1}{27} e^{3x} + c \\
 = \frac{1}{27} e^{3x} (9x^2 - 6x + 2) + c
 \end{array}$$

*Voorbeeld 2:*  $\int \ln x dx$

$$\begin{array}{l}
 u = \ln x \quad v' = 1 \\
 \downarrow \quad \searrow + \\
 \frac{1}{x} \quad x \\
 \int \quad \rightarrow \quad dx
 \end{array}
 \quad
 \begin{array}{l}
 = \ln x \cdot x - \int \frac{1}{x} \cdot x dx \\
 = x \ln x - \int 1 dx \\
 = x \ln x - x + c
 \end{array}$$

*Voorbeeld 3:*  $\int \sin 2x \cdot e^{-x} dx$

$$\begin{array}{l}
 u' = \sin 2x \quad v' = e^{-x} \\
 \downarrow \quad \searrow + \\
 2 \cos 2x \quad -e^{-x} \\
 \downarrow \quad \searrow - \\
 -4 \sin 2x \quad +e^{-x} \\
 \int \quad \pm \quad dx
 \end{array}
 \quad
 \begin{array}{l}
 = +\sin 2x \cdot (-e^{-x}) - 2 \cos 2x \cdot e^{-x} + \int -4 \sin 2x \cdot e^{-x} dx \\
 = -e^{-x}(\sin 2x + 2 \cos 2x) - 4 \int \sin 2x \cdot e^{-x} dx \\
 \Leftrightarrow 5 \int \sin 2x e^{-x} dx = -e^{-x}(\sin 2x + \cos 2x) \\
 \Leftrightarrow I = -\frac{1}{5} e^{-x}(\sin 2x + \cos 2x) + c
 \end{array}$$

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Figure 6: <https://www.geogebra.org/m/b4XpwQYH>

### 4.2 Oefeningen

1. Los op

- |                                  |  |                                       |
|----------------------------------|--|---------------------------------------|
| 1. $\int x e^{2x} dx$            | 7. $\int x \cos x dx$                    | 13. $\int e^x \sin 2x dx$             |
| 2. $\int x e^{-3x} dx$           | 8. $\int x^2 \cos x dx$                  | 14. $\int_0^{\pi/4} x \sin 2x dx$     |
| 3. $\int_0^{\ln 2} x e^{-3x} dx$ | 9. $\int x \ln x dx$                     | 15. $\int \frac{x^3}{(x^2 + 2)^2} dx$ |
| 4. $\int \cos^{-1} x dx$         | 10. $\int x^5 \ln x dx$                  | 16. $\int \frac{\ln x}{x^7} dx$       |
| 5. $\int x 2^x dx$               | 11. $\int x \sin 10x dx$                 | 17. $\int e^{5x} \cos 3x dx$          |
| 6. $\int x^2 2^x dx$             | 12. $\int_1^9 \frac{\ln x}{\sqrt{x}} dx$ |                                       |

### 4.3 Oplossingen

1. Oplossingen oef 1

$$1.) \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C \quad 2.) -\frac{1}{9}e^{-3x} - \frac{1}{3}xe^{-3x} + C \quad 3.) \frac{7}{72} - \frac{1}{24}\ln 2 \quad 4.) x\cos^{-1}x - \sqrt{1-x^2} + C$$

$$5.) \frac{2^x}{\ln 2} \left( x - \frac{1}{\ln 2} \right) + C \quad 6.) \frac{2^x}{\ln 2} \left( x^2 - \frac{2x}{\ln 2} + \frac{2}{\ln^2 2} \right) + C \quad 7.) x\sin x + \cos x + C$$

$$8.) x^2 \sin x + 2x \cos x - 2 \sin x + C \quad 9.) \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \quad 10.) \frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C$$

$$11.) \frac{1}{100}\sin 10x - \frac{1}{10}x \cos 10x + C \quad 12.) 6 \ln 9 - 8 \quad 13.) \frac{1}{5}e^x \sin 2x - \frac{2}{5}e^x \cos 2x + C \quad 14.) \frac{1}{4}$$

$$15.) \frac{1}{2} \ln(x^2 + 2) + \frac{1}{x^2 + 2} + C \quad 16.) -\frac{1}{36x^6} - \frac{1}{6x^6} \ln x + C \quad 17.) \frac{5}{34}(\cos 3x)e^{5x} + \frac{3}{34}(\sin 3x)e^{5x} + C$$

