

Matematika

10.3.2024
X. III. MMXXIV

Pravac

Jednadžbe: EKSPPLICITNA $y = kx + l$ (može i $y = ax + b$)
 IMPLICITNA $Ax + By + C = 0$
 SEGMENTNA $\frac{x}{m} + \frac{y}{n} = 1$

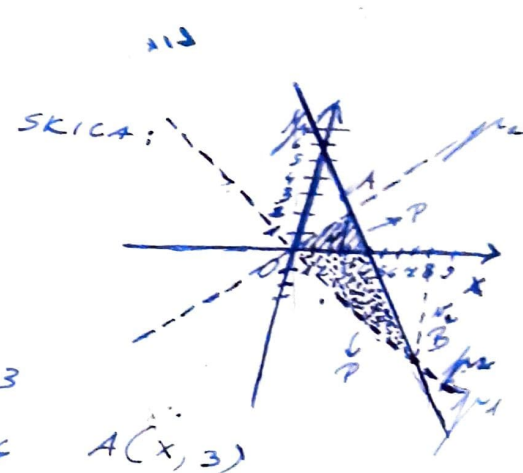
$m =$ odjciak na osi apscisa (x)
 $n =$ odjciak na osi ordinata (y)

$\varphi(x_1, y_1, x_2, y_2) \Rightarrow \tan \varphi = \left| \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \right|$

- I NASTAVNA JEDINICA: 33 Eliplicitni i implicitni oblik jedne pravca
- II NASTAVNA JEDINICA: 33 Segmentni oblik jedne pravca
- III NASTAVNA JEDINICA: 33 Dva drugi pravca Paralelnost i similitudnost pravca
- IV NASTAVNA JEDINICA: 33 Vektor smjera i vektor normale pravca
- V NASTAVNA JEDINICA: 33 Udaljenost točke od pravca. Simetrala luku
- VI NASTAVNA JEDINICA: 33 Pravac regresije
- VII NASTAVNA JEDINICA: 33 DODATAK

ZADATCI

I (str. 58.)
 24. $3x + 2y - 12 = 0 \rightarrow$ pravac 1
 $P = 6$



pravac 2 = P

$$3x + 2y - 12 = 0$$

$$2y = -3x + 12 \quad | :2 \Rightarrow y = -\frac{3}{2}x + 6$$

$$\Rightarrow 3x + 2y = 12 \quad | :3 \Rightarrow x + \frac{2y}{3} = 4 \quad | :4 \Rightarrow \frac{x}{4} + \frac{y}{6} = 1$$

$m = \frac{1}{4}$
 $n = 6$

$A(x, 3)$
 $x = 4 - \frac{2}{3} \cdot 3$
 $x = 2$

$P = P = 6$

(Slika ne odgovara istini, ali su izdali dva moguća rješenja)

I $A(2, 3)$
 II $B(x, -3)$
 $x = 4 + 2$
 $x = 6$

$P = \frac{a \cdot |w|}{2}$
 $P = \frac{m \cdot |w|}{2} \cdot 2$

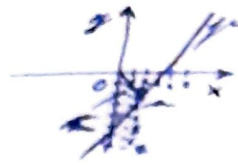
$|w| = \frac{12}{4}$
 $|w| = 3$

I pravac 2
 $y = \frac{3}{2}x$

pravac 2 $y = kx$
 $y = 2x$
 $3 = 2k$
 $k = \frac{3}{2}$

II pravac 2
 $y = kx$
 $k = -\frac{3}{6} = -\frac{1}{2}$

SKICA:



7. pravac $\rightarrow \frac{x}{3} + \frac{y}{6} = 1$

$n = 2$ $P_D = 7$

$P_D = |m \cdot m| : 2$

$P_D = |3 \cdot (-6)| : 2$

$P_D = 18 : 2$

$P_D = 9$

$P_D = \frac{a^2 - n^2}{2}$

$9 = \frac{3\sqrt{5} \cdot n_a}{2}$

$n_a = \frac{18}{3\sqrt{5}}$

$n_a = 2,08$

$a^2 = m^2 + n^2$

$a^2 = 9 + 36$

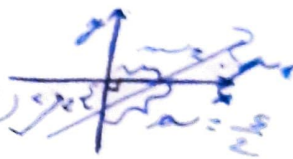
$a^2 = 45 / \sqrt{5}$

$a = \sqrt{45}$

$a = 3\sqrt{5}$

9. $2x + my + 2m = 0$
 $a = \frac{5}{2}$

SKICA:



