Lesson 1: Number puzzles

Goals

- Calculate a missing value for a number puzzle that can be represented by a linear equation in one variable, and explain (orally and in writing) the solution method.
- Create a number puzzle that can be represented by a linear equation in one variable.

Learning Targets

- I can solve puzzle problems using diagrams, equations, or other representations.

Lesson Narrative

In this introductory lesson, students solve and write number puzzles of the sort where you are given a series of operations on a number, and the final result, and have to find out the original number. These puzzles are good preparation for solving linear equations, where students have to perform operations on each side of the equation to isolate the variable. Students use representations of their choosing, such as line diagrams, bar models, and equations.

Building On

- Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Building Towards

- Solve linear equations in one variable.

Instructional Routines

- Collect and Display
- Compare and Connect
- Notice and Wonder

Student Learning Goals

Let's solve some puzzles!

1.1 Notice and Wonder: A Number Line

Warm Up: 5 minutes

The purpose of this warm-up is to introduce students to a number line diagram they will be using to represent addition and subtraction of integers in future lessons. To introduce this idea, students write a story and equation that a given number line diagram could represent.

Instructional Routines

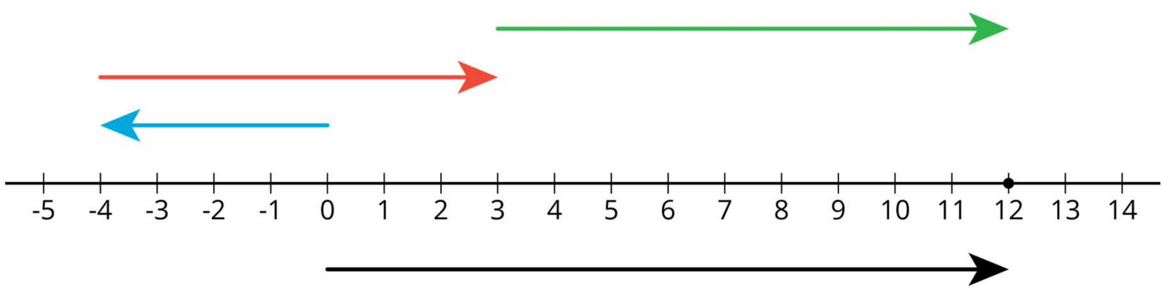
- Notice and Wonder

Launch

Tell students they are going to see a number line diagram and that their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students might notice:

- There are 3 arrows above the number line and 1 arrow below it.
- The blue arrow could represent -4.
- The green arrow could represent +9 or $\times$ 4
- The black arrow goes from 0 to 12.

Things students might wonder:

- What do the arrows represent?
- Why do the arrows above and below both start at 0 and end at 12?

Activity Synthesis

After students' notice and wonder ideas are displayed, tell them that the diagram is about money, and invite students to share a possible story and equation that the diagram represents. For example, someone owes £4. Then they earn £7 doing chores and another £9 helping out the neighbour with their yard. The equation would be $-4 + 7 + 9 = 12$.

1.2 Telling Temperatures

15 minutes

The purpose of this activity is for students to solve number puzzles using any representation they choose. Students then make sense of other representations for the same problems, starting with those of a partner. The whole-class discussion should focus on the strengths and weaknesses of different representations. For example, bar models only work for problems with all positive values, so you could use one for the distance puzzle, but a bar model would not work for the temperature puzzle.

Identify students using different strategies, such as number line diagrams, bar models, written out reasoning, and equations, to share during the whole-class discussion.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2. Give 5–6 minutes of quiet work time followed by partner discussion. During their discussion, partners explain their representations of the problems to one another, including any representations they started out with that didn't work. If partners used the same representation, ask them to find another representation they could use to solve the puzzle. Follow with a whole-class discussion.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, use colour to highlight connections between changing temperatures and positive and negative numbers on a number line.
Supports accessibility for: Visual-spatial processing Representing, Speaking, Listening: Collect and Display. During partner discussion, circulate and listen to students explain their representations of the problems to one another. Listen for the variety of ways students describe their number line diagrams, bar models, and equations. Record examples of student language and diagrams on a visual display. Continue to update the display, introducing mathematical vocabulary next to student language as students move through the activity. Remind students that they can borrow words, phrases or representations from the display to support their work. This will help students develop mathematical language about each representation.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

Students who write the equation $2x - 10 + 40 = 16$ for the temperature puzzle may incorrectly follow that up with $2x - 50 = 16$. By comparing their process with a different representation, help them notice that a decrease of 10 and an increase of 40 is, overall, an increase of 30.

