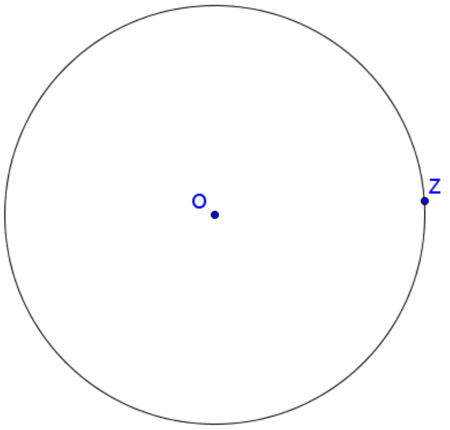
Task A-0: Angle at Centre and Angle at Circumference

Step 1

Create a circle.

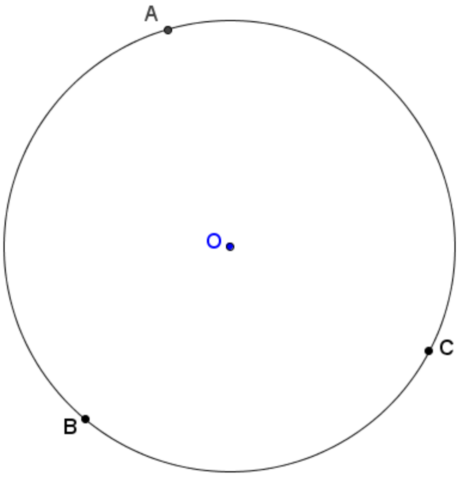
Rename the centre as O and the point on the circumference as Z.

Hide point Z.



Step 2

Create points A, B and C on the circumference.

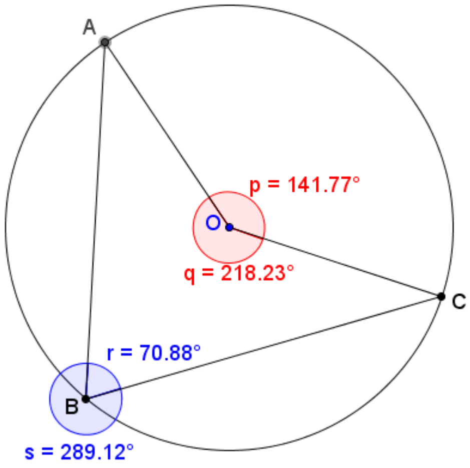


Step 3

Refer to the figure, draw the line segments and create ∠AOC, ∠ABC and their reflex angles.

Rename the angles as p, q, r and s as in the figure.

Set the colour of the angles at centre as red and the angles at circumference as blue.



Step 4

We want to show the pair p and r if r < s, otherwise show the pair q and s.

Right click on r, select object properties and choose the “Advanced” panel.
In the field “Condition to Show Object”, type “r<s”, do the same for the angle p

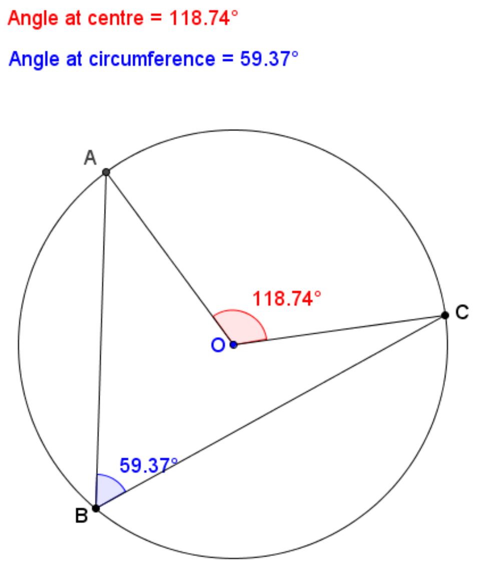
Right click on s, select object properties and choose the “Advanced” panel.
In the field “Condition to Show Object”, type “s<r”, do the same for the angle q

Step 5

Insert text ABC to show the sizes of ∠ABC and ∠AOC.

