Vektori

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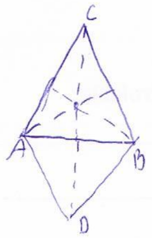
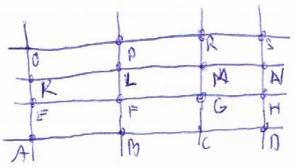
Točka T težište je trokuta ABC. Odredi zbroj vektora TA + TB + TC.



$$\vec{TA} + \vec{TB} + \vec{TC} = (\vec{TA} + \vec{TB}) + \vec{TC} =$$
$$= \vec{TD} + \vec{TC} = 0$$

Dana je pravokutna mreža kao na slici. Odredi:

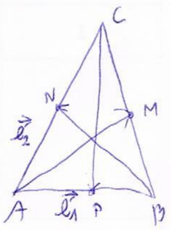


a) $\vec{AB} + \vec{KP} = \vec{AB} + \vec{AF} = \vec{AG}$

b) $\vec{AH} + \vec{SP} = \vec{AH} + \vec{HF} = \vec{AF}$

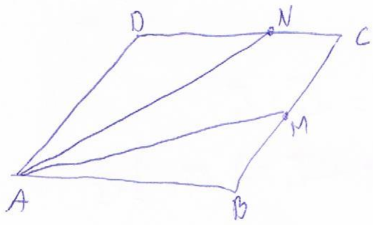
c) $\vec{CF} + \vec{RN} = \vec{CF} + \vec{FC} = \vec{0}$

Neka su M, N i P polovišta stranica BC, AC i AB trokuta ABC. Prikaži vektore AM, BN i CP kao linearne kombinacije vektora AB = e_1 i AC = e_2.



$$\vec{BC} = \vec{e_1} - \vec{e_2}$$
$$\vec{AM} = \vec{AB} + \vec{BM} = \vec{e_1} + \frac{1}{2}\vec{CB} = \vec{e_1} - \frac{1}{2}\vec{BC} =$$
$$= \vec{e_1} - \frac{1}{2}(\vec{e_1} - \vec{e_2}) = \frac{1}{2}\vec{e_1} + \frac{1}{2}\vec{e_2}$$
$$\vec{BN} = \vec{BA} + \vec{AN} = \frac{1}{2}\vec{e_2} - \vec{e_1}$$
$$\vec{CP} = \vec{CA} + \vec{AP} = -\vec{AC} + \frac{1}{2}\vec{AB} = \frac{1}{2}\vec{e_1} - \vec{e_2}$$

Točka M polovište je stranice BC , a točka N stranice CD paralelograma ABCD. Prikaži vektore AB i AD kao linearne kombinacije vektora AM i AN.



$$\vec{AB} + \frac{1}{2}\vec{AD} = \vec{AM} \quad / \cdot 2$$
$$\vec{AD} + \frac{1}{2}\vec{AB} = \vec{AN} \quad / \cdot (-1)$$

$$2\vec{AB} + \vec{AD} = 2\vec{AM}$$
$$-\vec{AD} - \frac{1}{2}\vec{AB} = -\vec{AN} \quad / +$$

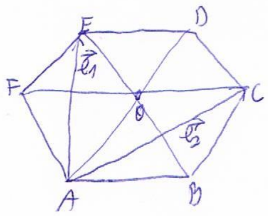
$$\frac{3}{2} AB = 2\vec{AM} - \vec{AN} \quad / \cdot \frac{2}{3}$$
$$\vec{AB} = \frac{4}{3}\vec{AM} - \frac{2}{3}\vec{AN}$$

$$\vec{AB} + \frac{1}{2}\vec{AD} = \vec{AM}$$
$$\vec{AD} + \frac{1}{2}\vec{AB} = \vec{AN} \quad / \cdot (-2)$$

$$\vec{AB} + \frac{1}{2}\vec{AD} = \vec{AM}$$
$$-2\vec{AD} - \vec{AB} = -2\vec{AN} \quad / +$$

$$-\frac{3}{2}\vec{AD} = \vec{AM} - 2\vec{AN} \quad / \cdot \left(-\frac{2}{3}\right)$$
$$\vec{AD} = -\frac{2}{3}\vec{AM} + \frac{4}{3}\vec{AN}$$

Točke A, B, C, D, E i F vrhovi su pravilnog šesterokuta. Ako je AF = e_1 , AC = e_2 , prikaži vektore AB , AD i AE kao linearnu kombinaciju vektora e_1 i e_2 .



$$\vec{AB} = \vec{e_2} + \vec{CB} = \vec{e_2} + \vec{OA} = \vec{e_2} + \vec{OF} + \vec{FA} = \vec{e_2} - \vec{AB} - \vec{e_1}$$

$$= \frac{1}{2}\vec{e_2} - \frac{1}{2}\vec{e_1}$$

$$\vec{AD} = \vec{AC} + \vec{CD} = \vec{e_1} + \vec{AF} = \vec{e_1} + \vec{e_2}$$

$$\vec{AE} = \vec{AB} + \vec{BE} = \frac{1}{2}\vec{e_2} - \frac{1}{2}\vec{e_1} + 2\vec{e_1} = \frac{3}{2}\vec{e_1} + \frac{1}{2}\vec{e_2}$$

Ako za tri jedinična vektora a , b i c vrijedi a + b + c = 0, koliko je a * b + b * c + c * a?