Student Task Statement

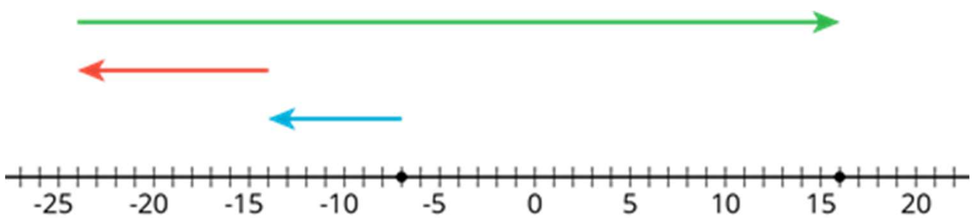
Solve each puzzle. Show your thinking. Organise it so it can be followed by others.

1. The temperature was very cold. Then the temperature doubled.
   Then the temperature dropped by 10 degrees. Then the temperature increased by 40 degrees. The temperature is now 16 degrees. What was the starting temperature?

2. Lin ran twice as far as Diego. Diego ran 300 m farther than Jada. Jada ran $\frac{1}{3}$ the distance that Noah ran. Noah ran 1 200 m. How far did Lin run?

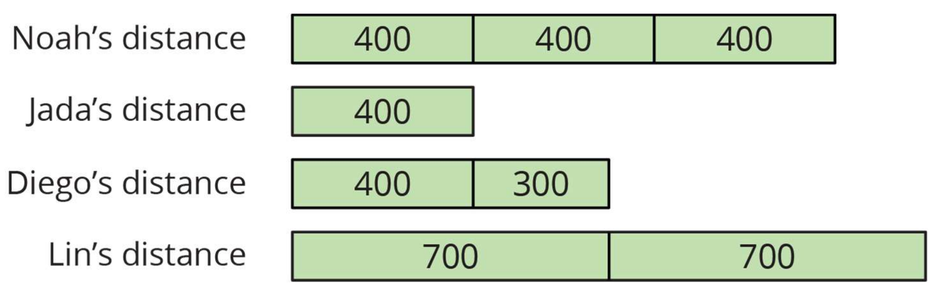
Student Response

1. -7 degrees Celsius. Sample responses:



Or $2t - 10 + 40 = 16$. Or Starting temperature $= (16 - 40 + 10) \times \frac{1}{2}$.

1. 1 400 metres. Sample response:



Or $\left(\frac{1}{2}x - 300\right) \times 3 = 1\,200$. Or Lin's distance = $2\left(\frac{1}{3}(1\,200) + 300\right)$

Activity Synthesis

The goal of this discussion is for students to make connections between different representations of problems and, more centrally to this unit as a whole, to see equations as an efficient way to represent problems.

Select previously identified students to share their representations. Record and display a visual of the representations for each problem. If not shared by students, make sure at least one equation representation for each problem is included. Once multiple representations from the class are displayed, ask 2–3 students to explain which one(s) they prefer and why. If not brought up in discussion, note that some representations, such as bar models, do not work all the time. For example, a bar model may be unsuitable for the temperature puzzle due to the negative values in the problem. Other representations, such as equations, can work for almost any type of problem.