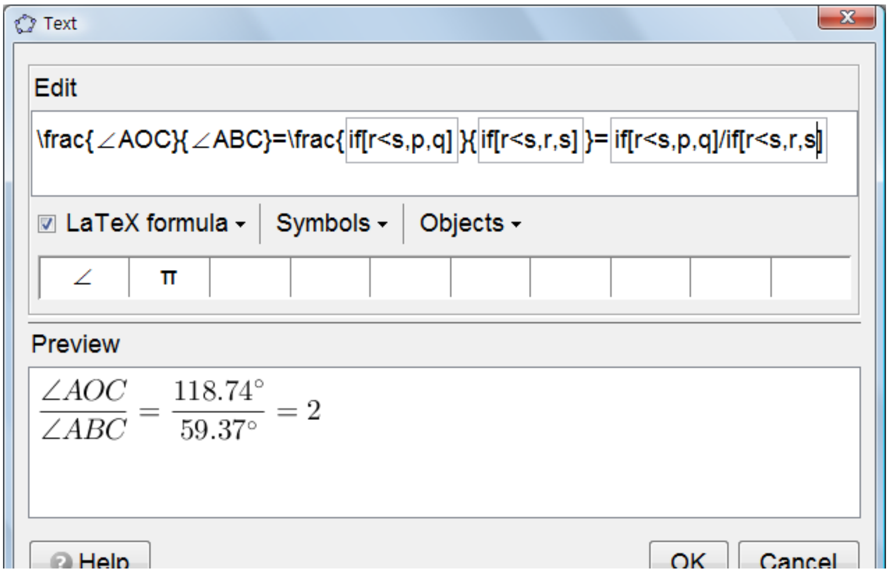
The size of ∠ABC is r if r < s , otherwise it is s. This can be written as if[r<s, r, s].

The size of ∠AOC is the corresponding “partner” of r and s. If r < s , it is p, otherwise it is q, i.e. if[r<s, p, q].



Step 6

Insert text ABC to show the ratio between ∠ABC and ∠AOC. To create a fraction, we use the LaTeX command **\frac{numerator}{denominator}**