$2x + my + 2m = 0 / : m$

$\frac{2x}{m} + y = -2 / : (-2)$

$\frac{x}{-m} + \frac{y}{-2} = 1$
 $n = -2$

VIDI SE n IZ
SEGMENTA DOG OBLIKA

$[(\frac{2}{2})^2 + (\frac{4}{2})^2] = (\frac{5}{2})^2$
 $(\frac{9}{4} + \frac{16}{4}) = \frac{25}{4}$
 $m = \pm \frac{3}{2}$

iz jednačine:

$m = \sqrt{(\frac{5}{2})^2 - (-2)^2}$

$m = \sqrt{\frac{25}{4} - \frac{16}{4}}$

$m = \sqrt{\frac{9}{4}}$

$m = \pm \frac{3}{2}$

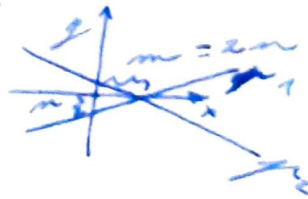
OBVEZNO!

21. $2x + ay - 5 = 0$

$m = 2m$

$a = ?$

SKICA:



$2x + ay = 5 / : 5$

$\frac{x}{\frac{5}{2}} + \frac{y}{\frac{5}{a}} = 1$

$m = \frac{5}{2}$

$n = \frac{m}{2}$

$n = \frac{5}{4}$

$n = \frac{5}{a}$

$\frac{5}{a} = \frac{5}{4}$

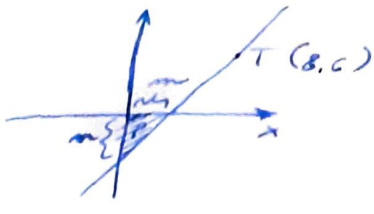
$a = 4$

$a_1 = 4$
 $a_2 = -4$

ZBOG
DVIJE
MOGUCNOSTI

14. $T(8,6)$
 $P=12$
 $\mu \dots ?$

SKICA:



$$\frac{m \cdot n}{2} = P$$

$$y = kx + l$$

$$Ax + By + C = 0$$

$$m \cdot n = 24$$

$$\frac{x}{m} + \frac{y}{n} = 1$$

$$m = \frac{24}{n}$$

$$\frac{8}{\frac{24}{n}} + \frac{6}{n} = 1$$

$$\frac{8n}{24} + \frac{6}{n} = 1 \quad | \cdot 3n$$

$$n^2 + 18 = 3n$$

$$n^2 - 3n + 18 = 0$$

$$\begin{aligned} n_1 &= 6 \\ n_2 &= -3 \\ m_1 &= 4 \\ m_2 &= -8 \end{aligned}$$

$$(3 \pm \sqrt{9-36})$$

$$3 \pm \sqrt{9-36}$$

$$\frac{3+9}{2}$$

$$\frac{3-9}{2}$$

$$\frac{12}{2} = 6$$

$$\frac{-6}{2} = -3$$

$$\begin{aligned} \mu_1 \dots \frac{x}{6} + \frac{y}{6} &= 1 \\ \mu_2 \dots \frac{x}{-8} + \frac{y}{-3} &= 1 \end{aligned}$$

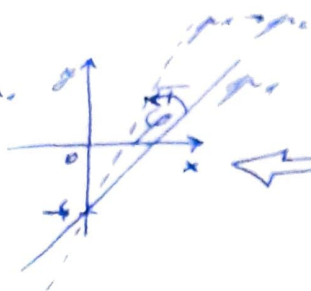
III (str. 68. i 69.)