1.3 Making a Puzzle

15 minutes

In this task, students create their own number puzzle to trade with a partner to solve. The purpose of this task is for students to practice writing and solving multi-step number puzzles and compare their representations with the representations of others to decide which are more efficient. While these problems are phrased using the words "number puzzle," it is important to note the mathematical work students are doing here thinking about, creating, and solving situations that are, essentially, linear equations in one variable, even if not all students are using equations to represent them.

While students are sharing representations, identify partners who solved at least one of their puzzles using different representations to share during the whole-class discussion. If possible, select groups who used an equation.

Instructional Routines

- Compare and Connect

Launch

Keep students in the same groups. Give 5 minutes for students to write their own puzzle and make a representation of their solution before trading their puzzle with a partner to solve. Make sure students write their puzzle and solution in such a way that when they trade, their partner cannot see the solution.

If both partners created the same (or very similar) representation for their solutions, ask them to work together to create a different representation. If they created different representations, ask partners to discuss which one they prefer and to be ready to explain why during the whole-class discussion.

Representing, Conversing: Compare and Connect. Before students create their own number puzzle, use this routine to give students the opportunity to identify and explain the correspondences between different strategies for representing the equation $3d + 5 - 2 = 15$. Invite students to demonstrate their strategy using a visual or numerical representation. Display one example of each representation to discuss. In pairs or groups, ask students to compare their strategies. Ask students to discuss how the strategies are the same or different, and then share with the whole class. This will increase students' meta-awareness and use of language for comparisons of mathematical representations.
Design Principle(s): Optimise output (for comparison); Maximise meta-awareness

Student Task Statement

Write another number puzzle with at least three steps. On a different piece of paper, write a solution to your puzzle.

Trade puzzles with your partner and solve theirs. Make sure to show your thinking.

With your partner, compare your solutions to each puzzle. Did they solve them the same way you did? Be prepared to share with the class which solution strategy you like best.

Student Response

Answers vary. Sample response: The temperature was very cold. Then the temperature dropped by 5 degrees. Then the temperature doubled.
The temperature is now 40 degrees. What was the starting temperature?

Equation: $2(x - 5) = 40$

Answer: 25 degrees

Are You Ready for More?

Here is a number puzzle that uses maths. Some might call it a magic trick!

1. Think of a number.
2. Double the number.
3. Add 9.
4. Subtract 3.
5. Divide by 2.
6. Subtract the number you started with.
7. The answer should be 3.

Why does this always work? Can you think of a different number puzzle that uses maths (like this one) that will always result in 5?

Student Response

Explanations vary. Sample response: Using x to represent any number, the steps in the puzzle form the expression $\left(\frac{2x+9-3}{2}\right) - x$, which equals 3.

Activity Synthesis

Select partners previously identified to share a puzzle with the class and the two representations they created. Ask which representation they prefer and why. If students do not bring it up in their explanations, ask which of their representations was the most efficient one for solving the puzzle.

During this discussion, students may ask you to state which representation is best and, if so, it is important to note that there is no one correct answer for the "best representation." The "best representation" is the one that makes sense to the student and helps them solve the problem. However, as problems grow more and more complex, students are likely to find that certain representations are more useful for solving problems than others.

Lesson Synthesis

Ask students to think about what their number line diagrams, bar models, and equations represented in each of the activities. Guide them in seeing that stories with an unknown quantity usually involve actions, like the temperature rising, earning money by doing chores, or relationships, like Diego's distance being half of Lin's distance and 300 m more than Jada's. Ask them to think about the puzzle they wrote in the last activity and whether they described actions or relationships. Invite their opinions about which representations best represent actions and which best represent relationships.

Tell students to think about the expression $x + 5$. Ask: "What could this mean in terms of two numbers being related to each other? What could this represent as an action?" (One number is 5 more than the other. A sample action: the temperature increases by 5 degrees.)

If time allows, ask students to make up another puzzle that describes actions if they previously chose relationships, and vice versa, and to represent their puzzles with a diagram and an equation. Display diagrams and equations for all to see.

