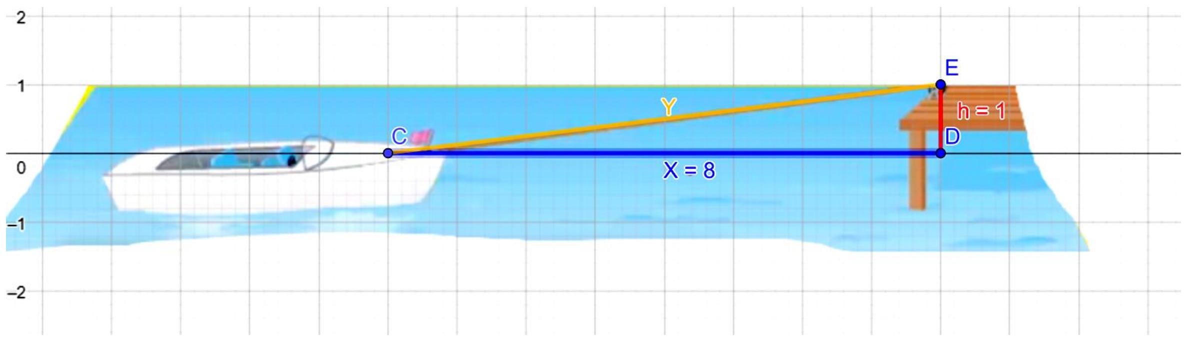
Example 3:

A boat is pulled in to a dock by a rope with one end attached to the front of the boat and the other end passing through a ring attached to the dock at a point 1 m higher than the front of the boat. The rope is being pulled through the ring at the rate of 1 m/sec. How fast is the boat approaching the dock when 8 m of rope are out?

Solution:

Step 1: Draw a picture.



The boat is approaching to the dock. This distance is unknown and let x denote that distance. It is known that the pulley is 1 meter higher than the front of the boat and let h denote this height. It is the constant value.

y denotes the length of the rope that the boat is pulled. From the figure above it can be noted that the angle $\sphericalangle CDE$ is the right angle.

Since the rope is being pulled at the rate of 1 m/sec, we know that $\frac{dy}{dt} = -1\ m/sec$. It is the negative value because the length of the rope is shorter and shorter by pulling the boat (it is shorter for 1 meter per second).

If the boat is apart 8 m from the dock, it is needed to find how fast the boat is approaching to the dock, i.e. the rate of change in the distance d between the boat and the dock per second. We need to find $\frac{dx}{dt} = ?\ m/s$ when x is 8m.

Note that both x and y are functions of time, and the height h is the constant.

Step 2: From the right triangle CDE we can use the **Pythagorean Theorem** to write an equation relating x and y (h =1m):

$$y^2 = x^2 + 1^2$$

Reminder
If h is a constant then $\frac{dh}{dt} = 0.$

Step 3: Differentiating this equation with respect to time and using the fact that the derivative of a constant is zero, we arrive at the equation:

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We can use the Pythagorean theorem to determine the lenght **y** when x=8 m, and the height is 1 m . Solving the equation:

$$y^2 = x^2 + h^2$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt} + 0$$

$$\frac{2y}{2x}\frac{dy}{dt} = \frac{2x}{2x}\frac{dx}{dt}$$

$$\frac{2 \cdot 8.06m}{2 \cdot 8\,m} \cdot (-1\,m/s) = \frac{dx}{dt}$$

$$\frac{dx}{dt} = -\frac{8.06}{8}m/s = -1.011m/s$$