## 2. Uitgewerkte oefeningen

extra oefeningen met oplossingen

Figure 7: <https://www.geogebra.org/m/b4XpwQYH>

## 5 Partieelbreuken

### 5.1 Voorbeelden

$T(x) = x$

$N(x) = x^2 + 3x - 4$

$$f(x) = \frac{x}{x^2 + 3x - 4} = \frac{\frac{1}{5}}{x-1} + \frac{\frac{4}{5}}{x+4}$$

**Berekening onbepaalde integraal:**

$$\int f(x)dx =$$

$$\int \frac{x}{x^2 + 3x - 4} dx =$$

$$\int \frac{\frac{1}{5}}{x-1} + \frac{\frac{4}{5}}{x+4} dx =$$

$$\frac{1}{5} \ln(|x-1|) + \frac{4}{5} \ln(|x+4|) + c_2$$

Figure 8: <https://www.geogebra.org/m/j5JTq5y8>

### 5.2 Oefeningen

1. Bereken:

$$\begin{array}{lll}
1. \int \frac{1}{x^2 + 3x} dx & 7. \int \frac{2x^3 - x^2 - 10x - 4}{x^2 - 4} dx & 12. \int \frac{2x + 1}{x^2 + 1} dx \\
2. \int \frac{x - 5}{x^2 - 2x - 8} dx & 8. \int \frac{5x - 17}{x^2 - 6x + 9} dx & 13. \int \frac{x^2 + 2}{x(x^2 + 6)} dx \\
3. \int \frac{1}{x^2 - a^2} dx & 9. \int \frac{2x^2 + 7x + 3}{x^2 + 1} dx & 14. \int \frac{-x + 6}{(x + 3)^2} dx \\
4. \int \frac{x - 1}{x^2 - 4} dx & 10. \int \frac{2x^2 - x + 20}{(x - 2)(x^2 + 9)} dx & 15. \int \frac{2x - 3}{x^2 + 9} dx \\
5. \int \frac{x - 1}{x^2 + 4} dx & 11. \int \frac{x^4}{x^4 - 16} dx & 16. \int \frac{x^2 + 2x - 1}{x^3 - x} dx
\end{array}$$

### 5.3 Oplossingen

1. Oplossingen oef 1

$$1.) \frac{1}{3} \ln|x| - \frac{1}{3} \ln|x+3| + C \quad 2.) \frac{7}{6} \ln|x+2| - \frac{1}{6} \ln|x-4| + C \quad 3.) \frac{1}{2a} \ln|x-a| - \frac{1}{2a} \ln|x+a| + C$$

$$4.) \frac{1}{4} \ln|x-2| + \frac{3}{4} \ln|x+2| + C \quad 5.) \frac{1}{2} \ln(x^2 + 4) - \frac{1}{2} \tan^{-1} \frac{1}{2}x + C$$

$$6.) x + \frac{1}{4} \ln|x-1| - \frac{9}{4} \ln|x+3| + C \quad 7.) x^2 - x - 3 \ln|x-2| + \ln|x+2| + C$$

$$8.) 5 \ln|x-3| + \frac{2}{x-3} + C \quad 9.) 2x + \frac{7}{2} \ln(x^2 + 1) + \tan^{-1} x + C$$

$$10.) 2 \ln|x-2| - \frac{1}{3} \tan^{-1} \frac{x}{3} + C \quad 11.) x + \frac{1}{2} \ln|x-2| - \frac{1}{2} \ln|x+2| - \tan^{-1} \frac{x}{2} + C$$

$$12.) \tan^{-1} x + \ln(x^2 + 1) + C \quad 13.) \frac{1}{3} \ln|x^3 + 6x| + C \quad 14.) -\ln|x+3| - \frac{9}{x+3} + C$$

$$15.) \ln(x^2 + 9) - \tan^{-1} \frac{x}{3} + C \quad 16.) \ln|x| + \ln|x-1| - \ln|x+1| + C$$