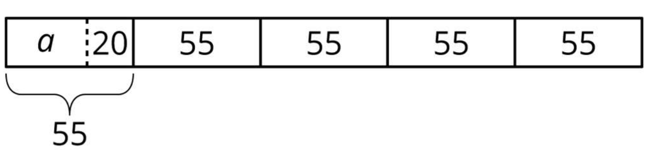
1.4 Seeing the Puzzle

Cool Down: 5 minutes

Student Task Statement

Andre and Elena are reading the same book over the summer. Andre says he has read $\frac{1}{5}$ of the book. Elena says she has read 20 more pages than Andre. If Elena is on page 55, how many pages are in the book?

Lin has drawn a diagram to solve this question. Find her error.



Student Response

Answers vary. Sample response: The diagram shows Elena read $\frac{1}{5}$ of the book. In order to represent that Andre read $\frac{1}{5}$ of the book, subtract 20 from Elena's 55 pages to get that Andre has read 35 pages. If 35 pages is $\frac{1}{5}$ of the book, then the book must be 5×35, or 175 pages long.

Student Lesson Summary

Here is an example of a puzzle problem:

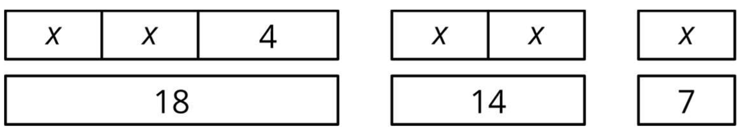
Twice a number plus 4 is 18. What is the number?

There are many different ways to represent and solve puzzle problems.

- We can reason through it.

  Twice a number plus 4 is 18.
  Then twice the number is $18 - 4 = 14$.
  That means the number is 7.

- We can draw a diagram.



- We can write and solve an equation.
  $2x + 4 = 18$
  $2x = 14$
  $x = 7$

Reasoning and diagrams help us see what is going on and why the answer is what it is. But as number puzzles and story problems get more complex, those methods get harder, and equations get more and more helpful. We will use different kinds of diagrams to help us understand problems and strategies in future lessons, but we will also see the power of writing and solving equations to answer increasingly more complex mathematical problems.

Lesson 1 Practice Problems

1. **Problem 1 Statement**

   Tyler reads $\frac{2}{15}$ of a book on Monday, $\frac{1}{3}$ of it on Tuesday, $\frac{2}{9}$ of it on Wednesday, and $\frac{3}{4}$ of the remainder on Thursday. If he still has 14 pages left to read on Friday, how many pages are there in the book?

   Solution

   180 pages

2. **Problem 2 Statement**

   Clare asks Andre to play the following number puzzle:

   - Pick a number
   - Add 2
   - Multiply by 3
   - Subtract 7
   - Add your original number

   Andre's final result is 27.
   Which number did he start with?

   Solution

   Andre's starting number was 7.

   $3(x + 2) - 7 + x$ simplifies to $4x - 1$.
   $4x - 1 = 27$ has solution $x = 7$.

3. **Problem 3 Statement**

   In a basketball game, Elena scores twice as many points as Tyler. Tyler scores four points fewer than Noah, and Noah scores three times as many points as Mai. If Mai scores 5 points, how many points did Elena score? Explain your reasoning.

   Solution

   22 points. Noah scores 15 points, which means Tyler scores 11 points, and Elena scores twice as many points as Tyler.

4. **Problem 4 Statement**

Select **all** of the given points in the coordinate plane that lie on the graph of the linear equation $4x - y = 3$.

a. (-1,-7)

b. (0,3)

c. $\left(\frac{3}{4}, 0\right)$

d. (1,1)