`\frac{∠AOC}{∠ABC}=\frac{ if[r<s,p,q] }{ if[r<s,r,s] }= if[r<s,p,q]/if[r<s,r,s]`

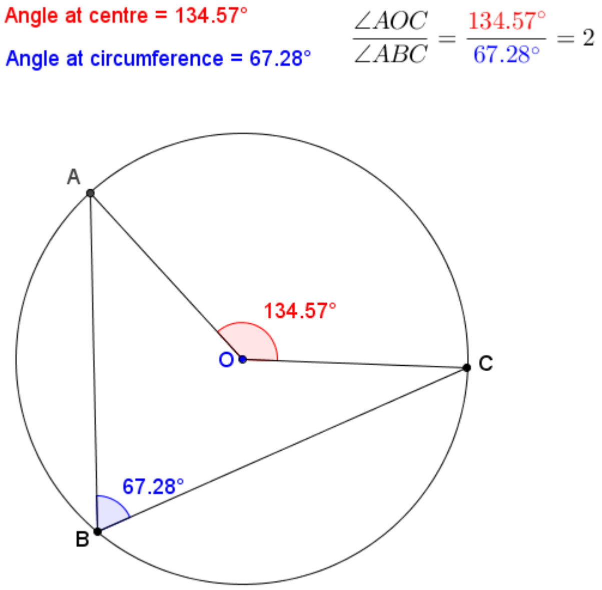
☑ LaTeX formula

$$\frac{\angle AOC}{\angle ABC} = \frac{118.74°}{59.37°} = 2$$

Step 7

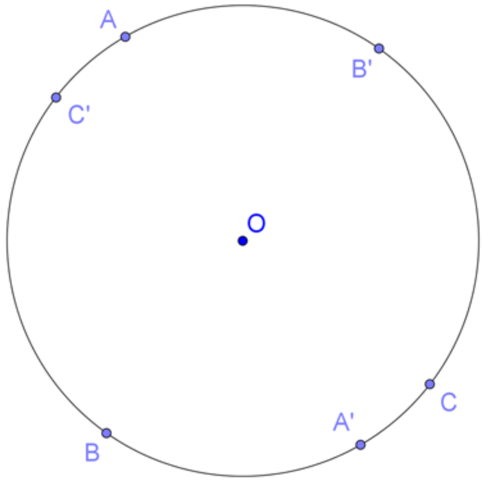
To change the colour of the numerator and denominator, we enclose the numerator by \red{ } and the denominator by \blue{ }.

`\frac{\red{ If[r < s, p, q] }}{\blue{ If[r < s, r, s] }}`



~ End of Task A-0 ~

Task A: Angle at Centre and Angle at Circumference



Step 1
Create a circle.

Rename the centre as O and the point on the circumference as Z.
Hide point Z.

Step 2
Create points A, B and C on the circumference.

Create their opposite points by using “Reflect Object about Point” .

Step 3
To create a point P that act as a “phantom” of A and snap to the point F, we use the command “DynamicCoordinates”.

The format of the command is **DynamicCoordinates[<Point A>, <Number X>, <Number Y>]**
When the new point moves, A moves with it and the coordinates of the new point are (X, Y) and usually X and Y depends on the coordinates of A.

At the **input** area at the bottom of the screen, type

x-coordinate of P

y-coordinate of P

P depends on A

```
P = DynamicCoordinates[
                         A,
                            If[Distance[A, C'] < 0.2, x(C'), x(A)],
                                              If[Distance[A, C'] < 0.2, y(C'), y(A)]
                      ]
```

Step 4
To create a point P that act as a phantom of A and snap to **points B' and C'**, we replace the x(A) and y(A) in the original definition of P by
If[Distance[A, B'] < 0.2, x(B'), x(A)] and If[Distance[A, B'] < 0.2, y(B'), y(A)] respectively.

Right click on P and edit its definition as

If[Distance[A, B'] < 0.2, y(B'), y(A)]

If[Distance[A, B'] < 0.2, x(B'), x(A)]

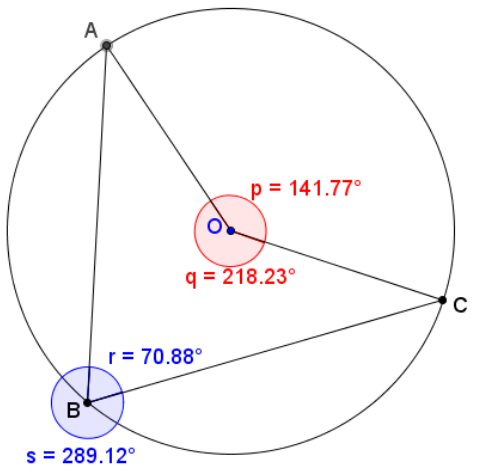
```
DynamicCoordinates[
                      A,
                        If[Distance[A, C'] < 0.2, x(C'), x(A)],
                                              If[Distance[A, C'] < 0.2, y(C'), y(A)]
                      ]
```

** In short, type

```
P = DynamicCoordinates[ A,
                         If[Distance[A, C'] < 0.2, x(C'), If[Distance[A, B'] < 0.2, x(B'), x(A)]],
                         If[Distance[A, C'] < 0.2, y(C'), If[Distance[A, B'] < 0.2, y(B'), y(A)]]
                          ]
```

Step 5
Create a point Q that act as a phantom of C and snap to points B'.

At the **input** area, type
Q = DynamicCoordinates[C, If[Distance[C, B'] < 0.2, x(B'), x(C)], If[Distance[C, B'] < 0.2, y(B'), y(C)]]



<u>Step 6</u>
Hide points A, A', B', C and C'.
Rename P as A and Q as C.
Change the color of A, B, and C to black.