2. Uitgewerkte oefeningen

## oefeningen partieelbreuken

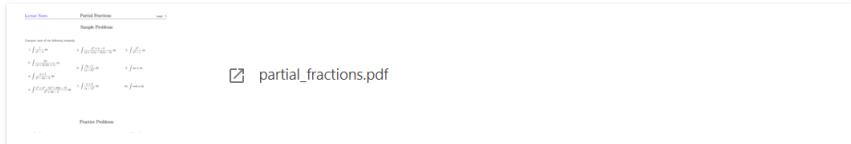


Figure 9: <https://www.geogebra.org/m/j5JTq5y8>

## 6 Alle technieken door elkaar

**1–80 ■ Evaluate the integral.**

1.  $\int \frac{2x+5}{x-3} dx$

3.  $\int \sin^2 x \cos^3 x dx$

5.  $\int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx$

7.  $\int \frac{\sqrt{x-2}}{x+2} dx$

9.  $\int \ln(1+x^2) dx$

11.  $\int_0^1 (1+\sqrt{x})^8 dx$

13.  $\int \frac{x}{x^2-2x+2} dx$

15.  $\int \frac{\sqrt{9-x^2}}{x} dx$

17.  $\int x^2 \cosh x dx$

19.  $\int \frac{\cos x}{1+\sin^2 x} dx$

21.  $\int_0^1 \cos \pi x \tan \pi x dx$

23.  $\int e^{3x} \cos 5x dx$

25.  $\int \frac{dx}{x^3+x^2+x+1}$

27.  $\int x^5 e^{-x^3} dx$

29.  $\int \frac{1}{\sqrt{9x^2+12x-5}} dx$

31.  $\int \sqrt[3]{x}(1-\sqrt{x}) dx$

33.  $\int \frac{x}{x^4+2x^2+10} dx$

35.  $\int \sin^2 x \cos^4 x dx$

37.  $\int \frac{x}{1-x^2+\sqrt{1-x^2}} dx$

39.  $\int \frac{e^x}{e^{2x}-1} dx$

2.  $\int e^{x+e^x} dx$

4.  $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$

6.  $\int_1^2 x^3 \ln x dx$

8.  $\int \frac{x}{(x+2)^2} dx$

10.  $\int \frac{\sqrt{1+\ln x}}{x \ln x} dx$

12.  $\int_0^{\pi/4} \tan^3 x \sec^4 x dx$

14.  $\int x \sin^{-1} x dx$

16.  $\int \frac{x}{x^2+3x+2} dx$

18.  $\int \frac{x^3+x+1}{x^4+2x^2+4x} dx$

20.  $\int \cos \sqrt{x} dx$

22.  $\int \frac{e^{2x}}{1+e^x} dx$

24.  $\int \cos 3x \cos 5x dx$

26.  $\int x^2 \ln(1+x) dx$

28.  $\int \tan^2 4x dx$

30.  $\int x^2 \tan^{-1} x dx$

32.  $\int \frac{dx}{e^x - e^{-x}}$

34.  $\int \frac{1}{x+\sqrt[3]{x}} dx$

36.  $\int \frac{1}{\sqrt{5-4x-x^2}} dx$

38.  $\int \frac{1+\cos x}{\sin x} dx$

40.  $\int \frac{1}{x^3-8} dx$

41.  $\int_{-1}^1 x^5 \cosh x dx$

43.  $\int_{-3}^3 |x^3 + x^2 - 2x| dx$

45.  $\int \cot x \ln(\sin x) dx$

47.  $\int \frac{x}{(x^2+1)(x^2+4)} dx$

49.  $\int x \sqrt[3]{x+c} dx$

51.  $\int \frac{1}{x+4+4\sqrt{x+1}} dx$

53.  $\int (x^2+4x-3) \sin 2x dx$

55.  $\int \frac{x}{\sqrt{16-x^4}} dx$

57.  $\int \cot^3 2x \csc^3 2x dx$

59.  $\int \frac{e^{\arctan x}}{1+x^2} dx$

61.  $\int t^3 e^{-2t} dt$

63.  $\int \sin x \sin 2x \sin 3x dx$

65.  $\int \sqrt{\frac{1+x}{1-x}} dx$

67.  $\int \frac{x+a}{x^2+a^2} dx$

69.  $\int \frac{x^4}{x^{10}+16} dx$

71.  $\int x \sec x \tan x dx$

73.  $\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$

75.  $\int \frac{\arctan \sqrt{x}}{\sqrt{x}} dx$

77.  $\int \frac{1}{e^{3x}-e^x} dx$

79.  $\int \frac{dx}{x\sqrt{2x-25}}$

42.  $\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\sin x \cos x} dx$

44.  $\int_0^{\pi/4} \cos^5 \theta d\theta$

46.  $\int \frac{1+e^x}{1-e^x} dx$

48.  $\int \frac{dx}{4-5 \sin x}$

50.  $\int e^{\sqrt[3]{x}} dx$

52.  $\int \frac{x^3+1}{x^3-x^2} dx$

54.  $\int \sin x \cos(\cos x) dx$

56.  $\int \frac{x^3}{(x+1)^{10}} dx$

58.  $\int (x+\sin x)^2 dx$

60.