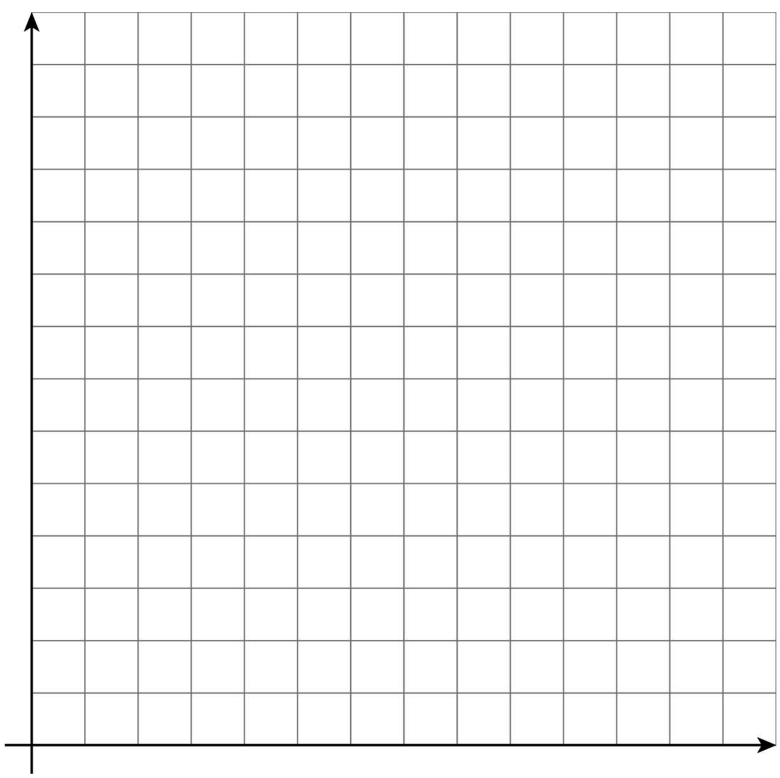
e. (2,5)

f. (4,-1)

Solution ["A", "C", "D", "E"]

5. **Problem 5 Statement**

A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 feet long, followed by the nested carts (so 0 nested carts means there's just the starting cart). The store measured a row of 13 nested carts to be 23.5 feet long, and a row of 18 nested carts to be 31 feet long.



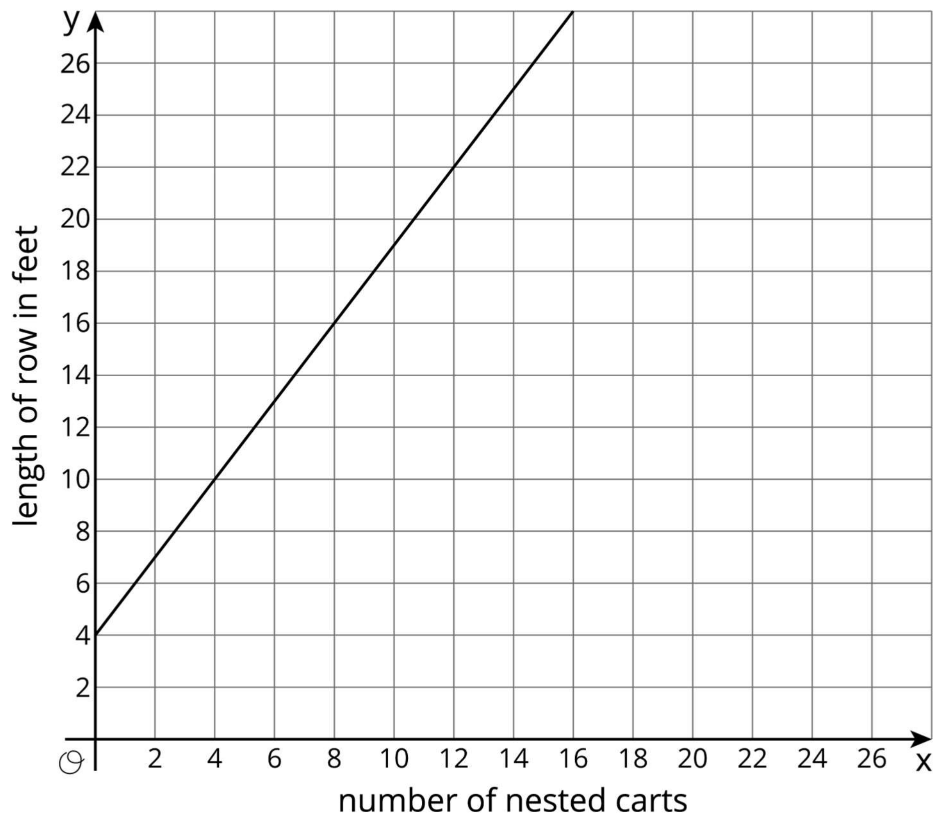
a. Create a graph of the situation.

b. How much does each nested cart add to the length of the row? Explain your reasoning.

c. If the store design allows for 43 feet for each row, how many total carts fit in a row?