<u>Step 7</u>
Refer to the figure, draw the line segments and create ∠AOC, ∠ABC and their reflex angles.

Rename the angles as p, q, r and s as in the figure.
Set the colour of the angles at centre as red and the angles at circumference as blue.

<u>Step 8</u>
We want to show the pair p and r if r < s, otherwise show the pair q and s.

Right click on r, select object properties and choose the "Advanced" panel.
In the field "Condition to Show Object", type "r<s", do the same for the angle p

Right click on s, select object properties and choose the "Advanced" panel.
In the field "Condition to Show Object", type "s<r", do the same for the angle q

<u>Step 9</u>
Insert text ABC to show the sizes of ∠ABC and ∠AOC.

The size of ∠ABC is r if r < s , otherwise it is s. This can be written as if[r<s, r, s].

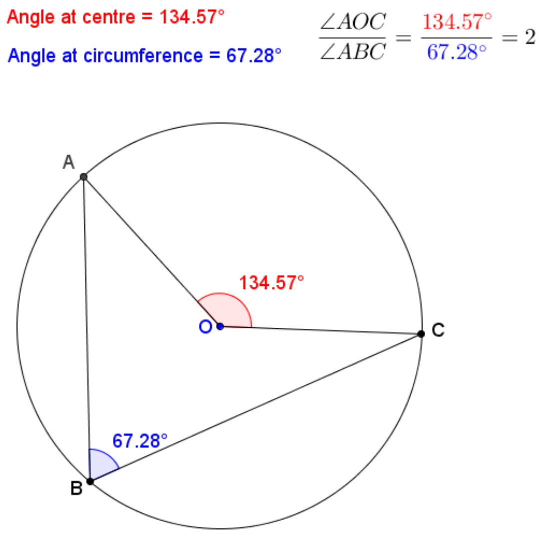
The size of ∠AOC is the corresponding "partner" of r and s. If r < s , it is p, otherwise it is q, i.e. if[r<s, p, q].

<u>Step 10</u>
Insert text ABC to show the ratio between ∠ABC and ∠AOC. To create a fraction, we use the LaTeX command **\frac{numerator}{denominatior}**

\frac{∠AOC}{∠ABC}=\frac{if[r<s, p, q]}{if[r<s, r, s]}= if[r<s, p, q]/if[r<s, r, s]

<u>Step 11</u>
To change the colour of the numerator and denominator, we enclose the numerator by \red{ } and the denominator by \blue{ }.

\frac{red{if[r<s, p, q]}}{\blue{if[r<s, r, s]}}



~ End of Task A ~

Task A-1: Angle at Centre and Angle NOT at Circumference

Step 1

Repeat Step 1 of Task A.

Step 2

Create a free point D on the plane. Then, create a line passing through the centre O and D. Mark one of the points of intersection of the line and the circle as E.

Hide the points D, E and the line.

Step 3

Create a point B with the dynamic coordinates depending on point D, while snapping to point E. **(Exercise)**

Step 4

Follow the rest of steps in Task A to complete the dynamic worksheet to show one constraint of the theorem by the counter-examples.

~ End of Task A-1 ~

Task A-2: Angle at Centre and Angle at Circumference of an ELLIPSE

Step 1

Create an ellipse. Label the mid-point of the foci as O.

Repeat all the other steps in Task A.

~ End of Task A-2 ~

Task A-3: Angle at Centre and Angle at "Circumference" of a SQUARE

Step 1

Create a square by using the "Regular Polygon" tool. Label the centre of the square as O.

Step 2

Create Sq as a list of item holding the four segments of the square using the bracket "{}". Define points A, B, C as "point[Sq]".

Repeat all the other steps in Task A.

~ End of Task A-3 ~

Think about it:

Student QQ claims that if $\beta = 2\alpha$, D must be the centre of the circle. Do you agree? Explain your answer.