1.1) $x - y + 5 = 0 \rightarrow \mu_1$
 $x + 2y - 4 = 0 \rightarrow \mu_2$

$$\begin{aligned} \mu_1 \rightarrow y &= x + 5 & k_1 &= 1 \\ \mu_2 \rightarrow y &= -\frac{1}{2}x + \frac{4}{2} & k_2 &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \varphi(\mu_1, \mu_2) &= \text{tg } \varphi = \left| \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \right| \\ \text{tg } \varphi &= \left| \frac{-\frac{1}{2} - 1}{1 + 1 \cdot (-\frac{1}{2})} \right| \\ \text{tg } \varphi &= \left| \frac{-\frac{3}{2}}{1 - \frac{1}{2}} \right| \\ \text{tg } \varphi &= \left| \frac{-\frac{3}{2}}{\frac{1}{2}} \right| \\ \text{tg } \varphi &= 3 \\ \varphi &= 71^\circ 34' \end{aligned}$$

9. $x + 2y - 4 = 0 \rightarrow p_1$
 $T(3,3)$
 $\varphi = ?$



SKICA JE
 NEISPRAVNA ($l=2$, na -6)
 TRAVAC JE TROUGLA TADAJNE!

$p_1 \rightarrow 2y = -x + 4 / :2$
 $y = -\frac{1}{2}x + 2$

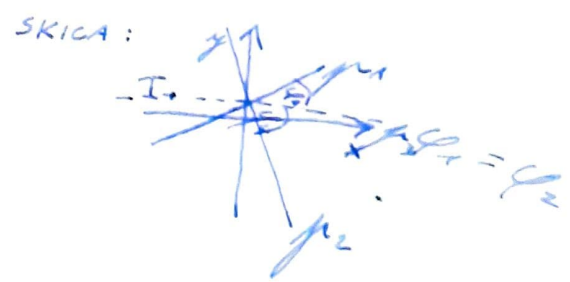
$p_2 \rightarrow 3 = k_2 x + l_2$
 $k_1 = k_2$ (circled)

$k_1 = -\frac{1}{2}$
 $k_1 = k_2 = 2$
 $k_2 = ?$
 $k_2 = \frac{3-2}{3}$
 $k_2 = \frac{1}{3}$

$\text{tg } \varphi = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$
 $\text{tg } \varphi = \left| \frac{\frac{1}{3} + \frac{1}{2}}{1 + \frac{1}{3} \cdot \left(\frac{1}{2}\right)} \right|$
 $\text{tg } \varphi = \left| \frac{\frac{5}{6}}{1 + \frac{1}{6}} \right|$
 $\text{tg } \varphi = \left| \frac{\frac{5}{6}}{\frac{7}{6}} \right|$
 $\text{tg } \varphi = 1$
 $\varphi = 45^\circ$

POBUKA!
 8. zadatak se
 rješava na
 isti način, samo
 je nul-točka
 u pitagori.
 NUL-TOČKA JE
 SPECIJNE S OSI X (circled)

10. $T(-4,1)$
 $p_1 \rightarrow x - y + 1 = 0$
 $p_2 \rightarrow 4x + y - 33 = 0$
 $\varphi_1 = \varphi_2$
 $p_3 \dots = ?$



$x - y + 1 = 0$
 $y = x + 1$
 $4x + y - 33 = 0$
 $y = -4x + 33$

$\text{tg } \varphi_1 = \text{tg } \varphi_2$
 $\left| \frac{k_3 - k_1}{1 + k_3 k_1} \right| = \left| \frac{k_3 - k_2}{1 + k_3 k_2} \right|$

1. SLUČAJ
 $\frac{k_3 - k_1}{1 + k_3 k_1} = \frac{k_3 - k_2}{1 + k_3 k_2}$

2. SLUČAJ
 $\frac{k_3 - k_1}{1 + k_3 k_1} = \frac{-k_2 + k_3}{1 + k_3 k_2}$



$$p_3(x) \Rightarrow y = 3x + 13$$

$$p_3(2) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$

12. Paralelni (nepřeseknutí): $p_1 \parallel p_3, p_2 \parallel p_4$
 Kolomiti: $p_1 \perp p_2, p_1 \perp p_3, p_2 \perp p_4, p_3 \perp p_4$

