

DOMAĆI

1210. a)

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2} &= \\ &= \lim_{n \rightarrow \infty} \frac{n^3 + 3n^2 + 3n + 1 - n^3 + 3n^2 - 3n + 1}{n^2 + 2n + 1 + n^2 - 2n + 1} = \\ &= \lim_{n \rightarrow \infty} \frac{6n^2 + 2}{2n^2 + 2} = \lim_{n \rightarrow \infty} \frac{2(3n^2 + 1)}{2(n^2 + 1)} = \lim_{n \rightarrow \infty} \frac{3n^2 + 1 /: n^2}{n^2 + 1 /: n^2} = \\ &= \lim_{n \rightarrow \infty} \frac{3 + \frac{1}{n^2}}{1 + \frac{1}{n^2}} = 3\end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

v)

$$\begin{aligned}\lim_{n \rightarrow \infty} \left(\frac{2n^2}{2n+3} + \frac{1-3n^3}{3n^2+1} \right) &= \\ &= \lim_{n \rightarrow \infty} \left(\frac{2n+3 - 6n^4 - 9n^3 + 6n^4 + 2n^2}{6n^3 + 9n^2 + 2n + 3} \right) = \\ &= \lim_{n \rightarrow \infty} \frac{-9n^3 + 2n^2 + 2n + 3 /: n^3}{6n^3 + 9n^2 + 2n + 3 /: n^3} = \\ &= \lim_{n \rightarrow \infty} \frac{-9 + 2 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}}{6 + 9 \cdot \frac{1}{n^2} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}} = -\frac{9}{6} = -\frac{3}{2}\end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^3} = 0$$

1211. b)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{(n+2)! - (n+1)!}{(n+3)!} &= \\ &= \lim_{n \rightarrow \infty} \frac{(n+2)(n+1)! - (n+1)!}{(n+3)(n+2)(n+1)!} = \\ &= \lim_{n \rightarrow \infty} \frac{(n+1)!(n+2-1)}{(n+3)(n+2)(n+1)!} = \\ &= \lim_{n \rightarrow \infty} \frac{n+1}{n^2 + 2n + 3n + 6} = \\ &= \lim_{n \rightarrow \infty} \frac{n+1/:n^2}{n^2 + 5n + 6/:n^2} = \\ &= \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{1}{n^2}}{1 + 5 \cdot \frac{1}{n} + 6 \cdot \frac{1}{n^2}} = 0 \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

1213. đ)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{(1+5+9+\dots+(4n-3))}{2(n+1)} - n &= \\ S_n &= \frac{n}{2}(a_1 + a_n) - \text{suma aritmetičkog niza} \\ S_n &= \frac{n}{2}(1 + 4n - 3) = \frac{n}{2}(4n - 2) = \frac{n}{2}2(2n - 1) = 2n^2 - n \\ &= \lim_{n \rightarrow \infty} \frac{2n^2 - n}{2n + 2} - n = \\ &= \lim_{n \rightarrow \infty} \frac{2n^2 - n - 2n^2 - 2n}{2n + 2} = \\ &= \lim_{n \rightarrow \infty} \frac{-3n/:n}{2n + 2/:n} = \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \frac{-3}{2 + 2 \cdot \frac{1}{n}} = -\frac{3}{2}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$