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\vec{b} + 2\vec{a}\vec{c} + 2\vec{b}\vec{c} = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\vec{b} + \vec{a}\vec{c} + \vec{b}\vec{c}) = 0$$

$$1 + 1 + 1 + 2(\vec{a}\vec{b} + \vec{a}\vec{c} + \vec{b}\vec{c}) = 0$$

$$\vec{a}\vec{b} + \vec{a}\vec{c} + \vec{b}\vec{c} = -\frac{3}{2}$$

Odredi kut između vektora a i b ako je IaI = 2IbI te ako je vektor 2a + b okomit na vektor a – 3b

$$(2\vec a+\vec b)\perp(\vec a-3\vec b) \Rightarrow (2\vec a+\vec b)(\vec a-3\vec b)=0$$

$$2\vec a^2-6\vec a\vec b+\vec a\vec b-3\vec b^2=0$$

$$2|\vec a|^2-6\vec a\vec b+\vec a\vec b-3|\vec b|^2=0$$

$$2\cdot(2|\vec b|)^2-5(2|\vec b||\vec b|\cos\angle(\vec a,\vec b))-3|\vec b|^2=0$$

$$8|\vec b|^2-10|\vec b|^2\cos\angle(\vec a,\vec b)-3|\vec b|^2=0\ /:|\vec b|^2$$

$$8-10\cos\angle(\vec a,\vec b)-3=0$$

$$-10\cos\angle(\vec a,\vec b)=-5$$

$$\cos(\angle(\vec a,\vec b))=\frac{1}{2}$$

$$\angle(\vec a,\vec b)=60^\circ$$

Odredi vektor b okomit na vektor a = -2 i + j ako je IbI = √5

$$\vec b\perp\vec a \Rightarrow \vec a\cdot\vec b=0 \Rightarrow -2b_x+b_y=0 \Rightarrow b_y=2b_x$$

$$|\vec b|=\sqrt{5}=\sqrt{b_x^2+b_y^2}=\sqrt{5}\ /^2$$

$$b_x^2+4b_x^2=5$$

$$b_x^2=1$$

$$b_x=\pm1$$

$$b_y=\pm2$$

$$\vec b=\pm(\vec i+2\vec j)$$

Ako su A (1,1), B (4, -1) , D (3,4) vrhovi paralelograma ABCD, koliki je kut između dijagonala paralelograma?

$$\vec{AB} = (4-1)\vec{i} + (-1-1)\vec{j} = 3\vec{i} - 2\vec{j}$$

$$|\vec{AB}| = \sqrt{9+4} = \sqrt{13}$$

$$\vec{AD} = (3-1)\vec{i} + (4-1)\vec{j} = 2\vec{i} + 3\vec{j}$$

$$|\vec{AD}| = \sqrt{4+9} = \sqrt{13}$$

$$\vec{AB} \perp \vec{AD}$$

$$\boxed{\varphi = 90^\circ}$$

Odredi jedinični vektor okomit na vektor AB ako je A (-2,3) , B (-4,2).

$$\vec{AB} = (-4+2)\vec{i} + (2-3)\vec{j} = -2\vec{i} - \vec{j}$$

$$\vec{e} \perp \vec{AB} \Rightarrow \vec{e} \cdot \vec{AB} = 0$$

$$\vec{e_x} \cdot (-2) + e_y \cdot (-1) = 0$$

$$-2e_x - e_y = 0 \Rightarrow e_y = -2e_x$$

$$\vec{e} = 1$$

$$\sqrt{e_x^2 + e_y^2} = 1 \;/^2$$

$$e_x^2 + (-2e_x)^2 = 1$$

$$5e_x^2 = 1$$

$$e_x^2 = \frac{1}{5}$$

$$e_x = \pm\frac{1}{\sqrt{5}} \qquad e_y = \mp\frac{2}{\sqrt{5}}$$

$$\vec{e}_1 = \frac{1}{\sqrt{5}}\vec{i} - \frac{2}{\sqrt{5}}\vec{j}$$

$$\vec{e_2} = -\frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$$