Solution

a.



b. 1.5 feet. Explanations vary. Sample response: The gradient, which can be found with the calculation $\frac{31-23.5}{18-13}$ tells the rate of change, or amount that each nested cart adds.

c. 26 nested carts, or 27 carts total. Explanations vary. Sample response: We can subtract 4 feet from 43 feet for the starting cart and then divide by 1.5 to find the number of nested carts that will fit. We can use a table and repeatedly add 1.5. There are 12 more feet from 31 to 43, so 12 ÷ 1.5, or 8 more carts, can be added to 18.

6. **Problem 6 Statement**

Triangle A is an isosceles triangle with two angles of x degrees and one angle of y degrees.

a. Find three combinations of x and y that make this sentence true.

b. Write an equation relating x and y.

c. If you were to sketch the graph of this linear equation, what would its gradient be? How can you interpret the gradient in the context of the triangle?

Solution

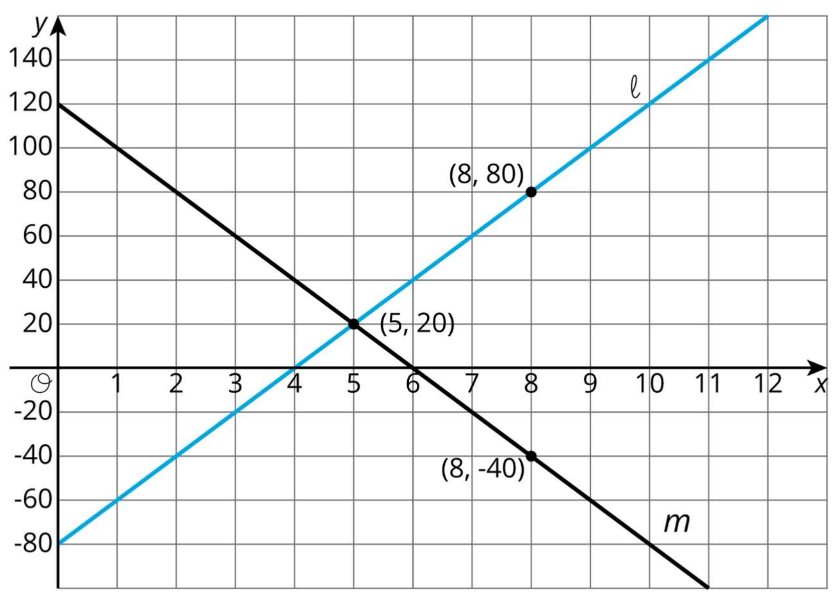
a. Answers vary; the key constraint is that the three angles must sum to 180 ($x + x + y = 180$). For example, $x = y = 60$, or $x = 30$ and $y = 120$ or $x = 45$ and $y = 90$.

b. $2x + y = 180$

c. -2. In the context of the triangle, for every 1 degree increase of x, y decreases by 2 degrees.

7. **Problem 7 Statement**

Consider the following graphs of linear equations. Decide which line has a positive gradient, and which has a negative gradient. Then calculate each line's exact gradient.



Solution

The line ℓ moves up on the y-axis as it moves to the right, so it has a positive gradient. m has a negative gradient since it moves downwards. The gradient of ℓ is $\frac{80-20}{8-5} = \frac{60}{3} = 20$. The gradient of m is $\frac{-40-20}{8-5} = \frac{-60}{3} = -20$.