$$\int \frac{dx}{x(x^4+1)}$$

62.  $\int \frac{\sqrt[3]{t}}{1+\sqrt[3]{t}} dt$

64.  $\int_1^3 |\ln(x/2)| dx$

66.  $\int \frac{x \ln x}{\sqrt{x^2-1}} dx$

68.  $\int \sqrt{1+x-x^2} dx$

70.  $\int \frac{x+2}{x^2+x+2} dx$

72.  $\int \frac{x}{x^4-a^4} dx$

74.  $\int \frac{1}{1+2e^x-e^{-x}} dx$

76.  $\int \frac{\ln(x+1)}{x^2} dx$

78.  $\int \frac{1+\cos^2 x}{1-\cos^2 x} dx$

80.  $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}} dx$

# Mat104 Fall 2002, Integration Problems From Old Exams

Compute the following integrals

$$(1) \quad \int \frac{\sin^5 x}{\cos x} dx$$

$$(2) \quad \int \frac{dx}{(4+x^2)^{5/2}}$$

$$(3) \quad \int \sin(\sqrt{1+x}) dx$$

$$(4) \quad \int \arctan(x) dx$$

$$(5) \quad \int \cos^4 x dx$$

$$(6) \quad \int_0^{\pi/2} \frac{\cos x}{4 - \sin^2 x} dx$$

$$(7) \quad \int \frac{\ln(1 + \ln x)}{x} dx$$

$$(8) \quad \int x^2 \arctan x dx$$

$$(9) \quad \int_{-1}^2 \frac{dx}{(4+2x+x^2)^{5/2}}$$

$$(10) \quad \int x \sin(x^2) e^{x^2} dx$$

$$(11) \quad \int \frac{dx}{\sqrt{x^2 + 25}}$$

$$(12) \quad \int \frac{2+x}{\sqrt[3]{x+2+x}} dx$$

$$(13) \quad \int \frac{3x^2}{x^2 + x - 2} dx$$

$$(14) \quad \int \frac{\cos \sqrt[3]{x}}{\sqrt[3]{x}} dx$$

$$(15) \quad \int \frac{dx}{\sqrt{x^2 + 2x}}$$

$$(16) \quad \int \frac{x^2 + 3x - 3}{(x+1)(x^2 + 6x + 10)} dx$$

$$(17) \quad \int \frac{dx}{x\sqrt{1-x^2}}$$

$$(18) \quad \int x^3 e^{x^2} dx$$

$$(19) \quad \int x^2 \ln x dx$$

$$(20) \quad \int \frac{x^3}{\sqrt{1-x^2}} dx$$

$$(21) \quad \int \tan^4 \theta d\theta$$

$$(22) \quad \int \frac{x+1}{x^2 + 4x + 13} dx$$

$$(23) \quad \int_0^{\pi/2} \frac{\cos x}{\sin^2 x + 5 \sin x + 6} dx$$

$$(24) \quad \int \frac{e^{x/2}}{1+e^x} dx$$

$$(25) \quad \int \frac{2x^2 + 5x + 10}{x^3 + 2x^2 + 10x} dx$$

$$(26) \quad \int (x-2)\sqrt{9-x^2} dx$$

$$(27) \quad \int_1^{\sqrt{e}} \frac{\arcsin(\ln x)}{x} dx$$

$$(28) \quad \int_0^1 xe^{-x} dx$$

$$(29) \int (\ln x)^2 dx$$

$$(30) \int \frac{\sin x}{\sqrt{1 + \cos x}} dx$$

$$(31) \int \frac{x^2}{x^6 - 1} dx$$

(Hint: Try a substitution first.)

$$(32) \int \sin^5 x \cos^2 x dx$$

$$(33) \int \frac{1 + e^x}{1 - e^x} dx$$

$$(34) \int_1^e \sin(\ln x) dx$$