$$2x - 4y + 3 = 0 \rightarrow p_1 \rightarrow y = \frac{1}{2}x + \frac{3}{4}$$

$$x + y - 1 = 0 \rightarrow p_2 \rightarrow y = -x - 1$$

$$x - 2y + 8 = 0 \rightarrow p_3 \rightarrow y = \frac{1}{2}x + 4$$

$$3x + 3y + 10 = 0 \rightarrow p_4 \rightarrow y = -x - \frac{10}{3}$$

$$x - y + 5 = 0 \rightarrow p_5 \rightarrow y = x + 5$$

$$2x + y = 0 \rightarrow p_6 \rightarrow y = -2x$$

$$k_1 = \frac{1}{2}$$

$$k_2 = -1$$

$$k_3 = \frac{1}{2}$$

$$k_4 = -1$$

$$k_5 = 1$$

$$k_6 = -2$$

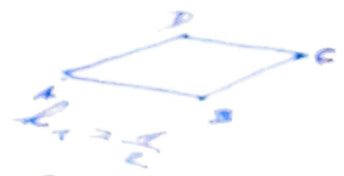
13. Obrazlivě káito je útvarodit
 p_1, p_2, p_3 a p_4 paralelogram

$$p_1 \dots x - 2y + 3 = 0 \rightarrow y = \frac{1}{2}x + \frac{3}{2}$$

$$p_2 \dots 3x + 2y - 1 = 0 \rightarrow y = -\frac{3}{2}x + \frac{1}{2}$$

$$p_3 \dots 2x - 4y - 4 = 0 \rightarrow y = \frac{1}{2}x - \frac{1}{2}$$

$$p_4 \dots 3x + 2y + 9 = 0 \rightarrow y = -\frac{3}{2}x - \frac{9}{2}$$



$p_1 \parallel p_3$
 $p_2 \parallel p_4$
 na nekom mysta u ramini tuore
 paralelogram.

Kutub imareti doqru
prava

15.3.2022

$p_1 \rightarrow k_1 x + b_1 = y$
 $p_2 \rightarrow y = k_2 x + b_2$

Δ (perpendikular)
 \Downarrow
 $\text{tg } \gamma = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$

$p_1 \perp p_2 \rightarrow k_1 = -\frac{1}{k_2}$
 $p_1 \parallel p_2 \rightarrow k_1 = k_2$

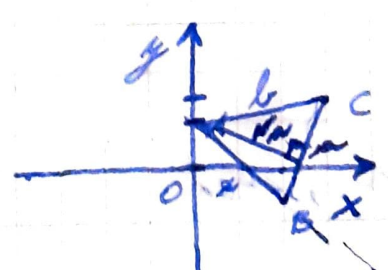
21. $p_1 \rightarrow 2x + 4y + 11 = 0$
 $4y = -2x - 11 / :4$
 $y = -\frac{1}{2}x - \frac{11}{4}$

$k_1 = -\frac{1}{2}$ $k_2 = -\frac{11}{4}$
 $k_2 = ?$ $k_2 = 0$

$p_1 \perp p_2$
 $k_1 = -\frac{1}{k_2}$
 $-\frac{1}{2} = -\frac{1}{k_2}$
 $k_2 = 2$

$p_2 \rightarrow y = 2x$

26. $A(0, 2)$
 $B(4, -1)$
 $C(6, 3)$



$p_1 \rightarrow y = \frac{11}{6}x + 2$
 $p_2 \rightarrow y = -\frac{3}{4}x + 2$
 $p_3 \rightarrow y = 2x + b$

$b = \frac{y_2 - y_1}{x_2 - x_1}$

$v_a = ?$
 $v_a = kx + d$

$a \perp v_a$
 $k_3 \perp k_{v_a}$
 $k_3 = -\frac{1}{k_{v_a}}$
 $2 = -\frac{1}{k_{v_a}}$
 $k_{v_a} = -\frac{1}{2}$

$k_{v_a} = k_1 = k_2$
 $k_{v_a} = 2$

$v_a = -\frac{1}{2}x + 2$

$\text{tg } \gamma = \left| \frac{\frac{5}{2} - 2}{1 + \frac{5}{2} \cdot 2} \right|$