$$(35) \int e^{\sqrt{x}} dx$$

$$(36) \int \frac{dx}{(4 - x^2)^{3/2}}$$

$$(37) \int_2^3 \frac{e^{1/x}}{x^2} dx$$

$$(38) \int \frac{\sqrt{x^2 - 4}}{x^3} dx$$

$$(39) \int \frac{x - 1}{x^3 + x} dx$$

$$(40) \int \frac{dx}{x^2 \sqrt{x^2 + 4}}$$

$$(41) \int \sin(\sqrt{x}) dx$$

$$(42) \int \frac{dx}{x(1 - x)^2}$$

$$(43) \int \frac{(x - 5)(\sqrt{x - 1} + 3)}{\sqrt{x - 1} + 2} dx$$

$$(44) \int (2x + 3) \ln x dx$$

$$(45) \int \frac{\sqrt{9 + x^2}}{x^2} dx$$

$$(46) \int \frac{x}{(x^2 + 1)(x + 1)} dx$$

$$(47) \int_0^1 (e^x + 1)^{20} e^x dx$$

$$(48) \int \frac{x^2}{x^2 + 4x + 5} dx$$

$$(49) \int \frac{x + 1}{x^2 + 2x + 3} dx$$

## Mat104 Fall 2002, Integration Problems From Old Exams

**Warning:** Many of these integrals can be done several different ways. If you choose a different method than I did, your answer may look quite different from the answer given here. The two different-looking answers may simply differ by a constant or perhaps they can be seen to be the same through the clever use of identities. If you believe that your answer is correct, but it does not match the one given here, consult your instructor! If you find errors, please let me know ([jmjohnso@math.princeton.edu](mailto:jmjohnso@math.princeton.edu)).

$$(1) \cos^2 x - \frac{\cos^4 x}{4} + \ln |\sec x| + C$$

$$(2) \frac{1}{16} \frac{x}{\sqrt{4+x^2}} - \frac{1}{48} \cdot \frac{x^3}{(\sqrt{4+x^2})^3} + C$$

$$(3) -2\sqrt{1+x} \cos \sqrt{1+x} + 2 \sin \sqrt{1+x} + C$$

$$(4) x \arctan x - \ln \sqrt{1+x^2} + C$$

$$(5) \frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C \text{ or, equivalently, } \frac{3x}{8} + \frac{\cos^3 x \sin x}{4} + \frac{3}{8} \cos x \sin x + C$$

$$(6) \frac{1}{4} \ln 3$$

$$(7) \ln(1 + \ln x) + (\ln x) \ln(1 + \ln x) - \ln x + C$$

$$(8) \frac{x^3 \arctan x}{3} - \frac{x^2}{6} + \frac{\ln \sqrt{1+x^2}}{3} + C$$

$$(9) \frac{1}{8\sqrt{3}}$$

$$(10) \frac{e^{x^2}}{4} (\sin x^2 - \cos x^2) + C$$

$$(11) \ln |x + \sqrt{x^2 + 25}| + C$$

$$(12) u^3 - 3u + \frac{3}{4} \ln |u - 1| + \frac{21}{8} \ln |u^2 + u + 2| + \frac{39}{4\sqrt{7}} \arctan \left( \frac{2u+1}{\sqrt{7}} \right) + C \text{ where } u = \sqrt[3]{x+2}$$

$$(13) 3x - 4 \ln |x+2| + \ln |x-1| + C$$

$$(14) 3\sqrt[3]{x} \sin \sqrt[3]{x} + 3 \cos \sqrt[3]{x} + C$$

$$(15) \text{Assume that } x > 0 \text{ for simplicity. In that case, the answer is } \ln |x+1+\sqrt{x^2+2x}| + C$$

$$(16) \ln(x^2 + 6x + 10) + \arctan(x+3) - \ln|x+1| + C$$

$$(17) -\ln \left| \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right| + C \text{ (if we use } -\ln |\csc \theta + \cot \theta| \text{ as our antiderivative for } \csc \theta.) \text{ Alternatively, we might use } \ln |\csc \theta - \cot \theta| \text{ as an antiderivative for } \csc \theta \text{ and this would give } \ln \left| \frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \right| + C \text{ instead.}$$

$$(18) \frac{1}{2}(x^2 e^{x^2} - e^{x^2}) + C.$$

$$(19) \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

$$(20) \frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2} + C$$

$$(21) \frac{\tan^3 \theta}{3} - \tan \theta + \theta + C$$

$$(22) \ln \sqrt{x^2 + 4x + 13} - \frac{\arctan((x+2)/3)}{3} + C$$

$$(23) \ln(9/8)$$

$$(24) 2 \arctan(e^{x/2}) + C$$

$$(25) \ln |x| + \ln \sqrt{x^2 + 2x + 10} + \frac{2}{3} \arctan\left(\frac{x+1}{3}\right) + C$$

$$(26) -\frac{(\sqrt{9-x^2})^3}{3} - 9 \arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C$$

$$(27) \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

$$(28) 1 - \frac{2}{e}$$

$$(29) x(\ln x)^2 - 2x \ln x + 2x + C$$

$$(30) -2\sqrt{1+\cos x} + C$$

$$(31) \frac{1}{6} \ln \left| \frac{x^3 - 1}{x^3 + 1} \right| + C$$

$$(32) \frac{2}{5} \cos^5 x - \frac{\cos^7 x}{7} - \frac{\cos^3 x}{3} + C$$

$$(33) x - 2 \ln |1 - e^x| + C$$

$$(34) \frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}$$

$$(35) 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

$$(36) \frac{x}{4\sqrt{4-x^2}} + C$$

$$(37) \sqrt{e} - \sqrt[3]{e}$$

$$(38) \frac{\operatorname{arcsec}(x/2)}{4} - \frac{\sqrt{x^2-4}}{2x^2} + C$$

$$(39) -\ln|x| + \ln\sqrt{x^2 + 1} + \arctan x + C$$

$$(40) -\frac{1}{4} \cdot \frac{\sqrt{x^2 + 4}}{x} + C$$

$$(41) -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$$

$$(42) \ln|x| - \ln|x-1| - \frac{1}{x-1} + C$$

$$(43) \frac{(\sqrt{x-1})^4}{2} + \frac{2(\sqrt{x-1})^3}{3} - 6(\sqrt{x-1})^2 + C$$

which can be simplified to  $\frac{(x-1)^2}{2} + \frac{2}{3}(x-1)^{3/2} - 6(x-1) + C$

or, simplifying further,  $\frac{x^2}{2} - 7x + \frac{2}{3}(x-1)^{3/2} + C$ .

$$(44) (x^2 + 3x) \ln x - \frac{x^2}{2} - 3x + C$$

$$(45) -\frac{\sqrt{9+x^2}}{x} + \ln \left| \frac{\sqrt{9+x^2}}{3} + \frac{x}{3} \right| + C$$

$$(46) \frac{1}{4} \ln(x^2 + 1) + \frac{1}{2} \arctan x - \frac{1}{2} \ln|x+1| + C$$

$$(47) \frac{(1+e)^{21}}{21} - \frac{2^{21}}{21}$$

$$(48) x - 2\ln(x^2 + 4x + 5) + 3\arctan(x+2) + C$$

$$(49) \frac{1}{2} \ln(x^2 + 2x + 3) + C$$